ICSV14
Cairns • Australia
9-12 July, 2007

14th International Congress on Sound & Vibration

A NUMERICAL STUDY ON PRACTICAL MEASUREMENT METHOD OF VIBRATION INTENSITY IN A PLATE WITH STANDING WAVE

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Abstract

It is important to gain information about the source of excitation and the resulting transmission paths when dealing with the response of actual structures to impacts. The Vibration Intensity (VI) method is an effective way of acquiring this information. It is also important to be able to establish VI even in the presence of background noise, such as that due to standing waves. A simplified VI formula was applied to practical VI measurement methods on actual structures. However the adoption of a simplified VI formula results in errors under certain conditions. Thus a simplified VI method was compared with the standard approach to investigate the vibration field of a plate subjected to an impulse and, simultaneously, a predominant standing wave. The VI direction angle error, with the simplification, was less than 30 degrees anywhere and was less than 10 degrees at loop points. Next, the results with and without predominant standing wave were compared for the various amplitude ratios, and the limit of detection of the impulse response were discussed. The VI direction angle errors decreased with increasing the amplitude ratios.

1. INTRODUCTION

In order to reduce the noise and vibration levels of mechanical structures, it is important to identify the vibration sources correctly. Vibration intensity (VI) method [1] is one of the effective methods to identify the vibration energy flow. The measurement signals on vibration energy transmission through mechanical structures are usually exposed to many kinds of noises including standing waves. It is also important to identify the vibration transmission paths from a vibration source to the points generating controversial vibration. One of the practical methods to measure the VI vector is 3-channel method [3]. The 3-channel method is based on the same simplified VI formula as 2x2-point finite difference method. However, such methods have some simplification errors [4].

The purpose of this research is to investigate the detection limit of the main impulse response based on a simplified VI formula in conditions with predominant standing wave using finite element method (FEM). First, the VI error with the simplification was discussed from viewpoint of VI direction angle in the vibration plate with the main impulse and the predominant standing wave. Next, the VI results between with and without predominant standing wave were compared. Then, the detection limit was discussed in terms of the amplitude ratio of the signal impulse to noise.

2. ANALYSIS METHOD

2.1 VI Formulation

The vibration energy transported by the flexural wave is expressed as a sum of each works done by the shear force, the bending moment and the twisting moment for unit width of a plate and unit time.

The x-direction component of VI based on the elastic theory for a thin plate in frequency domain is expressed as

$$VI_x(f) = B\left\{ F\left[\frac{\partial}{\partial x}\left(\nabla^2 w\right)\right]\cdot F^*\left[\frac{\partial w}{\partial t}\right] - F\left[\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right]\cdot F^*\left[\frac{\partial^2 w}{\partial x \partial t}\right] - (1-\nu)F\left[\frac{\partial^2 w}{\partial x \partial y}\right]\cdot F^*\left[\frac{\partial^2 w}{\partial y \partial t}\right]\right\} \tag{1}$$

where B is bending stiffness, ν is the Poisson's ratio, w is the out-plane displacement, f is frequency, $F[\ ]$ denotes Fourier transformation and * denotes the complex conjugate. Equation (1) consists of the real part and the imaginary part in frequency domain. The real part and the imaginary part in Eq. (1) are called the active component and the reactive component of VI, respectively. In order to obtain a VI vector based on Eq. (1), vibration data at 12 finite-difference points are required. This method is called 12-point method.

If one-dimensional free and far field is assumed, Eq. (1) is reduced to a simplified formula expressed as

$$VI_x(f) = -\frac{\sqrt{Bm}}{\pi f}\cdot\left\{F\left[\frac{\partial^2 w}{\partial t^2}\right]\cdot F^*\left[\frac{\partial^2 w}{\partial x \partial t}\right]\right\} \tag{2}$$

where m is mass density. In order to obtain a VI vector based on Eq. (2), only vibration data at 2x2 finite-difference points are required. This method is called 2x2-point method.

2.2 Analysis Model and Excitation Method

The FEM analysis model is shown in Fig. 1. The surroundings of the rectangular thin plate were simply supported. The x-y coordinate is set up as shown in Fig. 1. The size of the plate was 1000mm in x-direction and 500mm in y-direction. The thickness of the plate was 4.5mm. The size of one finite element was 10mm x 10mm. The material constants of the plate are shown in Table 1. The FEM analysis was performed using MSC. visual NATRAN for Windows. The transitional response analysis was employed.

A single impulsive force was applied in out-plane direction to cause a detection response of the plate. The plate was also excited by sine wave force to cause a background vibration with the frequency of 493 Hz to form the (3, 3) vibration mode of the plate. Excitation point was at the origin point as shown in Fig. 2. The amplitude of sine wave was varied to obtain various ratios of the amplitude of the single impulsive response to that of the standing wave. The signal to noise ratio, SNR, used in this study was expressed as

$$\mathrm{SNR} = 20\log\left(\frac{a}{b}\right) \qquad [\mathrm{dB}] \tag{3}$$

where a is the amplitude of acceleration of the impulsive response and b is that of a standing wave. In this research, SNR was used to investigate the detection limit of the VI under the existence of the predominant standing wave in the plate. All analyses were performed at 500 Hz in octave band.

Table 1. Material constants of the analysis model.

Parameter	Value
Young's modulus, E (GPa)	206
Density, ρ (kg/m^3)	7860
Poisson's ratio, ν	0.3

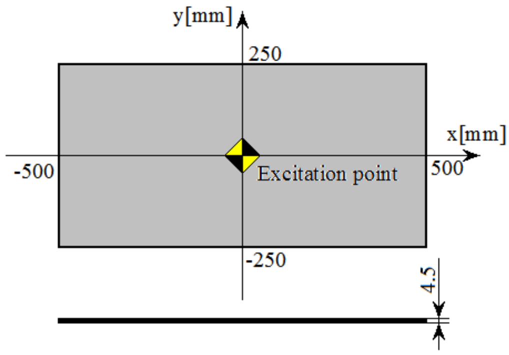


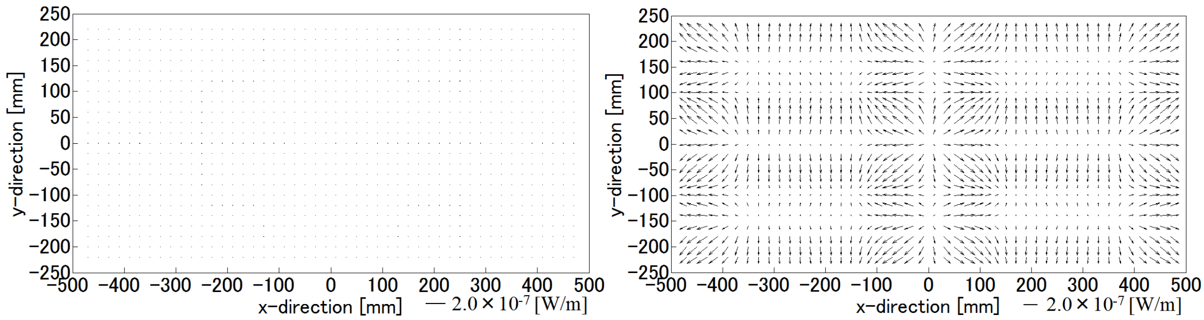
Figure 1. FEM analysis model.

3. RESULTS AND DISCUSSION

3.1 VI distribution with Standing Wave

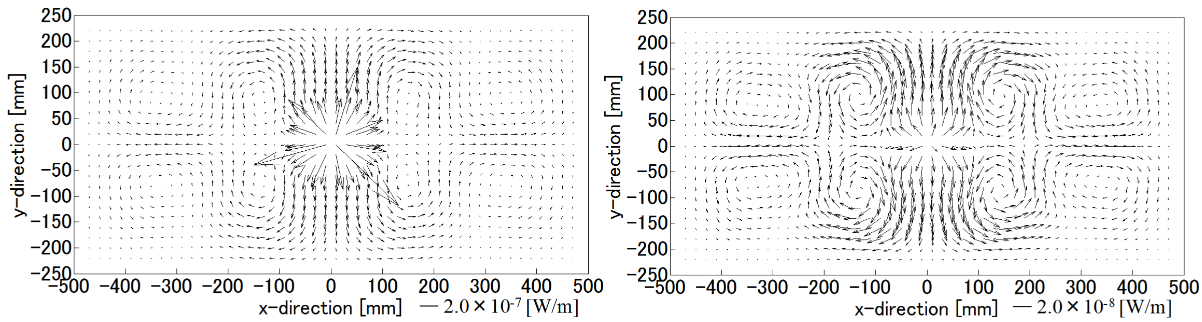
Figure 2 shows the active component of VI and the reactive component of VI when the plate was excited only by sine wave with natural frequency of (3,3) mode. The distribution of the reactive component of VI corresponded to that of (3,3) mode and the magnitude of the active component of VI was extremely small in comparison with the magnitude of the reactive component of VI. As shown in Fig. 2, the reactive component of VI was predominant when the standing wave was formed, and the vectors of reactive component of VI turn from loop points to the node points of the vibration mode as is known in VI theory.

Figure 3 shows typical active components of VI with impulse and the predominant standing wave at SNR of 14.8 dB. Large vibration power flew out from the excitation point. Vortex flow patterns were observed. There are some differences in VI distributions between VI of 12-point method and that of 2x2-point method. The 2x2-point method has error due to the formulation simplification. The simplification error will be discussed in the next section.



(a) Active component (b) Reactive component

Figure 2. VI distribution with predominant standing wave.



(a) 12-point method (b) 2x2-point method

Figure 3. VI distribution with impulse and predominant standing wave (SNR=14.8dB).

3.2 VI Simplification Error

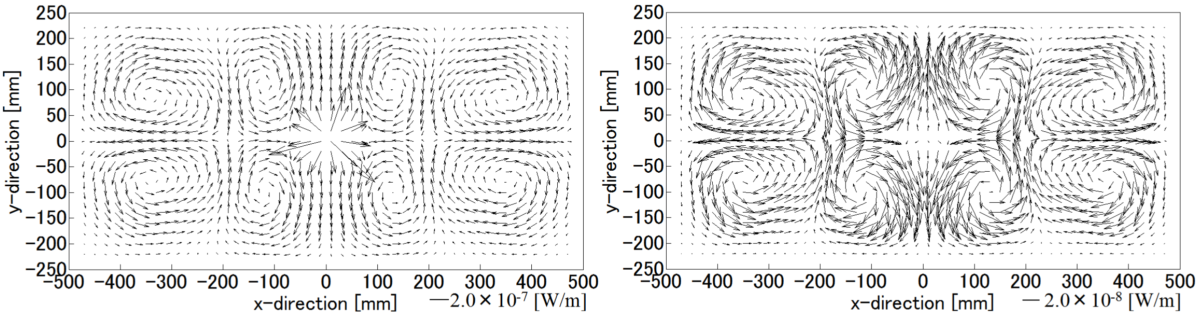
Figure 4 and 5 show VI distribution with impulse and predominant standing wave at SNR of 9.02 dB and that of 21.2 dB, respectively. As SNR is low, it is not easy to identify the vibration energy source of the impulse response with the 2x2-point method.

The adoption of the VI simplified formula yields an error, and it is defined as VI simplification error. VI error was estimated by comparing VI calculated with the 2x2-point method $VI_{2\times 2}$ with that calculated with the 12-point method VI_{12}. We focus on VI direction angle error *AE*, which is expressed as

$$\text{(VI direction angle error)} \equiv AE = \left|\theta_{2\times 2} - \theta_{12}\right| \tag{4}$$

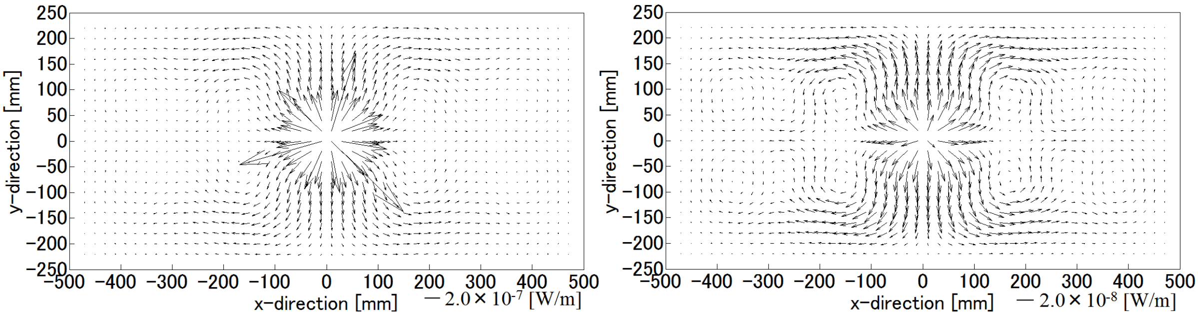
where $\theta_{2\times 2}$ is the direction angle of VI calculated with 2x2-point method and θ_{12} is the direction angle of VI calculated with 12-point method. The direction angle θ of VI is the angle of VI vector measured from x-axis and in the clockwise direction.

Figure 6 shows VI direction angle error *AE* against SNR at some loop points and node point of vibration mode. VI direction angle error was not constant with respect to SNR and the value of direction angle error was different in position. The VI direction angle error was less than 30 degrees anywhere and was less than 10 degrees at loop points. VI direction angle can be measured with satisfactory accuracy at loop point in comparison with at node point.



(a) 12-point method (b) 2x2-point method

Figure 4. VI distribution with impulse and predominant standing wave (SNR=9.02dB).



(a) 12-point method (b) 2x2-point method

Figure 5. VI distribution with impulse and predominant standing wave (SNR=21.2dB).

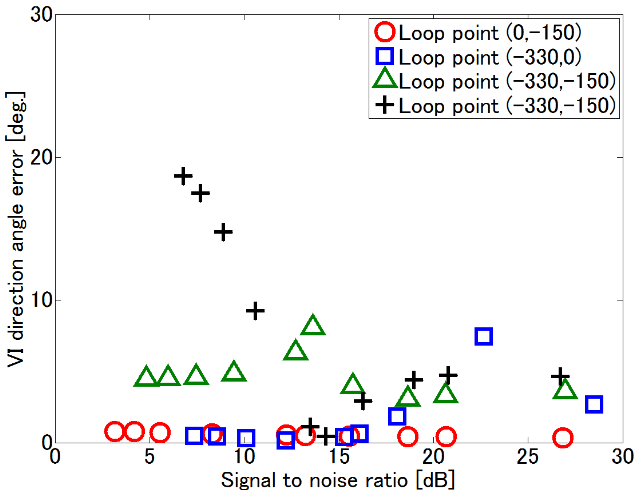


Figure 6. VI direction angle error at loop points and node point of the vibration mode.

3.3 Detection Limit of VI Measurement

Figure 7 shows VI distributions with 12-point method without and with the predominant mode. Since the 12-point method expresses the full set of terms in the theoretical equation of VI, it can be said that the standing wave does not affect the active component of VI calculated with 12-point method. However, VI distribution with the standing wave was different from that without the standing wave. Then, introducing the simplified method, 2x2-point method, the active component of VI is affected not only by the formula simplification but also by the reactive component of VI, which is originated by the predominant mode.

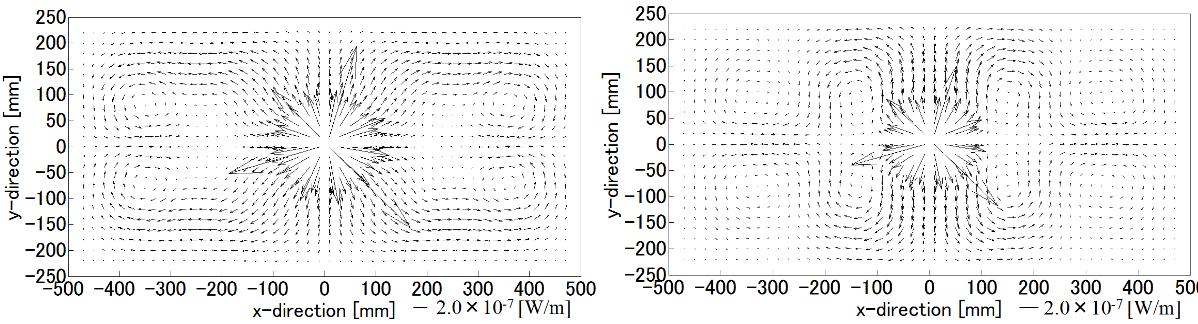
VI direction angle error *AE* was redefined to include the effects of the standing wave to the VI detection limits, as

$$\text{(VI direction angle error)} \equiv AE = \left|\theta - \theta_0\right| \tag{5}$$

where θ denotes VI direction angle calculated by 2x2-point method in case with the predominant standing wave in a plate, and θ_0 denotes VI direction angle calculated by 12-point method in a plate without predominant standing wave.

Figure 8 shows VI direction angle error *AE* of Eq. (5) against SNR in the area that VI vectors were relatively large. The VI direction angle error at node (-160, -150) was larger than that at loop (0, -150) and it decreased with increasing SNR. Figure 8 also shows VI direction angle error at some other points such as loop point (-330, -150) and node points (0, -110) and (-330, -110). The magnitude of VI direction angle error was different in each point. Although the reason for these results is not clear, the results of all points had similar tendency that VI direction angle error decreased with increasing SNR.

Figure 9 shows the probability that VI direction angle error *AE* was less than 30 degrees. As shown in Fig. 9, the probability of VI direction angle error less than 30 degrees became about 0.5 at SNR of 18.9 dB. Figure 10 shows VI direction angle error against the ratio of the magnitude of VI to the maximum VI, VI/VI_{max}, in case of SNR of 9.02 dB and in case of SNR of 21.2 dB. In case of SNR of 9.02 dB, the VI direction angle error of VI dispersed widely even for large VI magnitude ratio. On the other hand, in case of SNR of 21.2 dB, the VI direction angle error of large-magnitude VI vectors was comparatively small. So, VI based on the simplified method is roughly useful for larger SNR than 18.9 dB.



(a) Without standing wave (b) With standing wave (SNR=14.8dB)

Figure 7. VI distribution calculated by 12-point method without and with standing wave

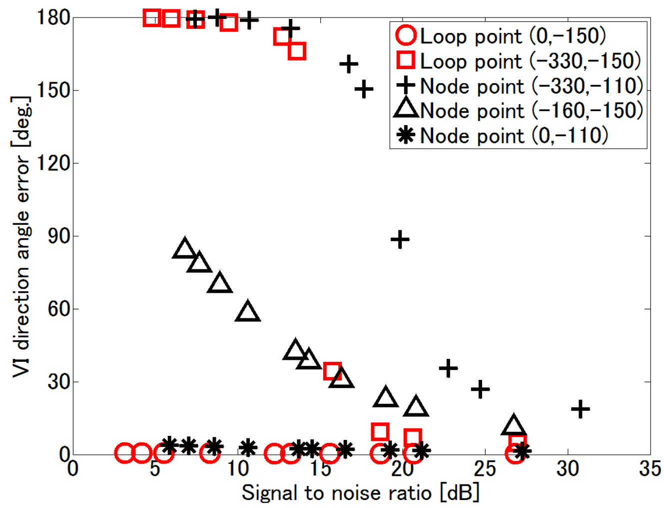


Figure 8. VI direction angle error vs. SNR at some points.

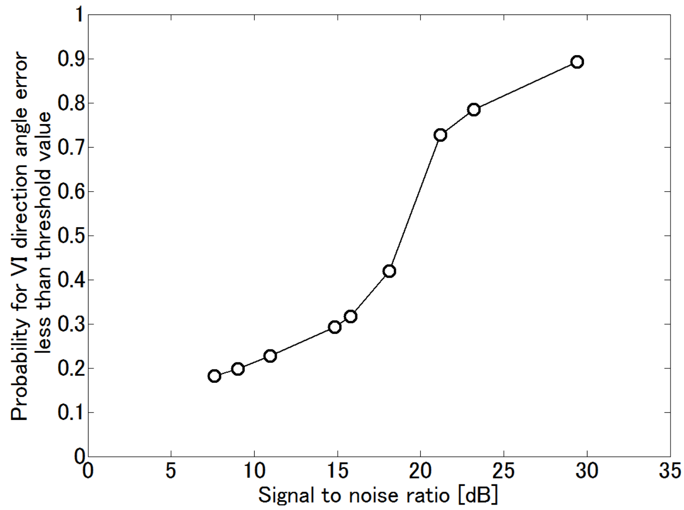
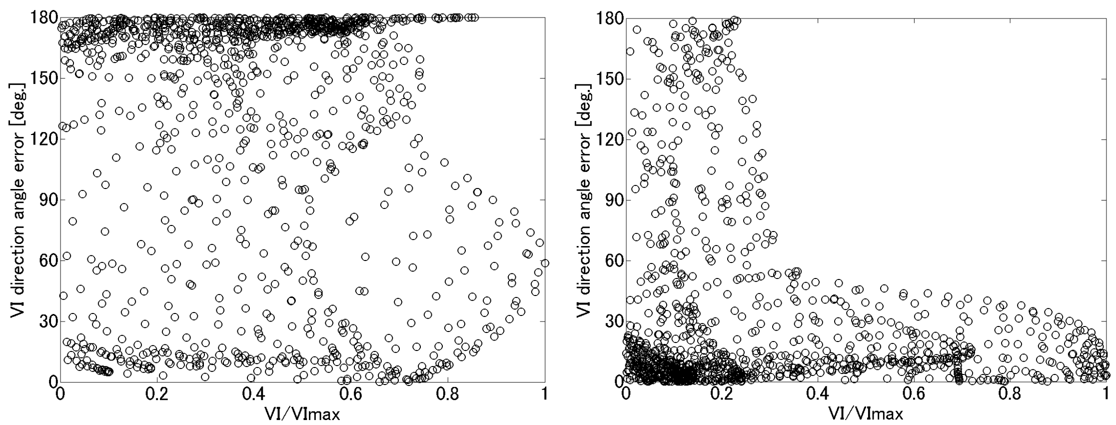


Figure 9. Probability for VI direction angle error less than 30 degrees.



(a) SNR of 9.02 dB

(b) SNR of 21.2 dB

Figure 10. VI direction angle error in a plate with standing wave vs.
ratio of magnitude of VI to VI maximum (2x2-point method).

4. CONCLUSIONS

In this paper, first, VI errors arising from the assumption of one-dimensional free and far field were discussed in the vibration field of a plate with impulse and standing wave. Next, the results between with and without standing wave were compared for the various ratios SNR of the amplitude of the main impulse response to that of the standing waves and the detection limit of the impulse response was discussed. The main conclusions are as follows.

(1) At loop points, the direction angle error of VI simplified formula was less than 10 degrees. At node points, however, the VI direction angle error increased with decreasing SNR. VI direction angle can be measured with satisfactory accuracy at loop points in comparison with at node points.

(2) In large-magnitude VI vectors, VI direction angle error arising from the existence of the predominant mode was comparatively small for larger SNR than 18.9 dB. So, the detection of VI based on the simplified formula requires such large SNR.

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