



# CONTRAST FUNCTION BASED FOURTH-ORDER STATISTICS AND WIENER FILTERING FOR SEPARATION OF CONVOLUTIVE MIXTURES

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## Abstract

In this paper, we consider the problem of blind separation of sources mixed by a convolutive system, expressed in the time-domain. We present a method based on fourth-order statistics and Wiener filtering. The technique is based on the minimization of two criteria based on the cancellation of the fourth-order cross cumulant between the contributions of all the estimated sources on each sensor, calculated by Wiener filtering. The matrix separation is updated iteratively by the contributions of the outputs recovered by Wiener filtering. These contributions are inserted in the separation procedure in order to get high separation quality of the data. Finally, simulation results illustrate the validity of our approach and show that it leads to improved separation performance.

# **1. INTRODUCTION**

Blind Source Separation (BSS) or Independent Component Analysis (ICA) is a basic and difficult problem in signal processing. It consists in retrieving a set of unknown source signals from the observation of their mixtures, assuming there is no information about the original source signals. Among many open issues, recovering the sources from their linear convolutive mixtures remains a challenging problem. Many solutions have been addressed in the frequency-domain, particularly for the separation of non-stationary audio signals. In the BSS of stationary signals, two problems occur in the time-domain. It has been proved [1] that convolutive mixtures are separable, that is, the independence of the outputs insures the separation of the sources, up to a few indeterminacies. However, the meaning of the independence is not the same in the convolutive and instantaneous contexts. Blind source separation methods depend on the nature of the criterion to measure the statistical mutual independence between the output signals.

In this paper, our approach is based on the maximization of a criterion using the fourth-order cumulant matrices between the estimated outputs and the projections of these outputs on each sensor. Several solutions have been addressed in the case of instantaneous mixtures [2,3]

based on fourth-order cumulants and using a reference signal. Other global approaches are based on the maximization of a criterion called a contrast function [4,5]. In this case, several approaches have been generalized through the use of reference signals. The first works have been restricted to the case of instantaneous source mixtures [6] while results concerning convolutive mixtures have been presented more recently [7]. The second problem comes from the inherent indeterminacy of the definition of the source in the BSS model. Indeed, any linear transform of a source can also be considered as a source and there is an infinity of separations that can extract sources. Some constraints can be added either on the source signals or the separating system (Minimal Distortion Principle [8]). In [8], one proposition is to choose the separator which minimizes the quadratic error between sensors and outputs, also known as Wiener filtering. Our approach is based on the calculation of fourth-order cumulant matrices and using the contributions of all sources on each sensor in order to diagonalize these matrices jointly. In [9,10], we used the mutual information of outputs as an independence criterion and we proved that testing the independence between the contributions of all sources on the same sensor at same time index n also leads to separability, we simplified the independence criterion using Wiener filtering. In this paper, the source separation matrix is updated iteratively by the projection of the outputs on the sensors calculated by Wiener filtering.

In section 2, we introduce the model. In section 3, we describe our algorithm with added contributions of all sources on each sensor. An illustration of the separating algorithm and simulation results is also developed. A conclusion is given in section 4.

#### **2. MODELING THE PROBLEM**

Let us consider the BSS model of a convolutive mixture with N sources and N sensors illustrated in Figure 1. We assume that sources  $s_i(n)$  (i = 1,...,N) are mutually independent. Each sensor  $x_j(n)$  (i = 1,...,N) receives a linear convolution from each source  $s_i(n)$  at discrete time n. The goal is to recover the source processes  $s_i(n)$ ,  $i \in \{1,...,N\}$  using only the observation x(n). The relationship between the observations and the sources can be expressed through the following linear model:

$$x_{j}(n) = \sum_{i=1}^{N} a_{ji} * s_{i}(n)$$
(1)

Equation (1) can be also written in Z-transform as:

$$x(n) = [A(z)]s(n)$$
<sup>(2)</sup>

where  $a_{ij}$  represents the impulse response from source *i* to sensor *j*. '\*' denotes the linear convolution operator. The aim of BSS is to find filters with impulse responses  $b_{ij}$  between sensor *i* and output *j*, such that the output vector y(n) estimates the sources, up to a linear filter:

$$y_{j}(n) = \sum_{i=1}^{N} \sum_{k=-L}^{L} b_{ji}(k) x_{i}(n-k)$$
(3)

$$y(n) = [B(z)]x(n) \tag{4}$$

It is useful to define the global filter by the following impulse response:

$$G(z) = B(z)A(z)$$
<sup>(5)</sup>

We have then:

$$y_{j}(n) = \sum_{i=1}^{N} \sum_{k=-L}^{L} g_{ji}(k) s_{i}(n-k)$$
(6)



Figure 1. Mixing and demixing system

If  $y_i(n)$  is any linear filtering of one source, then the contribution of this source on the first sensor is calculated by a (possibly non causal) Wiener filter  $W_{Ii}(z)$  such that the quadratic error between  $x_I(n)$  and  $y_i(n)$ :

$$E(|x_1(n) - w_{1i} * y_i(n)|^2) \text{ is minimized}$$
(7)

Therefore, the purpose of using Wiener filtering is to minimize the mean-square error between their output and a required output (see Figure 2). In [9,10] we proved that using the projection of the outputs on the same sensor also leads to separability, without making an independence test of delayed outputs (for more details about Wiener filter, see [9,10]). The contribution of the source i on the *m*th sensor is thus given by :



Figure 2. Separation Model

## **3. CONTRAST FUNCTIONS**

#### 3.1 Separability

In specific cases, testing the independence between  $y_i(n)$  and  $y_j(n)$  is sufficient [6] to ensure the separation. For example, for i.i.d. normalized sources, the sum of fourth-order cumulants of the outputs is a contrast function [7] under a condition on separating filters [6]. For linear filtering of i.i.d. signals, the same result is obtained after a first step of whitening of the data. However, in a general case, delays must be introduced in the contrast function and the separability of convolutive mixtures is obtained only when the components of the output vector y(n) are independent in the sense of stochastic variables :  $y_i(n)$  and  $y_j(n-m)$  have to be independent for all discrete time delays m. For example, a solution is to minimize the criterion J:

$$J = \sum_{i \neq j} \sum_{m} I\left(y_i(n), y_j(n-m)\right)$$
(9)

where I represents the mutual information (10). I is nonnegative and equal to zero if and only if the components are statistically independent.

$$I(y) = \int_{R} p_{y}(y) \ln \left( \frac{p_{y}(y)}{\prod_{i=1}^{M} p_{yi}(y_{i})} \right) dy$$
(10)

The delays *m* can be taken in an *a priori* set [-K, ..., K], which depends on the degree of the filters corresponding to the whole mixing-separating system. The criterion (9) is computationally expensive. In [3], a gradient-based algorithm minimizes (9): at each time iteration, a random value of delay *m* is chosen and  $I(y_i(n), y_j(n-m))$  is used as the current separation criterion.

In previous papers [9,10], we studied the separability of the contributions of the sources *i* and *j*, projected on the sensor *m*,  $z_{mi}(n)$  and  $z_{mj}(n)$  (4) versus  $y_i(n)$  and  $y_j(n)$ . We showed that it was simpler and that no time delay (n-m) was needed. Testing  $I(y_i(n), y_j(n))=0$  and  $I(z_{mi}(n), z_{mj}(n))=0$ , ensures the separability, as well as the minimization of the mutual information of the outputs  $I(z_{mi}(n), z_{mj}(n))=0$ . The criterion is much more easier to test than the mutual information of delayed outputs as it was verified in an iterative way. Moreover the outputs are directly the contribution of the sources on the processed sensor.

#### 3.2 Separation criteria based on the fourth-order cumulants

In [10], we studied the separability of  $z_{ji}(n)$  and we used the mutual information such as an independence criterion but the proof of separability can be also exploited with another independence test. In [11], we used a fourth-order cumulant based method to compute the separation matrix B(z). We propose here to compare two other criteria based on the cancellation of fourth-order cross-cumulants between the Wiener outputs  $z_{ji}(n)$ :

$$C1 = \min\left(\sum_{m} \sum_{i,j,k,l \neq iiii} \left| cum(z_{mi}^{*}, z_{mj}^{*}, z_{mk}, z_{ml}) \right| \right)$$
(11)

$$C2 = \min\left(\sum_{m} \sum_{i,j,k,l\neq iiii} \left| cum(z_{mi}, z_{mj}^*, x_m, x_m^*) \right| \right)$$
(12)

As in [3,9], the criteria C1 and C2 are minimised in an iterative way. The convolutive separative filters B(z) are updated with the error term C1 or C2. A Wiener step is added at the

output of the separating phase. The criteria are minimized iteratively, with the following algorithm :

Initialization : y(n) = x(n)Repeat until convergence :

- Estimate the criteria C1 or C2 between the contributions :  $z_j(n) = W_i(z) y_j(n)$
- Update :  $B(z) \leftarrow B(z) \mu C1$
- Compute the outputs of the separating filters  $y_j(n) = B(z) x(n)$
- Compute the Wiener filters  $W_i(z)$ , and the new contributions :  $z_j(n) = W_i(z) y_j(n)$
- Replace :  $y(n) \leftarrow [z_{11}(n), z_{12}(n))]$

The convergence and the behaviour of the algorithm is investigated in the appendix. The performances are shown in figures 1 and 2 with simulation results. Each source (of 1500 samples) is constituted of the sum of a uniform random signal and a sinusoid. They are mixed with filters :

$$H(z) = \begin{bmatrix} 1 + 0.2z^{-1} + 0.1z^{-2} & 0.5 + 0.3z^{-1} + 0.1z^{-2} \\ 0.5 + 0.3z^{-1} + 0.1z^{-2} & 1 + 0.2z^{-1} + 0.1z^{-2} \end{bmatrix}$$
(13)

The quadratic errors between  $z_1(n)$  and the exact contribution are plotted in Figure 3 for each iteration. They are averaged on 50 realizations of the sources. For a common value of the parameter  $\mu$ =0.03, the black and blue curves respectively represent the MSE of criteria C1 and C2. The red curve represents the MSE of criterion C2 where x(n) are the whitening sensors. In this case, C2 is also a contrast function. This result has been obtained in [12] using the approach of reference signals. It shows good results for the convergence and the residual quadratic error. However, C1 is the more efficient algorithm in term of convergence speed. It can be explained by the fact that C1 aims at the minimization of more cross-cumulants that C2. For example, in a 2 source, 2 sensor scheme, C1 must cancel 6 cross-cumulants whereas C2 minimizes only 2 cross-cumulants.



Figure 3. MSE of the criteria C1 and C2

In Figure 3, the parameter  $\mu$  was optimal for C2 (with whitened sensors - red curve). The final MSE in that case is equal to 5 10<sup>-7</sup> (versus 8 10<sup>-4</sup> for C2 and 2 10<sup>-4</sup> for C1). In Figure 4, the parameter was optimal for each criterion in term of MSE and the speed convergence behaviour is compared. For an equal MSE of 3 10<sup>-7</sup>, we see that C1 (black curve  $\mu$ =0.01) has still the better convergence speed versus C2.



Figure 4. MSE of the criteria C1 and C2

#### **4. CONCLUSIONS**

In this paper, we have addressed the source separation problem in the case of convolutive mixture, expressed in the time-domain. We have developed an algorithm based on the fourth-order statistics and Wiener filtering. The method is based on the minimization of two criteria based on the cancellation of fourth-order cross cumulants between the contributions of all the estimated sources on each sensor, calculated by Wiener filtering. The matrix separation is updated iteratively by the contributions of the outputs recovered by Wiener filtering. We show that the implementation of such an approach allows appreciable improvement of the quality of the source separation. Simulation examples are given to demonstrate the performance of the proposed method. The test is easier and shows good results on simulation.

#### **5. APPENDIX**

Let be a 2 sources 2 sensors scheme. For sake of simplicity, we call here sources the two contributions on the first sensor. So,  $x_I(n)$  is equal to :  $x_1(n) = s_1(n) + s_2(n)$ .  $z_{mi}$  are the contributions of the sources on the first sensor calculated by Wiener filtering, they are given by :

$$z_{mi}(n) = \sum_{k=-L}^{L} w_{mi}(k) y_i(n-k)$$
(14)

The DFT of filters  $w_{mi}(k)$  are computed at frequency bin f as a function of the cross-spectra  $\gamma_{Y_jXI}(f)$  of  $x_1(n)$  and  $y_j(n)$ , and  $\gamma_{Y_j}(f)$  the spectra of  $y_j(n)$ :

$$W_{11}(f) = \frac{\gamma_{Y1X1}(f)}{\gamma_{Y1}(f)}; W_{12}(f) = \frac{\gamma_{Y2X1}(f)}{\gamma_{Y1}(f)}$$
(15)

The two first contributions are given in function of the global filter G(z):

$$Z_{11}(f) = \frac{\left|G_{11}(f)\right|^{2} \gamma_{s1}(f) + G_{11}(f)\overline{G}_{12}(f)\gamma_{s2}(f)}{\gamma_{r1}(f)} S_{1}(f) + \frac{\overline{G}_{11}(f)G_{12}(f)\gamma_{s1}(f) + \left|G_{12}(f)\right|^{2} \gamma_{s2}(f)}{\gamma_{r1}(f)} S_{2}(f)$$

$$Z_{12}(f) = \frac{\left|G_{21}(f)\right|^{2} \gamma_{s1}(f) + G_{21}(f)\overline{G}_{22}(f)\gamma_{s2}(f)}{\gamma_{r2}(f)} S_{1}(f) + \frac{\overline{G}_{21}(f)G_{22}(f)\gamma_{s1}(f) + \left|G_{22}(f)\right|^{2} \gamma_{s2}(f)}{\gamma_{r2}(f)} S_{2}(f)$$

$$(16)$$

The algorithm has two parts : a separating step followed by a Wiener filtering. The two parts are iteratively updated and exploits the second order statistics (decorrelation between  $s_1(n)$  and  $s_2(n)$ ) and fourth-order statistics (independence between  $s_1(n)$  and  $s_2(n)$ ). At the convergence of the algorithm : the two parts converge. Concerning the Wiener step, we have :

$$\frac{|G_{11}(f)|^2 \gamma_{s1}(f) + G_{11}(f)\overline{G}_{12}(f)\gamma_{s2}(f)}{\gamma_{\gamma_1}(f)} \longrightarrow G_{11}(f)$$

$$\frac{|G_{12}(f)|^2 \gamma_{s2}(f) + G_{12}(f)\overline{G}_{11}(f)\gamma_{s1}(f)}{\gamma_{\gamma_2}(f)} \longrightarrow G_{12}(f)$$

$$\begin{bmatrix} \overline{G}_{11}(f) - |G_{11}(f)|^2 \end{bmatrix} \gamma_{s1}(f) + \begin{bmatrix} \overline{G}_{12}(f) - |G_{12}(f)|^2 \end{bmatrix} \gamma_{s2}(f) = 0$$
  
$$\begin{bmatrix} \overline{G}_{21}(f) - |G_{21}(f)|^2 \end{bmatrix} \gamma_{s1}(f) + \begin{bmatrix} \overline{G}_{22}(f) - |G_{22}(f)|^2 \end{bmatrix} \gamma_{s2}(f) = 0 \qquad \Leftrightarrow \qquad G_{ij}(f) = 0; \ or \ G_{ij}(f) = 1$$

We can eliminate the trivial solutions  $(z_{Ij}(n)=0 \text{ or } x_I(n))$ .  $z_{Ij}(n)$  are not zero, knowing that the Wiener filter maximizes the correlation between the outputs of the separating step and the sensors.  $z_{Ij}(n)$  are not equal to the sensors, knowing that :

$$\sum_{m} \sum_{i,j,k,l \neq iiii} \left| cum(z_{mi}^*, z_{mj}^*, z_{mk}, z_{ml}) \right| = 0$$
(17)

It remains that:  $z_{11}(n) = s_1(n)$  and  $z_{12}(n) = s_2(n)$ . In the same way, if  $x_2(n) = F_{11}(s_1(n)) + F_{12}(s_2(n))$ , we obtain that  $z_{21}(n) = F_{11}(s_1(n))$  and  $z_{22}(n) = F_{12}(s_2(n))$ .

As only second-order is used, it means that we could recover any orthogonal signals  $s'_1(n)$ and  $s'_2(n)$  such that their sum is equal to the first sensor  $x_1(n) = s'_1(n) + s'_2(n)$  as the decomposition is not unique.

Suppose that we recover :  $z_{11}(n) = s'_1(n) = \alpha s_1(n) + \beta s_2(n)$  and  $z_{12}(n) = s'_2(n) = \gamma s_1(n) + \delta s_2(n)$  with parameters  $(\alpha, \beta, \gamma, \delta)$  such that  $s'_1(n)$  and  $s'_2(n)$  are orthogonal and  $x_1(n) = s'_1(n) + s'_2(n)$ .

The problem is then equivalent to an instantaneous mixture and the criterion

$$\sum_{i,j,k,l \neq iiii} \left| cum(z_{1i}^*, z_{1j}^*, z_{1k}, z_{1l}) \right|$$
 is zero only for  $z_{11}(n) = s_1(n)$  and  $z_{12}(n) = s_2(n)$ 

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