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ACOUSTIC HOLOGRAPHY AND COMPLEMENTARY BOUNDARY CONDITIONS

Vincent Martin[1], Thibault Le Bourdon[1]

[1]Institut Jean Le Rond d'Alembert, CNRS/Paris VI
Place de la gare de Ceinture 78210 St Cyr l'Ecole, France
vmartin@ccr.jussieu.fr
lbourdon@ccr.jussieu.fr

Abstract

Nearfield holography consists in measuring the acoustic pressure radiated near an extended sound source with the aim of deducing its vibratory velocity and, from this information, the pressure radiated everywhere else. This well-known procedure rests on hypotheses which are still not well mastered today, in particular with respect to boundary conditions. Because of this, no quantitative parameter can guarantee the reliability of the results obtained. At a time when industrialists must comply with strict norms, this question of guarantee is crucial. In the case of the acoustic radiation of an object such as a vehicle wheel, subjected to a vibratory stimulus, a holographic antenna placed beside the wheel observes only the front side and its rim while the rear side and the thickness of the tyre remain invisible. The invisible vibratory area manifests itself through information obtained partly from the complementary part of the source plane i.e., the plane containing the front of the wheel but without the front surface itself. Numerical simulations (with the help of the BIEM) in the unbounded 3D space show how the effect on the invisible area can be described by an impedance relation, thus resulting in a problem in the half-3D unbounded space. Usually, Neumann or Dirichlet boundary conditions are used on the complementary source plane. What are the consequences of mixed boundary conditions both on the hologram pressure and on the vibratory velocity identified? It will be shown that these boundary conditions modify the hologram pressure significantly and, thus, the reconstruction of the source velocity. This is, in fact, the starting point of the geometrical interpretation of nearfield holography, which originates from the field of active noise control.

1. INTRODUCTION

The question of the veracity of results obtained by acoustic holography can be raised in other fields of acoustics (e.g. active noise control). In this paper, it is studied in the case of the holography of a circular vibrating object such as, for example, a wheel. The configuration studied is shown in Figure 1, and the notion of veracity is approached through the influence of the environment of the object being unknown.

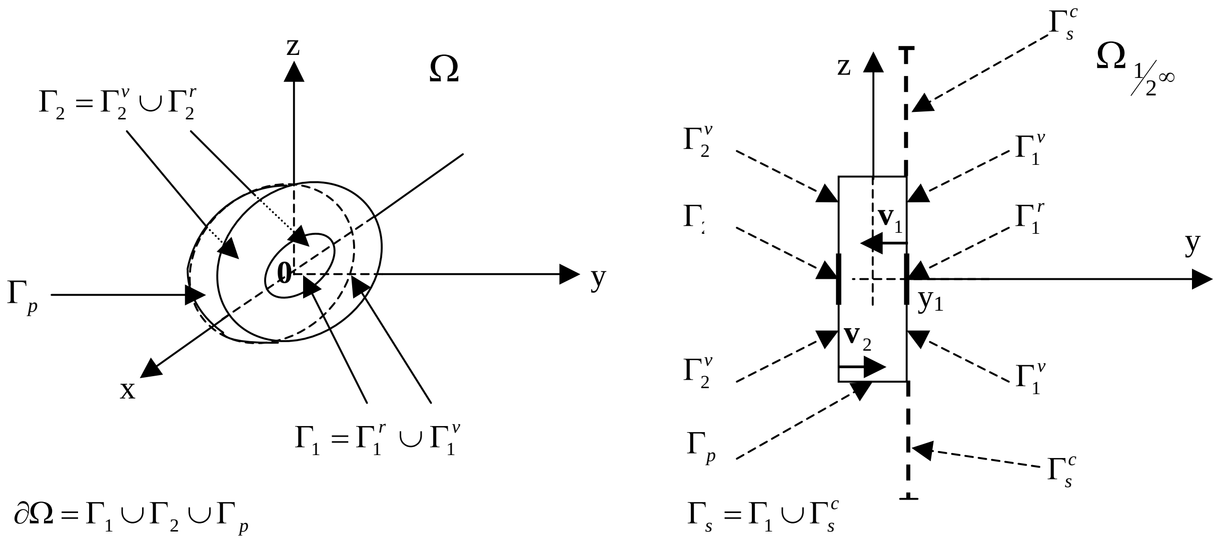


Figure 1. Configuration of a wheel.

To show how results obtained by acoustic holography are sensitive to the description of the environment of the vibrating objects, a numerical simulation of the experiment is carried out first : both sides Γ_1 and Γ_2 of the wheel are excited by a uniform and unitary vibratory velocity, and this object radiates in a unbounded 3D space (for the time being, the area Γ_p, representing the thickness of the tyre, does not have a vibratory velocity). The description rests on harmonic linear acoustics. In these conditions, the radiated pressure can be obtained with the integral equation method classically.

Then a numerical simulation of the holographic procedure is presented; the plane microphones array is placed parallel to the side of the vibrating object, the vibratory velocity of which is to be determined. Parallelism between the hologram plane and the source plane emerges from the first holographic procedures where the inversion needed to obtain the vibratory velocity was carried out with the Fourier transform [1, 2]. The method consists in considering the front surface of the vibrating object as belonging to the source plane, itself the boundary plane that limits the 3D unbounded half-space. To be rigorous, it is necessary to bring forward the whole rear environment of the wheel, which means the acoustic load behind as well as the vibration of the rear surface at Γ_2. This can be done by using the admittance both on the area of the front Γ_1 and on the area of its complementary part Γ_s^c on the source plane. This approach originates from a generic 1D problem where the equivalence of the operators in 1D space and in 1D half-space has been shown.

Thus, the holographic method rests on deducing the vibratory velocity of the front side Γ_1, from the hologram Γ_H, by considering only the 3D unbounded half-space. Here also the description of the field is dealt with by the integral equation method, which uses the admittance mentioned above.

In fact, this admittance is unknown and an arbitrary admittance is generally adopted. It seems natural here to consider a zero admittance since vibratory velocity v_2 (normal) on the rear is identical to that on the front v_1, and also provided the tyre is thin compared with the considered wavelength.

What is the consequence of the arbitrary nature of the admittance on the identified vibratory velocity v_1^{opt} compared with the real vibratory velocity v_1? The development of this approach leading to the conclusion that errors can exceed 100%, constitutes the purpose of this paper.

2. DIRECT PROBLEM IN A UNBOUNDED 3D SPACE

2.1 Analytical resolution by the integral method

The configuration in an 3D unbounded space imposes reference axes (see Figure 1); moreover P corresponds to a point source and Q to a receiver. With the presently chosen time convention $e^{j\omega t}$, and the operator $H = \Delta + k^2$, the acoustic pressure problem is defined by:

$$\begin{cases} H\ p = 0 & \text{in } \Omega_\infty \text{ (exterior to the wheel)} \\ \partial_n p = -i\rho\omega v_n & \text{on } \Gamma_1 \text{ and } \Gamma_2 \\ \partial_n p = 0 & \text{on } \Gamma_p \\ \text{Sommerfeld condition (radiation at infinity)} \end{cases} \tag{1}$$

The selected elementary solution $g_\infty(P,Q) = -e^{-jkr(P,Q)}/(4\pi\, r(P,Q))$ satisfies the Helmholtz equation with $+\delta(P-Q)$ as the right-hand member, and the condition of radiation at infinity. Without any source within the domain and by including the boundary conditions, the pressure $p(Q)$ at a point pertaining to the field Ω can then be written (the normal is directed towards the outside of the domain, therefore, entering the wheel):

$$p(Q) = i\,\rho\,\omega \int_{\Gamma_1 \cup \Gamma_2} v_n g_\infty(P,Q)\, dP + \int_{\Gamma_1 \cup \Gamma_2 \cup \Gamma_p} p(P)\, \partial_n g_\infty(P,Q)\, dP \tag{2}$$

where v_n is given in the direct problem. With the collocation method by meshing the surface in facets, the approximated numerical solution of (2) is sought, resulting in the matricial form:

$$\{p(Q_s \in \partial\Omega)\} = [\mathbf{I} - \mathbf{A}]^{-1}\, i\rho\omega \mathbf{B}\ \{v_n\} \tag{3}$$

where **A** is made up of the auto- and inter-influence terms (with principal value associated with the singularity of Green’s function and its first derivative), and **B** is the transfer of $\boldsymbol{v}_n$ on the wheel contour $\partial\Omega$ and

$$p(Q \in \Omega) = i\rho\omega(\mathbf{c}^t\ [\mathbf{I} - \mathbf{A}]^{-1}\ \mathbf{B} + \mathbf{d}^t)\{v_n\} \tag{4}$$

Line vectors $\mathbf{c}^t$ and $\mathbf{d}^t$ come respectively from the second and first terms of equation (2).

2.1 Definition of the conditions on the source plane via admittances

Pressure $p(Q \in \Omega)$ leads to v_n (here $\pm v_y$ on the source plane) by the discrete form

$$v_n = -\frac{i}{2\pi\ f\ \rho} \frac{p_1 - p_2}{\Delta y} \tag{5}$$

where p_1 and p_2 represent the pressures concerning two points separated from each other by Δy. The admittance β_s^c on the complementary surface Γ_s^c is thus $\beta_s^c = \rho c \dfrac{v_n}{p}$ and leads to the mixed passive condition $p_{,y} + ik\beta_s^c\, p = 0$; moreover the admittance β_1 on the source surface Γ_1 is $\beta_1 = \rho c \dfrac{v_n - v_1}{p}$ (which could be called active admittance) leading to the mixed boundary condition $p_{,y} + ik\beta_1\, p = -i\,\rho\,\omega v_1$ (following an elementary study on a 1D problem). It should be pointed out that, in practice, determining velocity v_n in the case of β_1 is not straightforward because the interior pressure of the wheel is not accessible to determine the normal acoustic velocity on the vibrating side of the wheel. An approximation has been found by calculating a velocity close to the vibrating side. It has appeared a posteriori that the ensuing error can be corrected efficiently in an empirical way. Finally we obtain $\beta = \beta_1 \cup \beta_s^c$. The results shown below were achieved with a wheel radius of 0.32m.

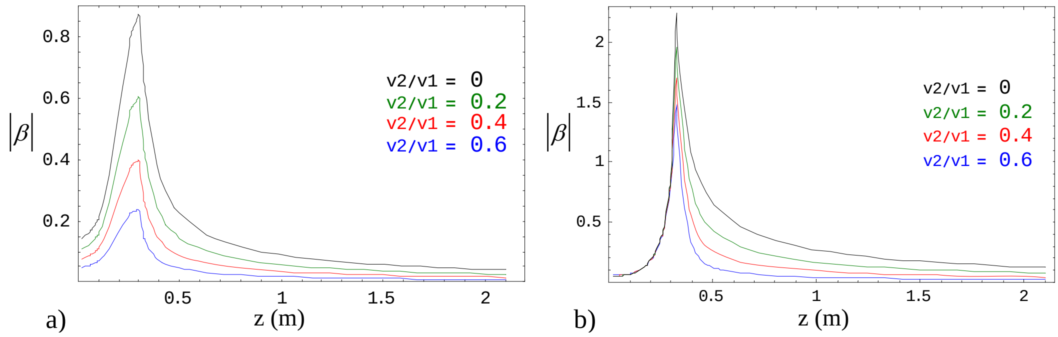


Figure 2. Absolute value $|\beta|$ - function of z (x = 0 and y = y_1) at 250 Hz for v_2/v_1 given and extreme thinness of the wheel at a), and of a thickness of 10 cm at b).

To become confident in these results, it can be shown that the Neumann boundary conditions are obtained i.e., $\beta_1 \approx \beta_s^c \approx 0$ when considering the very thin wheel and by imposing $v_2 = v_1 = 1$. Another reason to be confident will be presented in the following stage.

3. DIRECT PROBLEM IN A 3D UNBOUNDED HALF-SPACE

3.1 Analytical resolution by the integral method

The configuration in a 3D unbounded half-space is immediately deduced from the previous configuration. The acoustic pressure problem is thus defined in the following way:

$$\begin{cases} H\ p = 0 & \text{in } \Omega_{1/2\infty}\ (y \geq 0) \\ \partial_y p + i\,k\,\beta_1\, p = -i\rho\omega v_1 & \text{on } \Gamma_1 \\ \partial_y p + i\,k\,\beta_s^c\ p = 0 & \text{on } \Gamma_s^c \\ \partial_y p = 0 & \text{on } \Gamma_{ext} \\ \text{Sommerfeld condition (radiation at infinity)} & \end{cases} \quad (6)$$

The equation $\partial_y p = 0$ on Γ_{ext} comes from the shape of the admittance (see Figure 2) on the complementary part of the source plane. Indeed β_s^c tends towards 0, beyond a certain limit on the source plane, and a homogeneous Neumann condition is reached. The area concerned is denoted Γ_{ext} $(\equiv (plane\, x0z) - (\Gamma_S))$. This condition is essential for the finite meshing of the source plane.

The selected elementary solution $g_{1/2\infty}(P,Q) = -e^{-jkr(P,Q)}/(2\pi r(P,Q))$ satisfies the Helmholtz equation with $+\delta(P-Q)$ as the right-hand member, and the conditions of radiation at infinity. Without a source within the domain, and including the boundary conditions, the pressure $p(Q)$ at a point in the domain $\Omega_{1/2\infty}$ can then be written

$$p(Q) = i\rho\omega \int_{\Gamma_1} v_n\, g_{1/2\infty}(P,Q)\,dP + ik\int_{\Gamma_1} \beta_1\, p(P)\, g_{1/2\infty}(P,Q)]dP + ik\int_{\Gamma_s^c} \beta_s^c\, p(P)\, g_{1/2\infty}(P,Q)]dP \tag{7}$$

where v_n is given in the direct problem. The approximated solution of (7) is also obtained numerically with the collocation method. The matricial writing is as follows:

$$\{p(Q_s \in \partial\Omega)\} = \left[\mathbf{I} - i\,k\,\mathbf{A}(\beta_1,\beta_s^c)\right]^{-1} i\rho\omega\mathbf{B}\ \{v_n\} \tag{8}$$

where $\mathbf{A}(\beta_1,\beta_s^c)$ and $\mathbf{B}$ have the same significance as in (3). Therefore we obtain

$$p(Q\in\Omega) = i\rho\omega(i\,k\,\mathbf{c}^t(\beta_1,\beta_s^c)\left[\mathbf{I} - i\,k\,\mathbf{A}(\beta_1,\beta_s^c)\right]^{-1}\mathbf{B} + \mathbf{d}^t)\{v_n\} \tag{9}$$

where the line vectors $\mathbf{c}^t$ and $\mathbf{d}^t$ come respectively from the second and third terms and from the first term of the second member of the equation (7). Figures 3 and 4 show first the equivalence between the unbounded 3D model and half-3D unbounded model, with a certain degree of confidence, and afterwards the influence of a disturbance on the velocity or on the thickness of the wheel, on the pressure $p(\Gamma_s)$ and thus on $p(\Omega_{1/2\infty})$, or more precisely on the hologram pressure $p(\Gamma_H)$. Here, the hologram plane consists of 121 equidistant nodes (11×11), distributed symmetrically along the wheel axis, on a $1m \times 1m$ surface at a distance y_H from the source plane and parallel to the latter. The difference between the unbounded 3D model and the half-3D unbounded model (see Figure 4) may result from a numerical error or from an intrinsic error from the calculation of β_1. Here also, it is possible to remove empirically the error.

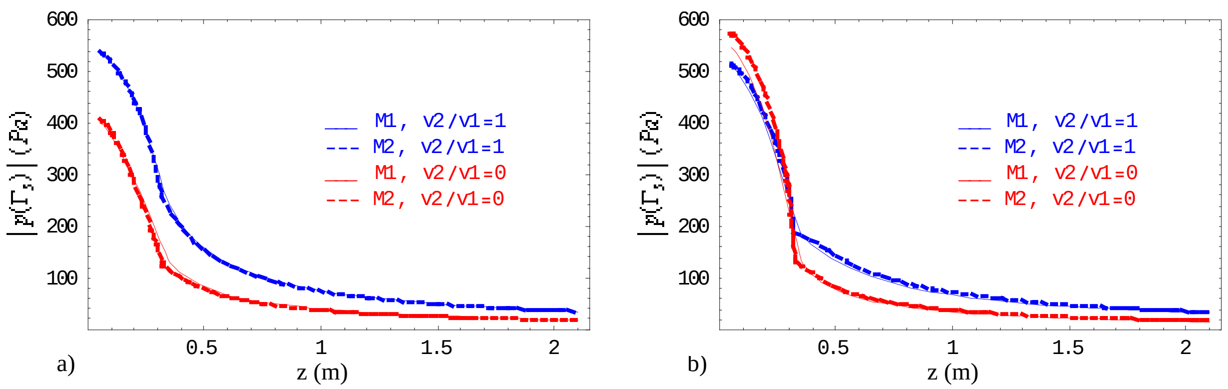


Figure 3. Absolute surface pressure $|p(\Gamma_s)|$ - function of z (x = 0 and y = y_1) at 250 Hz for v_2/v_1 given and an extreme wheel thinness at a), and a 10cm thickness at b). M1 corresponds to the unbounded 3D model and M2 to the half-3D unbounded one.

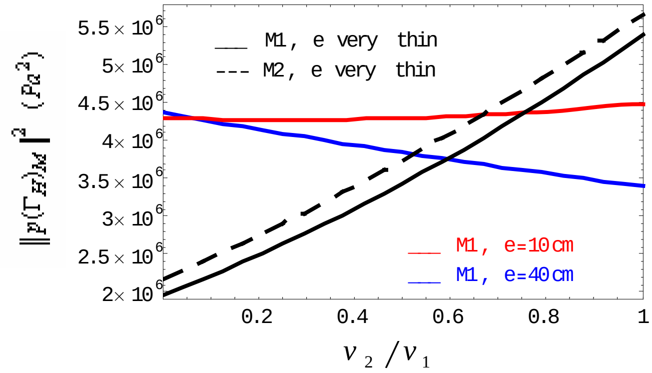


Figure 4. Squared absolute value $\| p(\Gamma_H)_M \|^2$ of the average pressure on Γ_H (y_H=0.12m) at 250Hz - function of v_2/v_1 for a given thickness, e, of the wheel . M1 corresponds to the unbounded 3D model and M2 to the half-3D unbounded one.

4. INVERSE PROBLEM

The aim of this study is to determine the vibratory velocity of the wheel using the nearfield acoustic holography method. What is sought is v_1^{opt} i.e., the vibratory field reconstructed on the front side Γ_1 of the wheel, knowing the pressure $p(\Gamma_H)$ on the hologram plane Γ_H facing the source. This section deals with the influence of mixed boundary conditions, revealed by the admittance of the source plane, upon the identified velocity.

4.1 Approximation of the inverse problem.

Formally only, determining velocity v_n starting from the knowledge of $p(\Gamma_H)$ in the domain $\Omega_{1/2\infty}$ requires the calculation of $p(\Gamma_s)$ on the source plane, which itself results from v_n. The discretized form of (7) specifies better the steps to follow than the continuous form. Indeed equation (7) leads to the matricial form

$$\{p(Q)\}_{N\times 1} = \mathbf{E}(\beta_1, \beta_s^c)_{N\times M} \; \{v_n\}_{M\times 1} \tag{10}$$

where $\mathbf{E}(\beta_1,\beta_s^c) = i\rho\omega(i\,k\,\mathbf{c}^t(\beta_1,\beta_s^c)\left[\mathbf{I} - i\,k\,\mathbf{A}(\beta_1,\beta_s^c)\right]^{-1}\mathbf{B} + \mathbf{d}^t)$; M is the number of points in mesh Γ_s and N is the number of points from meshing the hologram plane Γ_H.

Within the framework of the least square method and with appropriate hypotheses ($M \le N$ and rank(E)=M), it can be concluded that

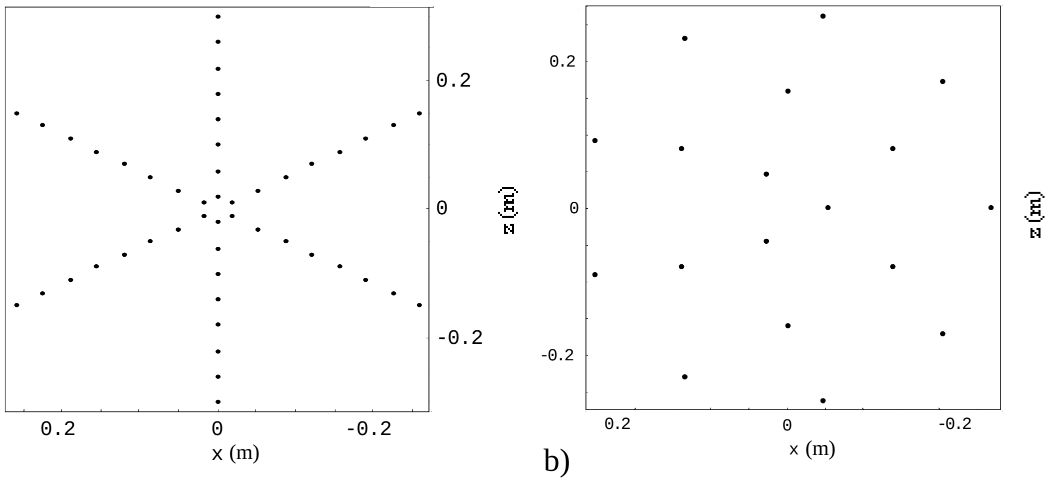
$$\{v_n\} \approx \left[\mathbf{E}^*(\beta_1,\beta_s^c).\mathbf{E}(\beta_1,\beta_s^c)\right]^{-1}\mathbf{E}^*(\beta_1,\beta_s^c)\{p(Q)\} \tag{11}$$

The vibratory velocity obtained is in fact the solution of the algorithm

$$\min_{\{v_n\}} \left\| \left[\mathbf{E}^*(\beta_1,\beta_s^c)\right]\{v_n(\Gamma_s)\} - \{p(\Gamma_H)\} \right\|_{\Gamma_H}^2 \tag{12}$$

4.2 Results of the reconstruction of the velocity v_n

The resolution of an inverse problem often generates instabilities generally arising from a poor conditioning of matrix **E**. These instabilities, usually not mastered a priori, can nonetheless be controlled empirically because they arise in general from a concentration of calculation points on the mesh and/or of symmetries between the various transfer functions i.e., of the position of the hologram plane with regard to the wheel as well as to the complementary part (poor conditioning here lies in the dependent lines and/or columns resulting from the near-identical transfer functions). Figure 5 shows the mesh of source plane (Γ_1 only) used for the direct problem and for the inverse problem which is well conditioned at 250 Hz, and for a distance between the hologram plane and the plane source of y_H = 0.12m.



a) b)

Figure 5. Mesh of Γ_1 for the direct problem, a), for the inverse problem, b).

Mesh b) is thus made in order to obtain a better distributed density of collocation points than in mesh a). In addition the hologram plane is dissymmetrical to the wheel axis. Lastly, in the case of a unitary stimulus v_1 uniformly distributed on the wheel, including a disturbance on the admittance of the source plane (evidenced by a disturbance on v_2/v_1 obtained by the unbounded 3D model) the reconstructed velocity v_1^{opt} is shown to present an error of up to 140% (see Figure 6).

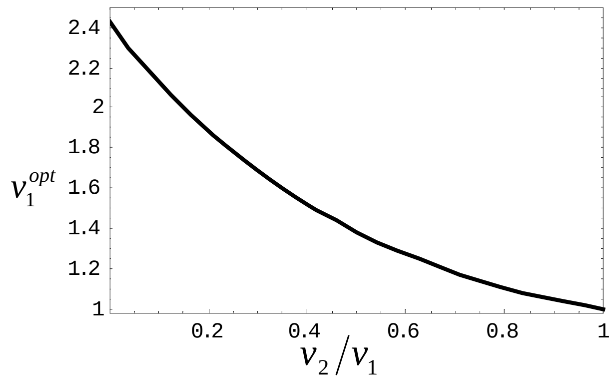


Figure 6. Reconstruction of v_1^{opt} (m/s) – function of v_2/v_1 with a thin wheel at 250Hz and y_H=0.12m.

5. CONCLUSION

Results achieved with the numerical simulations presented show that the conditions on the whole source plane (area Γ_1 and complementary part Γ_s^c) vary according to the rear acoustic load and to the disturbances arising from stimuli v_1 and v_2, and also according to the geometry of the system (shown here by a variation in the thickness of the wheel). Thus, each perturbation leads to an admittance on plane Γ_s. It is noteworthy that this local reaction (i.e., a relation between the pressure and the normal velocity at the same point) appears to be sufficient to describe phenomena in the 3D space into the 3D half-space. The pressure calculated on the hologram is also dependent on these disturbances.

In the case studied here, it was shown that, at a certain distance from the source, whatever the stimulus or the geometry, Neumann boundary conditions on the plane source but far from the surface source itself are definitely satisfactory. As has been observed, the influence of β is close to the source and decreases more or less quickly.

Under these conditions, the transfer of the rear acoustic load onto the source plane has a significant influence on the identified velocity, and it is difficult to trust results achieved via arbitrary boundary conditions. Is it possible to identify these boundary conditions as well as the vibratory velocity? To answer the question, we will take as a starting point the procedure initiated in [3] which, in an analytical and much easier situation, resorted to a geometrical method to improve model $\mathbf{E}(\beta_1, \beta_s^c)$ when erroneous, in order to guarantee the result obtained even when there is also an error on the objective $p(\Gamma_H)$.

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