ICSV14
Cairns • Australia
9-12 July, 2007

14th International Congress on Sound & Vibration

OPERATIONAL MODAL ANALYSIS BY USING TRANSMISSIBILITY MEASUREMENTS WITH CHANGING DISTRIBUTED LOADS.

Devriendt Christof, Patrick Guillaume, De Sitter Gert, Vanlanduit Steve

Vrije Universiteit Brussel
Department of Mechanical Engineering
Acoustics and Vibration Research Group
avrg@vub.ac.be • www.avrg.vub.ac.be

cdevrien@vub.ac.be

Abstract

Recently a new approach to identify modal parameters from output-only transmissibility measurements was introduced. In general, the poles that are identified from transmissibility measurements do not correspond with the system's poles. However, by combining transmissibility measurements under different loading conditions, it has been shown that model parameters can be identified. In the previous papers on this topic a single input situation was assumed. In order that the method should be useful for operational modal analysis, where there are in general a number of simultaneous sources, a generalization of the technique with a multiple input assumption is proposed. In this paper the extended technique is demonstrated and validated by means of an experimental test on a beam.

1. INTRODUCTION

1.1. Experimental and operational modal analysis

During the last decade modal analysis has become a key technology in structural dynamics analysis [1]. Experimental modal analysis (EMA) identifies a modal model, $[H(\omega)]$, from the measured forces applied to the test structure, $\{F(\omega)\}$, and the measured vibration responses $\{X(\omega)\}$,

$$\{X(\omega)\} = [H(\omega)]\{F(\omega)\} \tag{1}$$

with

$$[H(\omega)] = \sum_{m=1}^{N_m} \frac{\{\phi_m\}\{L_m\}^T}{i\omega - \lambda_m} + \frac{\{\phi_m\}^*\{L_m\}^H}{i\omega - \lambda_m^*} \tag{2}$$

and

$$\lambda_m = -\sigma_m + i\omega_{dm} \tag{3}$$

The modal model (2) expresses the dynamical behavior of the structure as a linear combination of N_m resonant modes. Each mode is defined by a damped resonant frequency, $f_{dm} = \omega_{dm}/2\pi$, a damping ratio, $\zeta_m = \sigma_m/|\lambda_m|$, a mode shape vector, $\{\phi_m\}$, and a modal participation vector, $\{L_m\}$. These modal parameters depend on the geometry, material properties and boundary conditions of the structure.

More recently, system identification techniques were developed to identify the modal model from the structure under its operational conditions using output-only data [2]. These techniques, referred to as operational modal analysis (OMA) or output-only modal analysis, take advantage of the ambient excitation due to e.g. traffic and wind. During an EMA, the structure is often removed from its operating environment and tested in laboratory conditions. The laboratory experimental situation can differ significantly from the real-life operating conditions. An important advantage of OMA is that the structure can remain in its normal operating condition. This allows the identification of more realistic modal models for in-operation structures. Frequency-domain output-only estimators start from power spectra. It can be shown that — assuming the operational forces to be white noise sequences — the power spectrum matrix or covariance matrix, $[S_X(\omega)] = \mathrm{cov}(\{X(\omega)\})$, satisfies

$$[S_X(\omega)] = \sum_{m=1}^{N_m} \frac{\{\phi_m\}\{K_m\}^T}{i\omega - \lambda_m} + \frac{\{\phi_m\}^*\{K_m\}^H}{i\omega - \lambda_m^*} - \frac{\{\phi_m\}\{K_m\}^T}{i\omega + \lambda_m} - \frac{\{\phi_m\}^*\{K_m\}^H}{i\omega + \lambda_m^*} \tag{4}$$

with $\{K_m\}$ the operational participation vectors, which depend on the modal participation vector, $\{L_m\}$, and the power spectrum matrix of the unknown operational forces.

1.2. Transmissibilities

In this paper attention will be paid to the use of transmissibilities to derive modal parameters [3]. Contrary to the classical approach of Section 1.1 no assumption about the nature of forces will be required. In general, it is not possible to identify modal parameters from transmissibility measurements. Transmissibilities, as used in this paper, are obtained by taking the ratio of two response spectra, i.e. $T_{ij}(\omega) = \frac{X_i(\omega)}{X_j(\omega)}$. By assuming a single force that is located in, say, the input degree of freedom (DOF) k, it is readily verified that the transmissibility reduces to

$$T_{ij}(\omega) = \frac{X_i(\omega)}{X_j(\omega)} = \frac{H_{ik}(\omega)F_k(\omega)}{H_{jk}(\omega)F_k(\omega)} = \frac{N_{ik}(\omega)}{N_{jk}(\omega)} \triangleq T_{ij}^k(\omega) \tag{5}$$

with $N_{ik}(\omega)$ and $N_{jk}(\omega)$ the numerator polynomials occurring in the transfer-function models $H_{ik} = \frac{N_{ik}(\omega)}{D(\omega)}$ and $H_{jk} = \frac{N_{jk}(\omega)}{D(\omega)}$. This situation is defined as the - Single Reference Single Input Case - (SRSI) with $X_j(\omega)$ the reference response. However in real operational conditions we mostly deal with a multiple input situation. In general an unknown number of simultaneous sources are exciting the structure, some of which may be distributed.

In this situation the transmissibility reduces to:

$$T_{ij}(\omega) = \frac{X_i(\omega)}{X_j(\omega)} = \frac{\sum_{k=1}^{n} H_{ik}(\omega)F_k(\omega)}{\sum_{k=1}^{n} H_{jk}(\omega)F_k(\omega)} = \frac{\sum_{k=1}^{n} N_{ik}(\omega)F_k(\omega)}{\sum_{k=1}^{n} N_{jk}(\omega)F_k(\omega)} \tag{6}$$

with n the number of forces applied to the structure. This last will be defined as the - Single Reference Multiple Input Case - (SRMI). Note that in both cases the common-denominator polynomial, $D(\omega)$, which roots are the system's poles, λ_m, disappears by taking the ratio of the two response spectra. Consequently, the poles of the transmissibility functions (5) and (6) equal the zeroes of transfer function $H_{jk}(\omega)$, i.e. the roots of the numerator polynomial $N_{jk}(\omega)$. So, in general, the peaks in the magnitude of a transmissibility function do not at all coincide with the resonances of the system. In Section 2 and Section 3 it will be shown that by combining transmissibility measurements under different loading conditions it is still possible to identify the modal parameters (i.e., resonant frequencies, damping ratios and mode shape vectors).

2. THEORETICAL RESULTS

2.1. Single Reference Single Input (SRSI)

In this case one can verify that the transmissibility function (5) does not depend on the amplitude of the force $F_k(\omega)$ and only depends on its location, indicated by k By making use of the modal model (2) between input DOF, k, and, say, output DOF, i,

$$H_{ik}(\omega) = \sum_{m=1}^{N_m} \frac{\phi_{im} L_{km}}{i\omega - \lambda_m} + \frac{\phi_{im}^* L_{km}^*}{i\omega - \lambda_m^*} \tag{7}$$

in the Laplace domain (obtained by replacing $i\omega$ by the Laplace domain variable s) one concludes that the limit value of the transmissibility function (5) for s going to the system's poles, λ_m, converges to

$$\lim_{s \to \lambda_m} T_{ij}^k(\omega) = \frac{\phi_{im} L_{km}}{\phi_{jm} L_{km}} = \frac{\phi_{im}}{\phi_{jm}} \tag{8}$$

and becomes independent of the location of the input DOF k of the (unknown) force. Consequently, the substraction of two transmissibility functions with the same output DOFs, (i, j), but with different input DOFs, (k, l) satisfies

$$\lim_{s \to \lambda_m} \left(T_{ij}^k(\omega) - T_{ij}^l(\omega) \right) = \frac{\phi_{im}}{\phi_{jm}} - \frac{\phi_{im}}{\phi_{jm}} = 0 \tag{9}$$

This means that the system's poles, λ_m, are zeroes of the rational function $\Delta T_{ij}^{kl}(\omega) \triangleq T_{ij}^k(\omega) - T_{ij}^l(\omega)$, and, consequently, poles of its inverse, i.e.

$$\Delta^{-1} T_{ij}^{kl}(\omega) \triangleq \frac{1}{\Delta T_{ij}^{kl}(\omega)} = \frac{1}{T_{ij}^k(\omega) - T_{ij}^l(\omega)} \tag{10}$$

2.2. Single Reference Multiple Input (SRMI)

Note that it is no longer possible to eliminate the forces $F_k(\omega)$, and so the transmissibility function (6) not only depends this time on the location but also on the amplitude of those forces. Never the less in equation (11) one can still verify that the above transmissibility functions for different forces (e.g. different locations of the forces, different number of forces or different amplitudes of the forces) still converge to the same unique value in the system's poles, and therefore the proposed technique is still applicable.

$$\lim_{s \to \lambda_m} T_{ij}(\omega) = \frac{\phi_{im} L_{1m}^T F_1(\omega) + \phi_{im} L_{2m}^T F_2(\omega) + ... + \phi_{im} L_{nm}^T F_n(\omega)}{\phi_{jm} L_{1m}^T F_1(\omega) + \phi_{jm} L_{2m}^T F_2(\omega) + ... + \phi_{jm} L_{nm}^T F_n(\omega)} = \frac{\phi_{im}}{\phi_{jm}} \tag{11}$$

Above remark shows the robustness of the method since it demonstrates that the procedure is not harmed by the fact that we have several forces exciting the structure simultaneously. These forces can be unknown and distributed. In order to use the transmissibility OMA procedure in operational conditions with a multiple input or distributed load it is sufficient that the loads change in position, amplitude or number of applied loads. These different loading conditions can be obtained by e.g a change in the ambient forces (e.g. change of wind-level/wind-direction) or in case of stationary ambient loads by adding artificial applied forces (e.g. impact hammer) in different locations.

3. EXTRACTING THE MODAL PARAMETERS

The correct system's poles can now easily be identified, by directly applying a frequency domain estimator [2] to the $\Delta^{-1}T_{ij}^{kl}(\omega)$ functions. In this way both the damped resonant frequencies and the damping ratios of each mode in the frequency range of interest will be obtained. However ,in general, only a subset of the poles of $\Delta^{-1}T_{ij}^{kl}(\omega)$ will correspond to the real system's poles. This can be verified by the fact that the rational function $\Delta^{-1}T_{ij}^{kl}$ can be rewritten as

$$\begin{aligned}
\Delta^{-1}T_{ij}^{kl}(\omega) &= \frac{1}{\frac{H_{ik}(\omega)}{H_{jk}(\omega)} - \frac{H_{il}(\omega)}{H_{jl}(\omega)}} \\
&= \frac{H_{jk}(\omega)H_{jl}(\omega)}{H_{ik}(\omega)H_{jl}(\omega) - H_{il}(\omega)H_{jk}(\omega)} \\
&= \frac{N_{jk}(\omega)N_{jl}(\omega)}{N_{ik}(\omega)N_{jl}(\omega) - N_{il}(\omega)N_{jk}(\omega)}
\end{aligned} \tag{12}$$

and so, the order of the polynomial, $N_{ik}(\omega)N_{jl}(\omega) - N_{il}(\omega)N_{jk}(\omega)$, can exceed the order of the common-denominator polynomial, $D(\omega)$. This means that $\Delta^{-1}T_{ij}^{kl}(\omega)$ can contain more poles than the system's poles only.

In order to reduce the risk to identify additional poles one can build a - virtual frequency response matrix - by using multiple transmissibility functions under different loading conditions.

$$[Hvirt(\omega)] = \begin{bmatrix} T_{ir}^k(\omega) & T_{ir}^l(\omega) & T_{ir}^m(\omega) \\ T_{jr}^k(\omega) & T_{jr}^l(\omega) & T_{jr}^m(\omega) \\ T_{or}^k(\omega) & T_{or}^l(\omega) & T_{or}^m(\omega) \end{bmatrix}^{-1} \tag{13}$$

To build the above matrix 9 different Transmissibility functions were used with 1 reference DOF, (r), and 3 output DOFs, (i, j, o), under 3 different loading conditions , (k, l, m).

The elements of this matrix are again rational functions with poles that correspond with the exact system poles. One can easily verify this. Consider the matrix in the case of 1 reference, 1 output DOF, 2 loading conditions and a second row equal to ones. The determinant of this matrix reduces to a above defined $\Delta^{-1}T_{ij}^{kl}(\omega)$ functions. By performing simultaneous a modal analysis on all elements of the above defined virtual frequency response matrix and imposing that all elements have a common denominator the real system poles can directly be identified.

In a second step it is also possible to obtain unscaled mode shape vectors by estimating the different transmissibility measurements $T_{ir}(\omega)$ (for $i = 1, \ldots, n$ with n the number of measured output DOFs and r a fixed chosen reference) and evaluate them in the above obtained system poles. This follows directly form equation (8). The value of the mode shape in the reference DOF is set to 1.

4. EXPERIMENTAL RESULTS

In order to investigate the efficiency of the proposed procedure, experiments have been done for a free-free beam shown in Figure 1. We can assume that, for small vibrations as encountered in our experiments, the system behaves linearly. 4 shakers were used to provide stationary noise excitation (from the left to right you see shaker 1 up to 4). A free run measurement of 40 seconds and a total of 10 Hanning windowed averages of 4096 discrete time samples have been taken for each experiment. The frequency resolution was 0.25Hz. A total of 11 accelerometers where equally distributed over the full length of the beam (the numbering starts from the left free end of the beam). In this paper modal parameters are identified with the least squares complex frequency domain estimator (lscf) [2]. A single reference case is employed; we will consider cross power spectra and transmissibility functions with reference to the first accelerometer only, leading to 10 cross power spectra and 10 transmissibility functions for each experiment. This experimental setup will allow us to identify the first 4 bending modes.

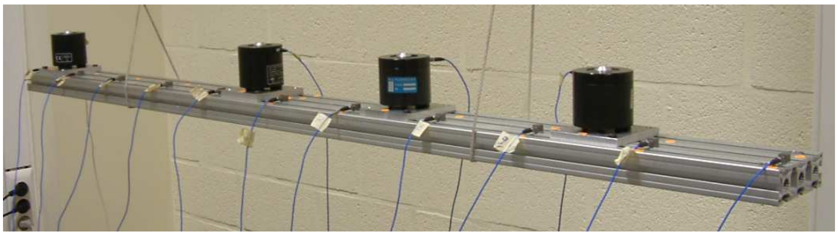

Figure 1. Experimental setup for the free-free beam structure

The necessary condition for applying the proposed transmissibility OMA approach is that the loading conditions are changing during the test. This can be achieved in several ways, as mentioned above (e.g. different locations of the applied forces, different number of forces or different amplitudes of the forces). In this experiment we will consider the last situation, 3 different loading conditions will be considered. Figure 2 shows the normalized amplitudes of the input signal of the 4 different shakers for the 3 different loading conditions.

Next, the transmissibility between different outputs is computed and compared in Figure 3 for the 3 different excitation conditions, T_{41}^{k}, T_{71}^{k} and T_{111}^{k} with $k = 1, \ldots, 3$. One notice that there are 4 frequencies where all 3 transmissibilities cross each other. These frequencies correspondent with the resonance frequencies of the first 4 bending modes of the beam, which is in agreement with the theoretical results of Section 2.

Starting from transmissibility measurements under different loading conditions, several $\Delta^{-1}T_{ij}^{kl}(\omega)$ functions — defined in (10) — can be computed. These functions together with

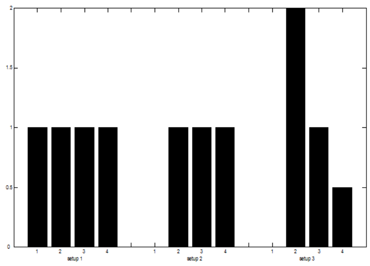

Figure 2. the normalized amplitudes of the input signal of the 4 different shakers used during the 3 different setups

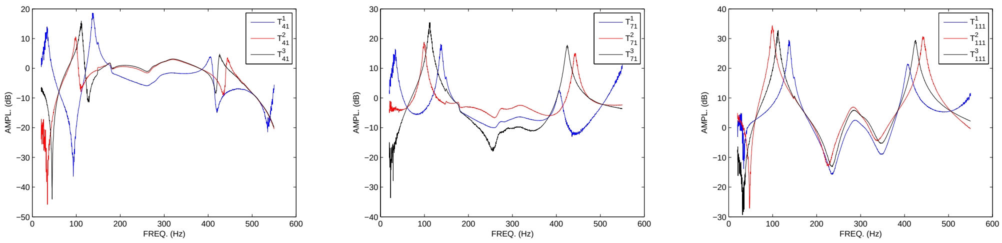

Figure 3. Transmissibility functions for the 3 different loading conditions with reference DOF 1 and output DOFs 4,7 and 11

their stabilization diagram, after applying the lscf estimator, are illustrated in Figure 4 for different combinations of the force locations k and l. It can be observed that most of the amplitude's peaks coincide with the resonant frequencies of the first 4 bending modes of the beam but additional stable poles are found. This is in agreement with the theoretical results where was stated that only a subset of the poles of the $\Delta^{-1}T^{kl}_{ij}(\omega)$ functions corespondent with the real system poles. This can easily be understood by the fact that only 2 transmissibility functions are used to calculate the $\Delta^{-1}T^{kl}_{ij}(\omega)$ functions and by looking to only 2 transmissibility functions in Figure 3 one notices that additional crossings occur and sometimes the transmissibility functions even coincide over a certain frequency band.

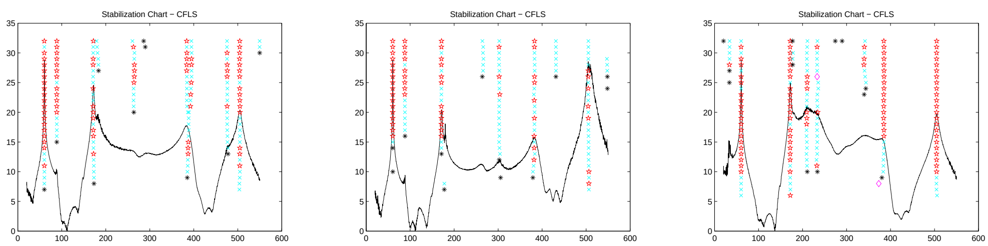

Figure 4. Transmissibility-based functions and their stabilization diagrams.

To reduce the risk to identify additional poles a different approach was proposed by calculating a virtual frequency response matrix . This new approach allows us to compare and evaluate more than 2 transmissibility functions at the same time.

The following virtual frequency response matrix was used.

$$[Hvirt(\omega)] = \begin{bmatrix} T_{71}^1(\omega) & T_{71}^2(\omega) & T_{71}^3(\omega) \\ T_{41}^1(\omega) & T_{41}^2(\omega) & T_{41}^3(\omega) \\ T_{111}^1(\omega) & T_{111}^2(\omega) & T_{111}^3(\omega) \end{bmatrix}^{-1} \tag{14}$$

The elements of this matrix are shown in Figure 5 and compared with the measured cross power spectra.

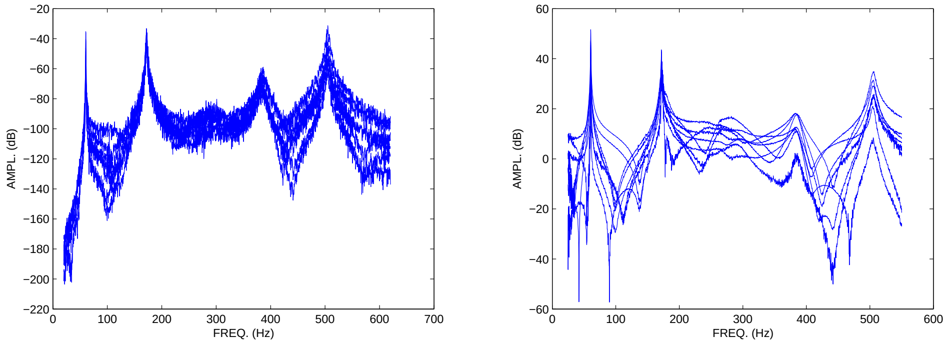

Figure 5. Cross power spectra (left) and elements of the virtual frequency response matrix (right).

By using the lscf estimator [2], on both the cross power spectra as well as the elements of the virtual frequency response matrix the modal parameters can be estimated. These functions together with their stabilization diagram are illustrated in Figure 6

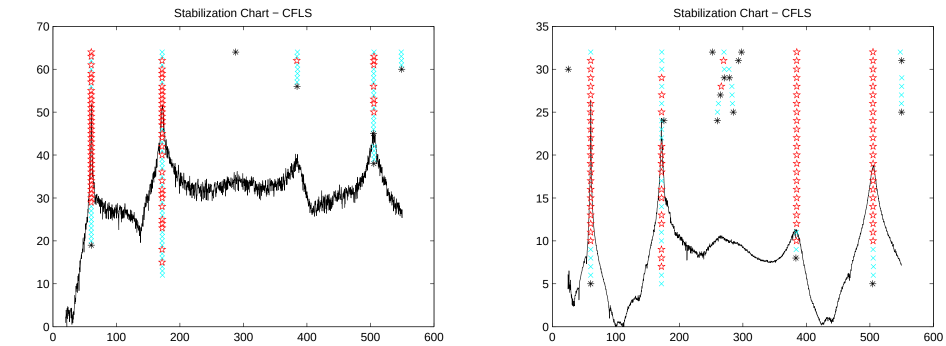

Figure 6. Stabilization diagram Cross power spectra (left) and the virtual frequency response matrix (right).

One notices that in this case the enhanced transmissibility approach results only in 4 stable poles that corresponded with the first 4 bending modes of the beam (Note: a lower modal order was needed to identify all 4 modes in the transmissibility approach in comparison with the classical OMA approach). In the frequency range of 250-300Hz the influence of a badly excited and badly measured first torsion mode is present (Both shakers and sensors are placed on the nodal line of this mode). The identified damped natural frequencies and damping ratios are summarized in Table 1.

Table 1. Comparison of the estimated damping ratios and damped natural frequencies obtained from the cross power spectra measurements and the transmissibility-based approach.

$\zeta(S)[\%]$	$\zeta(Hvirt)[\%]$	$f_d(S)$ [Hz]	$f_d(Hvirt)$ [Hz]
0.21	0.22	60.65	60.58
0.53	0.42	172.20	171.93
2.09	1.92	385.21	384.70
0.74	0.73	504.71	505.06

Once the poles are known, it is possible to derive the (operational) mode shapes directly from the transmissibilities measurements as was explained in the theory. The first 4 bending modes calculated by using transmissibilities measurements and the classical OMA appraoch are shown in Figure 4 together with their MAC values.

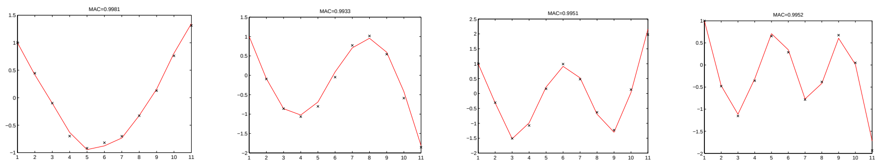

Figure 7. First 4 modes obtained by cross power spectra (x) and transmissibility approach (full line)

5. CONCLUSIONS

It has been shown in this paper that correct system's poles can be identified starting from transmissibility measurements only. A generalization of the technique to a multiple input assumption is proposed. An enhanced approach, by using a virtual frequency response matrix, was used in order to reduce the risk in identifying non physical poles. The theoretical results are verified by means of experimental data. Classical output-only techniques often require the operational forces to be white noise. This is not necessary for the proposed transmissibility-based approach. The unknown operational forces can be arbitrary (colored noise, swept sine, impact, ...) as long as they are persistently exciting in the frequency band of interest.

REFERENCES

[1] W. Heylen, S. Lammens, and P. Sas, Modal Analysis Theory and Testing. K.U.Leuven, Belgium, 1997.

[2] B. Cauberghe, Applied frequency-domain system identification in the field of experimental and operational modal analysis. PhD thesis, Department of Mechanical Engineering, Vrije Universiteit Brussel, Belgium, phd.avrg.be, 2004.

[3] P. Guillaume, C. Devriendt, and G. De Sitter, Identification of Modal Parameters from Transmissibility Measurements, 1st International Operational Modal Analysis Conference,April 2005; Copenhagen, Denmark.