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APPLICATION OF MODAL MODEL REDUCTION FOR HYBRID STRUCTURAL FINITE ELEMENT - ACOUSTIC WAVE BASED MODELS

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Abstract

This paper presents a newly developed hybrid simulation technique for coupled structural-acoustic analysis, which applies a wave based model for the acoustic cavity and a modally reduced higher order finite element model for the structural part. The resulting hybrid model benefits from the computational efficiency of the wave based method, while retaining the finite element method's ability to model the structural part of the problem in great detail. Application of this approach to the analysis of a cavity-backed plate assembly shows the improved computational efficiency as compared to classical finite element procedures and illustrates the potential of the hybrid method as a powerful tool for the analysis of coupled structural-acoustic systems.

1. INTRODUCTION

At present, the commonly used numerical simulation techniques for steady-state interior vibro-acoustic performance analysis are deterministic element-based methods, such as the Finite Element Method [1] (FEM). These methods use simple polynomial shape functions to approximate the dynamic variables within (small) elements of a discretisation of the problem domain. As wavelengths shorten with increasing frequency, the number of elements and subsequent computational efforts increase. As a result, the practical applicability of the element-based methods is limited to low frequency problems. Recently, the Noise and Vibration research group of the K.U.Leuven has developed an alternative deterministic simulation technique for the analysis of steady-state structural-acoustic problems. This so-called Wave Based Method [2] (WBM) has shown to be a highly efficient method for low- and mid-frequency analysis of problems of moderate geometrical complexity. In order to extend its applicability to problems of more complex geometries, a family of hybrid coupling approaches between the WBM and the FEM have been proposed [3, 4, 5]. A recent development involves a direct hybrid coupling between acoustic WB models and structural FE models, which is outlined in the present paper. Through the introduction of higher-order structural FE models and the use of classical modal reduction techniques for the structural FE models, a computationally efficient vibro-acoustic analysis procedure is obtained, as will be illustrated by means of a numerical validation example.

2. PROBLEM DEFINITION

The steady-state dynamic behaviour of a general 3D coupled structural-acoustic system, as shown in figure 1, is described by two physical variables: the acoustic pressure $p(\mathbf{r})$ in the internal acoustic cavity and the combined dynamic in-plane ($w_{x'}(\mathbf{r}')$, $w_{y'}(\mathbf{r}')$) and out-of-plane deformations ($w_{z'}(\mathbf{r}')$, $\theta_{x'}(\mathbf{r}')$, $\theta_{y'}(\mathbf{r}')$) in the structural domain Ω_s.

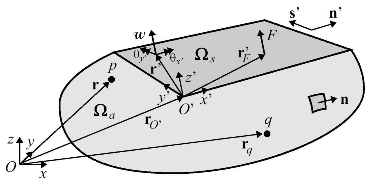


Figure 1. A 3D coupled structural-acoustic problem

2.1. Acoustic pressure field

The acoustic cavity V is filled with an acoustic fluid, with ambient fluid density ρ_a and speed of sound c and is excited at circular frequency ω by an acoustic point source q at position $\mathbf{r_q}(x_q, y_q, z_q)$. Under the assumption that the fluid in the cavity exhibits linear, inviscid and adiabatic behaviour, the Helmholtz equation governs the steady-state acoustic pressure $p(\mathbf{r})$ inside the cavity [6].One boundary condition needs to be specified at each point of the boundary in order to obtain a well-posed problem. On the parts Ω_p, Ω_v and Ω_Z of the boundary of the cavity respectively acoustic pressure, acoustic normal velocity or normal impedance boundary conditions are specified.

2.2. Structural vibrations

The part Ω_s of the acoustic boundary consists of a flexible plate assembly, made of a material with density ρ_s, Young's modulus E and Poisson ratio ν. The structure is excited harmonically by a point force F at position $\mathbf{r'_F}(x'_F, y'_F)$. Since the Helmholtz equation assumes the acoustic medium to be inviscid, the acoustic pressure only directly influences the out-of-plane dynamic deformations $w_{z'}(\mathbf{r}')$, $\theta_{x'}(\mathbf{r}')$ and $\theta_{y'}(\mathbf{r}')$ of the structure. To describe these deformations various plate bending theories are available. The Kirchhoff thin plate bending theory [7] is the most widely known, but the hybrid methodology applies equally well to other available plate bending theories, such as for example the more general Reissner-Mindlin theory [7]. The structural partial differential equation needs to be complemented with appropriate boundary conditions at each point on the edge Γ_s of the structural domain. Two of the possible types of boundary conditions are imposed kinematic and mechanical boundary conditions.

2.3. Structural-acoustic interactions

In a coupled structural-acoustic system, the acoustic pressure field and the structural displacements mutually influence each other as follows:

- **Influence of the structure on the acoustic pressure:** The vibrations of the plate act as a normal velocity excitation for the fluid in the cavity and result in an additional velocity continuity boundary condition for the fluid.
- **Influence of the acoustic pressure on the structure:** The acoustic pressure acts as a supplementary load on the structure. This load only affects the out-of-plane bending deformation

and results in an additional load term in the bending differential equations which is proportional to the acoustic pressure on the 'wetted' surface.

3. HYBRID MFE-WB METHOD

3.1. Modal FEM for uncoupled structural vibrations

The FEM determines an approximate solution to the problem described by the plate bending equations and the imposed structural boundary conditions by applying for each of the field variables an approximation of the type (1) in terms of simple polynomial shape functions $N_n(\mathbf{r}')$. These functions are locally defined within a discretisation of the problem domain into a large, but finite number of small, non-overlapping elements.

$$w_{z'}(\mathbf{r}') \approx \hat{w}_{z'}(\mathbf{r}') = \sum_{n=1}^{n_k} N_n(\mathbf{r}') w_{z',n} = \mathbf{N} \cdot \mathbf{w}_{\mathbf{z}'} \tag{1}$$

The polynomial shape functions do not fulfill the differential equations and may violate the imposed mechanical boundary conditions. The approximation errors are minimised by application of a Galerkin weighted residual formulation [1]. This results in a set of algebraic equations:

$$\mathbf{Z} \cdot \mathbf{d} = \mathbf{f_s} \tag{2}$$

with $\mathbf{Z}$ the structural dynamic stiffness matrix, $\mathbf{d}$ the vector containing the unknown nodal structural deformations and $\mathbf{f_s}$ the structural loading vector. Solution of (2) yields the deformation components in the nodes of the FE discretisation. The matrix $\mathbf{Z}$ is large, symmetric and sparsely populated and can be decomposed into frequency independent submatrices. These properties allow the use of very efficient solution algorithms to compute the unknown nodal displacements. A major advantage of the FEM is its versatility regarding geometrical complexity of the problem domain.

The application of the FEM for real-life engineering problems usually results in very large models, large memory requirements and long calculation times. However, the model sizes and subsequent computational efforts may be substantially reduced by using the modal reduction technique. In this approach, the displacement field $\mathbf{d}$ is written as a superposition of (some of) the normal modes $\mathbf{V_m}$ of the structure. The weighting factors of the different modes, the modal participation factors ψ_i, become the new unknowns of the model:

$$\tilde{\mathbf{Z}}_\mathbf{m} \cdot \Psi = \tilde{\mathbf{f}}_{\mathbf{s,m}} \tag{3}$$

with $\tilde{\mathbf{Z}}_\mathbf{m} = \mathbf{V_m}^T \cdot \mathbf{Z} \cdot \mathbf{V_m}$ and $\tilde{\mathbf{f}}_{\mathbf{s,m}} = \mathbf{V_m}^T \cdot \mathbf{f_s}$ the modally projected stiffness matrix and loading vector. A rule of thumb states that all modes with eigenfrequencies up to twice the maximum frequency of interest need to be taken into account to obtain an accurate prediction of the steady-state dynamic behaviour. Especially in the low- and mid-frequency range, where the modal densities are fairly low, this results in a significant reduction of the number of structural degrees of freedom. Moreover, since the system matrices become diagonal, the solution of the reduced system of equations for each frequency of interest requires very little computational effort.

3.2. WBM for uncoupled acoustic problems

The WBM, which is based on an indirect Trefftz approach, partitions the entire problem domain V into a small number of large, convex subdomains. Within these subdomains, the dynamic acoustic pressure $p(\mathbf{r})$ is written as a weighted sum $\hat{p}(\mathbf{r})$ of wave functions, which exactly satisfy the Helmholtz equation , but which may violate the imposed boundary conditions:

$$p(\mathbf{r}) \approx \hat{p}(\mathbf{r}) = \sum_{a=1}^{n_a} \Phi_a(\mathbf{r}) \cdot p_a + \hat{p}_q(\mathbf{r}) = \Phi \cdot \mathbf{p_a} + \hat{p}_q(\mathbf{r}) \tag{4}$$

with $\mathbf{p_a}$ an $(n_a \times 1)$ vector of unknown wave function contributions p_a and Φ an $(1 \times n_a)$ vector collecting the wave functions $\Phi_a(\mathbf{r})$. $\hat{p}_q(\mathbf{r})$ is a particular solution of the inhomogeneous Helmholtz equation For a detailed description of these functions used to describe the pressure in a bounded acoustic domain, the reader is referred to [2].

Since the pressure expansion (4) exactly satisfies the governing equation, the only error consists of the violation of the imposed boundary conditions. In order to obtain a numerical model for the n_a wave function contributions, this error is minimised by applying a weighted residual formulation, yielding a system of n_a equations in the n_a unknown wave function contributions p_a.

$$\mathbf{A_{aa}} \cdot \mathbf{p_a} = \mathbf{f_a} \tag{5}$$

Solution of this system for the unknown wave function contributions $\mathbf{p_a}$ and substitution of these results in (4) yields an approximation $\hat{p}(\mathbf{r})$ for the acoustic pressure response. The acoustic system matrix $\mathbf{A_{aa}}$ is fully populated with complex and frequency dependent elements. The major advantage of the WBM is the substantially smaller number of dofs required in comparison to the FEM. This property, combined with the enhanced convergence properties of the method, make the WBM a computationally more efficient simulation technique than the FEM and allow the WBM to be used for structural-acoustic analysis in the mid-frequency range. The requirement of convexity of the wave based subdomains imposes, however, a limitation to the practical applicability of the method for complex geometries.

3.3. Hybrid coupling strategy

In many practical examples (e.g., vibro-acoustic modelling of a car body), a complex structure and a geometrically simple acoustic cavity are in contact. An FE model is more suited to model the complex structure than the WBM. The acoustic cavity, on the other hand, can be modelled as a single (or a combination of a limited number of coupled) WB subdomain(s). As proposed by the authors in [8], the structural vibrations and the acoustic pressure field can be directly coupled in a hybrid (Modal) FE-WB model by enforcing the velocity continuity conditions on the acoustic model and by introduction of the acoustic pressure loading term in the structural equations. Combination of the residual formulations for both domains, application of a Galerkin approach and projection of the structural FEM model on a modal base yields the following matrix equation for the coupled vibro-acoustic system in terms of the wave function contributions $\mathbf{p_a}$ and the modal participation factors Ψ:

$$\left[\begin{array}{cc} \mathbf{A_{aa}} + \mathbf{C_{aa}} & j\omega\tilde{\mathbf{C}}^T_{\mathbf{as,m}} \\ \tilde{\mathbf{C}}_{\mathbf{as,m}} & \tilde{\mathbf{Z}}_{\mathbf{m}} \end{array}\right] \cdot \left\{\begin{array}{c} \mathbf{p} \\ \Psi \end{array}\right\} = \left\{\begin{array}{c} \mathbf{f_a} + \mathbf{f_{sa}} \\ \tilde{\mathbf{f}}_{\mathbf{s,m}} + \tilde{\mathbf{f}}_{\mathbf{as,m}} \end{array}\right\} \tag{6}$$

with $\tilde{\mathbf{C}}^T_{\mathbf{as,m}}$ the modally projected acoustic-structural coupling matrix and $\tilde{\mathbf{f}}_{\mathbf{as,m}}$ the modally projected structural-acoustic forcing vector due to the acoustic point source q. Until now, only first-order FE parts have been used for modelling the structural part in the hybrid methodology. However, the hybrid formulations (6) are not restricted to low-order models, but may also be applied for FE parts of higher order. The following section will discuss the application of second order (Modal) FEM models and discuss their performance.

4. NUMERICAL VALIDATION

4.1. Model description

The performance of the hybrid MFE-WB methodology is validated through the analysis of the coupled structural-acoustic problem shown in figure 4.1. The top boundary surfaces of a convex acoustic cavity consists of an assembly of three flat rectangular plates. The remaining boundaries of the acoustic cavity are acoustically rigid. The smallest rectangular bounding box enclosing the cavity has dimensions $L_x \times L_y \times L_z$= $1.5m \times 0.9m \times 1m$. The cavity is filled with air with an ambient fluid density $\rho_a = 1.1225\frac{kg}{m^3}$ and a speed of sound $c = 340\frac{m}{s}$. The three plates are made of steel ($E = 210GPa$,$\nu = 0.3$,$\rho = 7850\frac{kg}{m^3}$) and they have a thickness $t = 1.5mm$. The system is excited using a structural point force $F = 100N$, acting on the center plate at coordinates (x_F,y_F,z_F) = ($1m$,$0.6m$,$1m$).

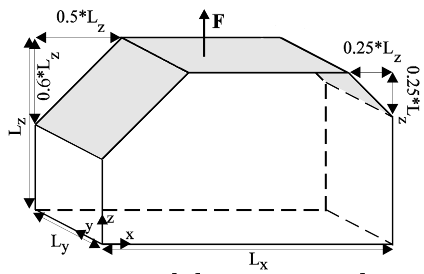


Figure 2. Validation example: cavity backed steel plate assembly

4.2. Coupled structural-acoustic models

The validation example in figure 4.1 is modelled using both pure FE models and hybrid WB-MFE models:

- **Pure FE models:** In order to study the convergence behaviour of the FEM both the structural plates and the acoustic cavity are modelled using linear and quadratic FEM models. The linear models combine 4-noded quadrilateral structural elements for the plates with 8-noded hexahedral elements for the acoustic cavity, while the quadratic FEM models are built using 8-noded quadrilateral structural and 20-noded hexahedral acoustic models. The details of the different quadratic FEM models, used in the comparison are given in table 1. h_{max} is the length of the longest side of a finite element in the discretisation and t_{solve} is the CPU time needed to solve the different models for a single frequency. $f_{max,a}$ and $f_{max,s}$ indicate the upper frequencies for which the FE models include at least 6 elements per acoustic or structural wavelength (rule of thumb). MSC.Nastran2005r2 is used as FE solver.
- **Hybrid WB-MFE models:** The hybrid models use the same structural parts as the pure FE descriptions. The acoustic cavity is modelled using a single acoustic WB domain. By varying the number of wave functions in the acoustic domain, a single structural FE model

Table 1. Properties of the quadratic FE models (structural QUAD8 + acoustic HEX20)

	$dofs_s$	$dofs_a$	h_{max} $[m]$	$f_{max,a}$ $[Hz]$	$f_{max,s}$ $[Hz]$	t_{solve} $[s]$
FE Quad 1	3768	8260	0.1000	1082	149	3.8
FE Quad 2	5622	16081	0.0750	1443	266	13.6
FE Quad 3	12990	63191	0.0500	2165	598	150.8
FE Quad 4	22494	139397	0.0375	2886	1063	695.4
FE Quad 5	34590	260459	0.0300	3608	1661	3408.9
FE Ref	139410	1513299	0.0150	7215	6643	/

can be used to create several hybrid models. All the quadratic structural models in table 1 are used to construct equivalent hybrid models. The number of modal vectors used in the structural base is determined according to the aforementioned rule of thumb, taking into account all structural modes up to twice the maximum frequency of interest. The routines to build and solve the hybrid and the associated WB models are implemented in Matlab 7.0. All calculations are performed on a 3GHz Intel-based Linux-system with 1 gigabyte of RAM.

4.3. Numerical results

To illustrate that the hybrid MFE-WB method accurately describes the vibro-acoustic coupling effects between the steel plates and the internal acoustic cavity, figure 3 shows a color map of the acoustic pressure and structural displacement amplitude at $150Hz$ obtained with a hybrid (left figure) and a FE model using direct dofs for both the structure and cavity (right figure). Both models use the same FE discretisation for the structure (quadratic FE model 5). The total time for solving the FE model is 3409 sec, while only 297 sec are needed to build and solve the hybrid model. The results show a good agreement between both models.

In the hybrid model, structural modes up to $300Hz$ (94 in total) are used in the modal base. If the same modal reduction is applied for the structure in the pure FE model, the solving time grows to 41114 sec. The same phenomenon has been observed by the authors in [8] for linear FE models combining a modal structural and a direct acoustic model. Based on this observation, the direct solving time is taken as calculation time for the pure FE models.

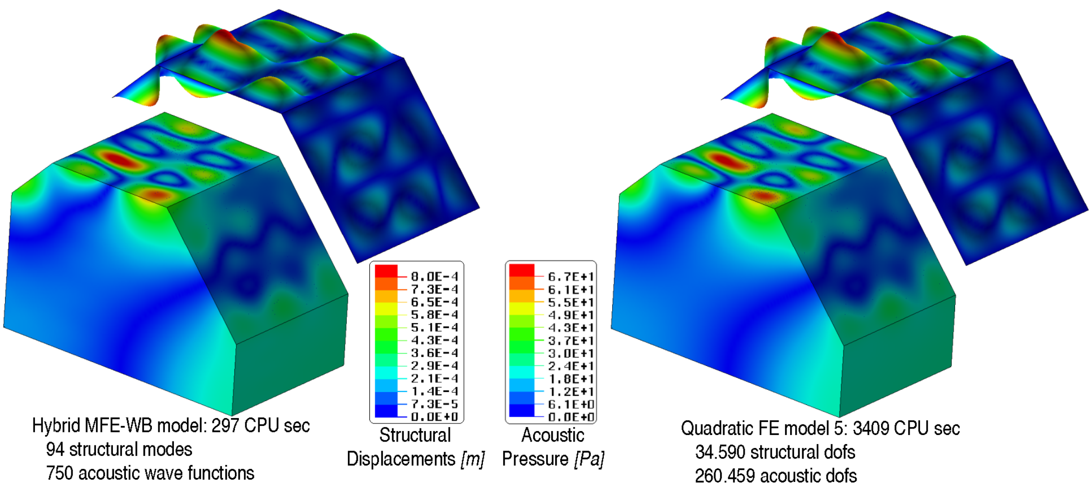


Figure 3. Color maps at $150Hz$ for the FEM (right) and the hybrid MFE-WB (left) method

In order to compare the computational efficiency of the hybrid method and the FEM a convergence analysis is performed. The acoustic pressure at $50Hz$ and $150Hz$ in 20 response points, uniformly distributed inside the acoustic cavity, is calculated for all the models described in section 4.2. The average relative prediction error with respect to the FE reference model listed in table 1 is plotted against the CPU times needed to solve the different models. Only frequency dependent operations are taken into account in the calculation time. For the FEM only the time needed to solve the system of equations is given. For the hybrid method the time needed to build the WB system matrix and the hybrid coupling matrices as well as the time needed to solve the system of equations are considered.

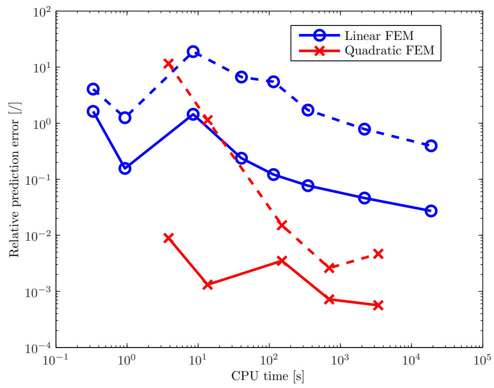
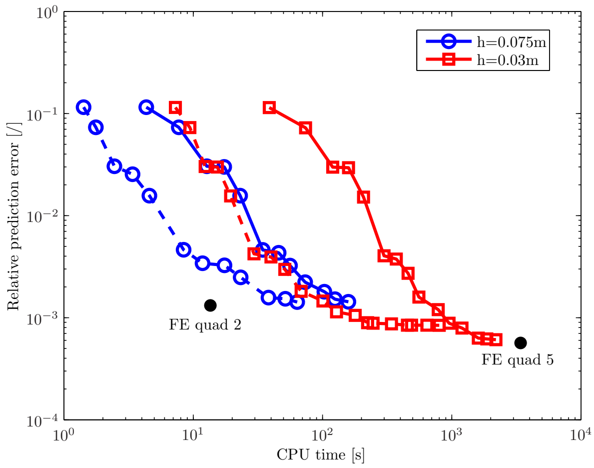
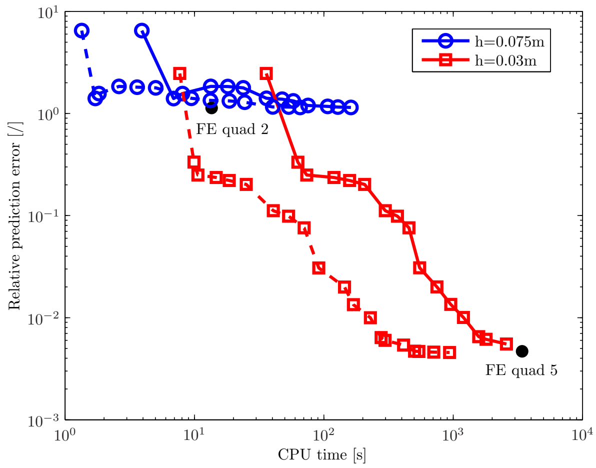


Figure 4. Convergence curves for linear and quadratic FEM at $50Hz$ (solid line) and $150Hz$ (dashed line)



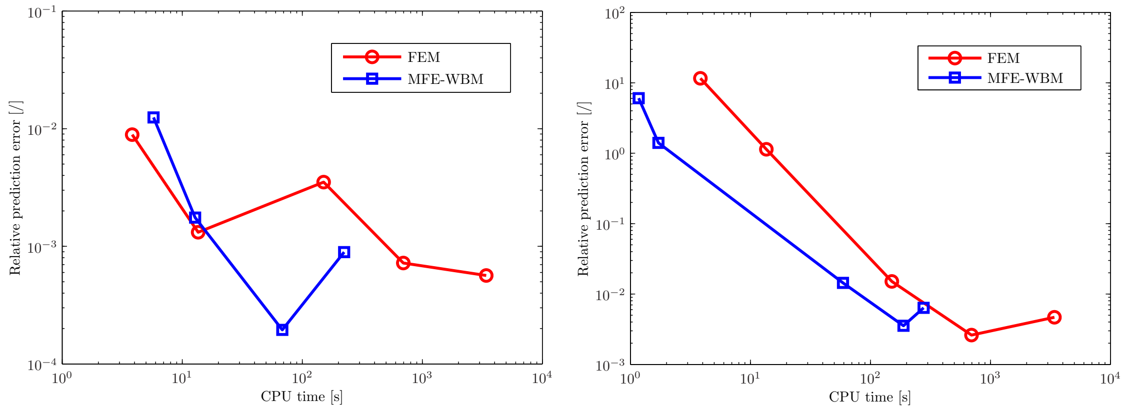
(a) $f = 50Hz$



(b) $f = 150Hz$

Figure 5. Convergence curves for the direct FE-WBM (solid line) and modal FE-WBM (dashed line)

Figure 4 compares the convergence rate of the different FE models. From this figure it is clearly seen that the quadratic models perform much better than their linear counterparts. This forms the basic motivation for the development of higher order hybrid methodologies. Figure 5 compares the convergence rate of different higher order hybrid FE-WB models applying a direct (FE-WBM, solid lines) and modally reduced (MFE-WBM, dashed lines) structural model with that of the corresponding quadratic FE models (• markers). The individual convergence curves for the hybrid models are calculated by combining a fixed structural model (using 30 modes at $50Hz$ and 94 modes at $150Hz$ for the MFE-WB models) with an increasing number of wave functions (ranging from 6 till 1376 acoustic dofs). The average structural FE mesh dimensions in the different hybrid models are $0.075m$ (○ marker, FE Quad 2) and $0.03m$ (□ marker, FE Quad 5). The convergence curves show that, as the number of wave functions increases, the prediction accuracy of the hybrid models increases steadily until some saturation is reached where the error remains constant. The saturation level is determined by the density of the structural FE mesh and it is similar to the error for a pure FE model with the same structural part. As frequency increases, the computational advantage of the hybrid methodology becomes more apparent. All the direct FE-WB models (except the most coarse one at $50Hz$) converge to the FE precision in a calculation time which is of the same order of magnitude as the pure FE predictions. The hybrid MFE-WB method converges much faster (up to a factor ≈ 10 for the model with $h = 0.03m$) to the same prediction accuracy. This is illustrated more clearly in figure 6, where the global convergence curves at $50Hz$ and $150Hz$ for the quadratic MFE-WBM are compared to those of the FEM. These curves are obtained by interconnecting the converged hybrid models using the different structural models, illustrating the effect of simultaneously refining both the structural and acoustic part of the models, like in the construction of the pure FE convergence curves.



(a) $f = 50Hz$ (b) $f = 150Hz$

Figure 6. Global convergence curves for the quadratic MFE-WBM and FEM

5. CONCLUSIONS

This paper describes a newly developed hybrid MFE-WB modelling technique for steady-state structural-acoustic problems. The motivation for the hybrid approach is the combination of the advantages of both techniques in a ’best of two worlds’-methodology. The complex structural part is described in great detail by the geometrically versatile FEM. The application of the WBM for the acoustic part results in favourable convergence properties. The hybrid method presented in this paper couples the higher order structural FE and acoustic WB models by directly enforcing the vibro-acousic interactions. The use of modal reduction techniques for the structural part results in a significant gain in CPU time while maintaining a comparable level of prediction accuracy for coupled structural-acoustic behaviour.

A comparison between the FEM and the hybrid MFE-WB method is made based on the analysis of a cavity-backed steel plate assembly. The results illustrate that the prediction accuracy of the hybrid models increases as the number of wave functions increases, until saturation is reached and the prediction error remains constant at a level similar to that of the FE predictions. The density of the structural FE model determines the maximum prediction accuracy. Especially for denser structural meshes, the hybrid method yields a higher accuracy in less computation time. These results illustrate the potential of the hybrid MFE-WB method as a powerful tool for the prediction of the dynamic behaviour of real-life coupled structural-acoustic systems.

Future research includes a further enhancement of the computational efficiency of the technique. Furthermore, the possibility of enforcing the structural-acoustic coupling in an indirect manner and the introduction of damping materials along the fluid-structure interaction surface will be explored.

6. ACKNOWLEDGEMENTS

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