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TWO MODELLING APPROACHES FOR PERIODIC RIB-STIFFEND PLATES TYPCIAL OF FLOOR ASSEMBLIES

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Abstract

Floors in lightweight framed construction are periodic orthotropic point connected plate-rib structures formed from orthotropic wood sheathing and a series of joists spaced at a regular interval. These structures are driven into motion by point excitations from people walking, dropping objects, etc. To accurately estimate flanking involving mechanical excitation one must predict the injected power using the mobility of the source and structure, and then estimate the resulting vibration distribution across the periodic floor structure. By examining two prediction approaches – SEA and an analytical solution for an assembly of finite sized plates – this paper shows that the more complex analytic solution gives a much better estimate of the vibration distribution, and is the only method capable of capturing the spatial dependence of the drive-point mobility. The analytic model is then used to identify the parameters of a joist floor that should be considered when designing a floor that has good low frequency impact sound insulation.

1. INTRODUCTION

This paper examines a periodic rib-stiffened plate and methods to predict the drive point mobility and velocity distribution. It integrates information presented in two earlier papers [1,2] and expands on the results and interpretations. Specifically, this begins by presenting a summary of an analytic model for orthotropic structures with arbitrarily spaced ribs. Measurements and predictions from the analytic model are compared to those from SEA where bending-only, and full-wave models are used. This gives insight into the possible transmission mechanisms at the plate-rib junctions and the limited importance of stop-bands at high frequencies.

Measured and predicted drive-point mobilities are examined next. While ordinary mobilities may provide a convenient upper and lower bound, they do not capture the spatial variation, which is very important in these structures. The analytic model presented shows good agreement with measured results. Finally, after demonstrating that statistical approaches do not adequately model a point excited periodic plate, the analytic model is used to conduct a preliminary parametric study to identify the obvious parameters to consider when designing for a joist floor for good low frequency impact sound insulation. Conclusions are then given.

2. THEORY

The ribbed plate is modelled as a number of plate elements coupled at a series of parallel junctions as shown in Figure 1. The plate to which the ribs are attached is considered to be formed from a series of smaller finite-sized plates defined by the plate/rib junctions. In this paper the portion of the plate between the ribs will be referred to as the “bay”. Thin plate theory is used so the effects of shear deformation and rotary inertia are not considered. Plate and ribs are treated similarly. Both are considered plate elements having a vibration response that is described by a series expansion assuming simply supported boundary conditions along the plate edges perpendicular to the junction line shown in Figure 2.

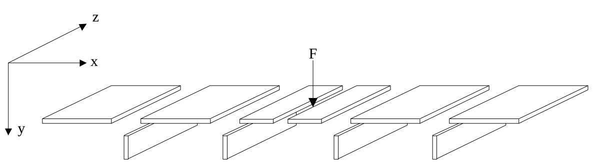

Figure 1: A ribbed plate is modelled as finite-sized plates coupled at a number of parallel plate junctions.

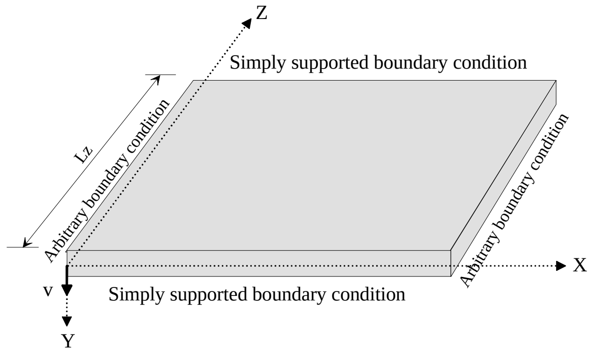

Figure 2: Boundary conditions applied to each plate.

Uncoupled edges of the plate and ribs parallel to the z-direction are assumed to have free boundary conditions. For a plate element parallel to the x-z plane, the solution is obtained by substitution of Eqn(1) into the equations of motion [3], [4]

$$v(x,z,t) = \sum_{n=1}^{\infty} A_n e^{jk_n x} \sin\left(\frac{n\pi}{L_z} z\right) e^{j\omega t} \tag{1}$$

In the Eqn(1), v represents the plate displacement, A_n the complex amplitude, k_n the wavenumber in x-direction and L_z the plate width in z-direction. In the model each junction consists of two plates and a stiffening rib coupled by a junction beam. The boundary conditions at the junctions are described by equilibrium and continuity conditions for the equations of motion and are identical to those given elsewhere [5].

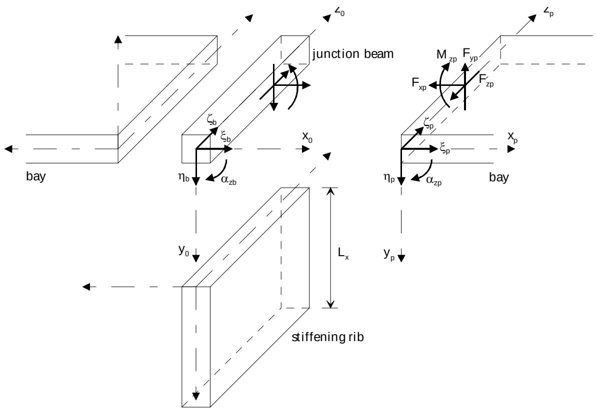

Figure 3: Joint sketch showing forces, moments, and displacements for the boundary conditions.

The plate structure is driven by a point force normal to the plate surface, and as shown in Figure 1, the excitation is taken into account by introducing an additional junction at the driving point location. This helps evaluation of displacement and continuity conditions at the source. This modeling approach will be referred to as “semi-modal finite plate” approach.

3. MEASUREMENT SETUP

A simple well-defined structure was chosen for the initial evaluation of the model. Both the plate and the ribs were cut from Plexiglas, which is a homogeneous and isotropic material with well-characterized material properties [4]. The plate has dimensions 2.42x1.21m with a thickness of 11.9mm. Plexiglas ribs, 18.7 mm thick, nominally spaced 40 mm on center, divided the 1.2x2.4 m plate into bays as shown in Figure 4 and Figure 7. Sixteen equally spaced bolts fastened each rib to the plate which reasonably approximate a line connection.

The boundary conditions assumed in the model formulation can be difficult to implement for the evaluated construction. The easiest to satisfy is the requirement for a free boundary at the uncoupled edge of a plate parallel to the global z-direction. This is achieved by mounting the plate vertically as shown in Figure 4. The simply supported boundary conditions for edges parallel to the x- and y-direction are difficult to achieve in practice. Differences in the boundary condition are discussed elsewhere [2], but will be most important at the low frequencies where there are low order modes.

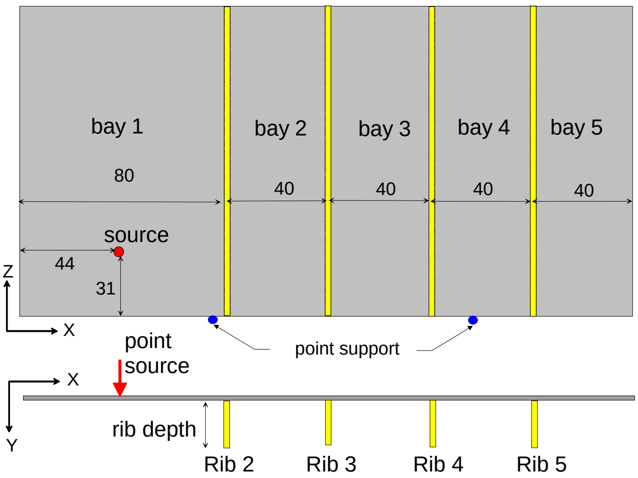

Figure 4: Sketch showing the periodic structure used to evaluate the model. All dimensions are given in centimetres.

4. MEASURED AND PREDICTED RESULTS

4.1 Velocity Level Difference

In this section the accuracy of the semi-modal finite plate approach, and SEA predictions is gauged by comparing the measured and predicted velocity level difference (VLD) between the source bay (Bay 1) and each of the four receiving bays (Bays 2 through 5). In this paper we restrict ourselves to a rib depth of 235 mm; data for additional rib depths are available elsewhere [1]. Here two SEA predictions using proprietary SEA software [6] are presented – one with all wave types considered, and the other where only bending is considered. In both cases the ribs are treated as plates.

Examining the error in the prediction for the VLD between Bay 1 (source bay) and Bay 2, shown in Figure 5, indicates that none of the methods have bias and all exhibit good agreement, although the finite plate model is slightly better. The important thing to note is that including wave types other than bending in the SEA model does not appreciably increase the accuracy of the VLD prediction between Bays 1 and 2. This is because attenuation at the first plate/rib junction is primarily due to bending.

Examining the errors in the predicted VLD for the other model it can be seen that the SEA prediction exhibits a bias – the VLD is consistently overestimated when only bending waves are considered. With four junctions separating the source and receiver bays the error is typically 20 dB or more. Whereas, the SEA prediction that includes all wave types exhibits much better agreement with a much smaller bias toward overestimation. This strongly suggests that there is high bending to bending attenuation at each junction, which accounts for the high VLD at the first junction. At successive junctions wave conversion from in-plane to bending plays prominently and is the dominant source for bending waves several junctions away from the source. This has previously been observed [7] in buildings when there are many junctions.

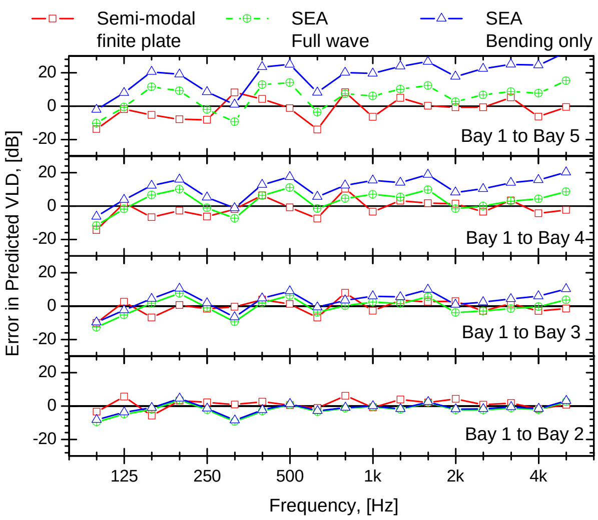

Figure 5: Error in the prediction over the building acoustics frequency range 100 through 5000 Hz. Data are for 235 mm deep ribs.

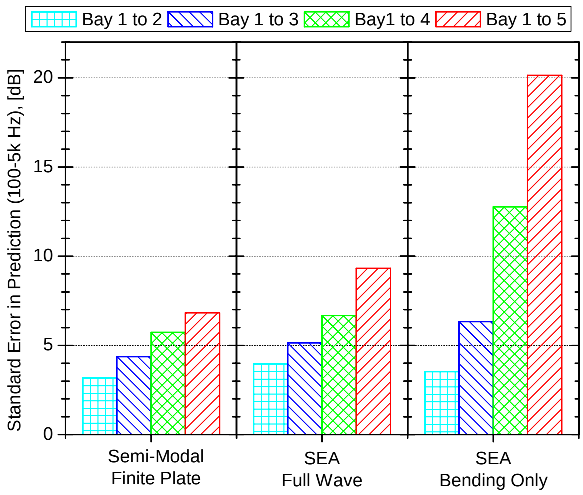

Figure 6: Standard error in the predicted VLD for the three modelling approaches when predicting the case with the 235mm deep ribs.

Figure 6 shows the standard error in the prediction for the range 100-5000 Hz for all three models. From the figure it is evident that for all modelling approaches errors increase with increasing number of junctions between the source and receiving bays. The figure also shows that the semi-modal finite plate approach provides the most accurate prediction. For this method the standard error is dominated by the low frequencies where the assumed boundary conditions are not fully realized.

4.1 Drive Point Mobility

The Plexiglas plate-rib assembly used for velocity measurements was also used to evaluate the drive point mobility as a function of location. First, to be evaluated experimentally is the dependence of the drive point mobility on the location of excitation relative to the ribs.

The mobility was measured in turn at each of the nine points (labeled A through I) shown in Figure 7 using a Wilcoxon F4 integrated shaker and impedance head. Estimates were corrected for the effect of the mass below force gauge (which is dominated by the mass of the mounting screw). This paper only considers the real part of the mobility because this is the component associated with power injection, and agreement between measured and predicted results were similar for both real and imaginary parts.

Figure 8 shows that the real part of the drive point mobility is a function of location. It is important to note that there are four measurement pairs (A&I, B&H, C&G, and D&F) which are symmetric about position E and have nominally identical distances (0, 5, 10, and 15 cm) to the nearest adjacent rib. There is no such symmetry and common distance to the perimeter of the ribbed-plate.

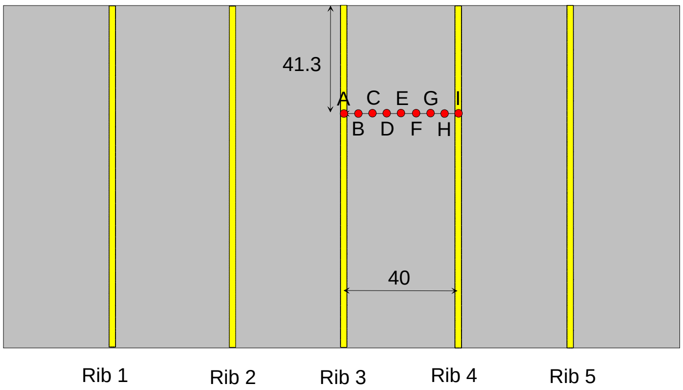

Figure 7: Sketch of the periodic structure used to evaluate the model. All dimensions are given in centimetres.

Important observations can be made from the results shown in Figure 8:

- Mobility is very similar for positions that have nominally identical distances to an adjacent rib. Distance to an adjacent rib is the dominant factor in determining the mobility – the distance to the plate edge is much less important.
- Mobility of points near the centre of the bay (more than 15 cm from a rib) is reasonably approximated by that of an uncoupled infinite plate of the same thickness.
- Mobility decreases as the distance to an adjacent rib decreases.
- Mobility for points immediately above a rib is reasonably approximated, below about 1000 Hz, by that of an uncoupled infinitely long beam of similar depth and width.

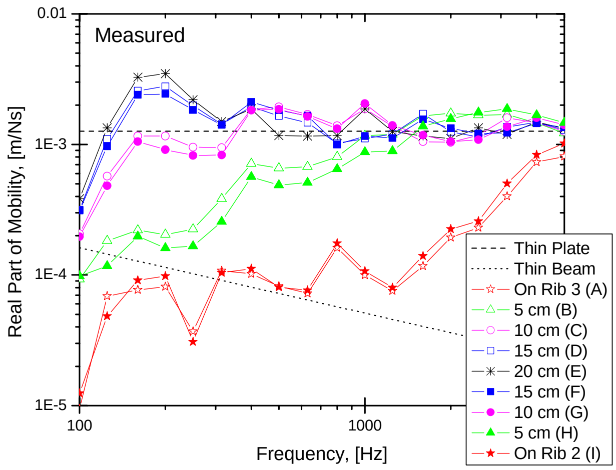

Figure 8: Measured mobility as a function of distance from a 235mm deep rib. Shown for comparison are ordinary mobility estimates.

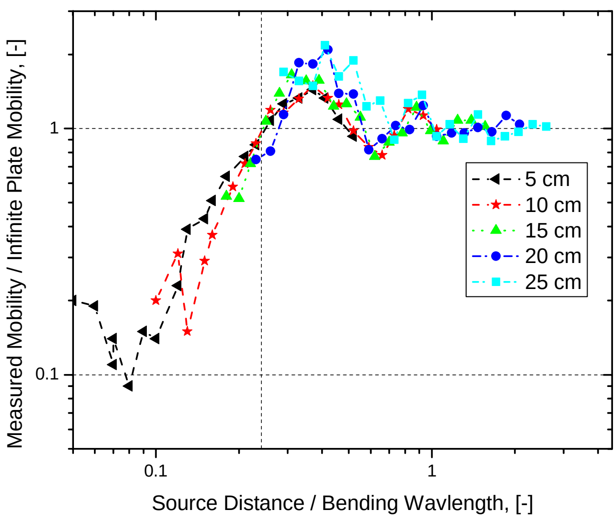

Figure 9: Normalised mobility as a function of the non-dimensional distance between the drive point and the 235mm deep rib.

To examine the importance of source position relative to a stiffening rib the mobilities of Figure 8 were normalized by that of an infinite plate, and plotted as a function of the source distance normalized by the bending wavelength. The results are shown in Figure 9. There are two very important observations.

- When the ratio of the source distance to bending wavelength is greater than unity the ribs have minimal effect – the plate can be considered as an infinite uncoupled plate.
- When this ratio is less than one-quarter (0.25) the ribs play prominently – the mobility is considerably less than that of an uncoupled infinite plate.

Lin and Pan [8] also observed these from theoretical calculations. For small enough values the mobility, as shown by Figure 8, will tend to that of a rib. For ratios between ratios 0.25 and 1.0, the normalized mobility will oscillate about unity (first greater than that of the infinite plate and then less, and so on) as it converges to unity.

Figure 10 and Figure 11 show the semi-modal finite plate approach correctly predicts the magnitude and the trends of the mobility when the drive point is 20 and 5 cm from a rib, respectively. Results of other positions [1] show similar good agreement.

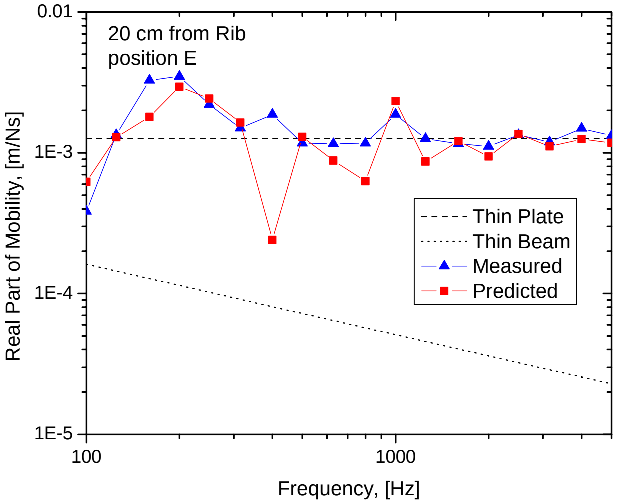

Figure 10: Measured and predicted mobility 20 cm from a 235mm deep rib. Shown for comparison are ordinary mobility estimates.

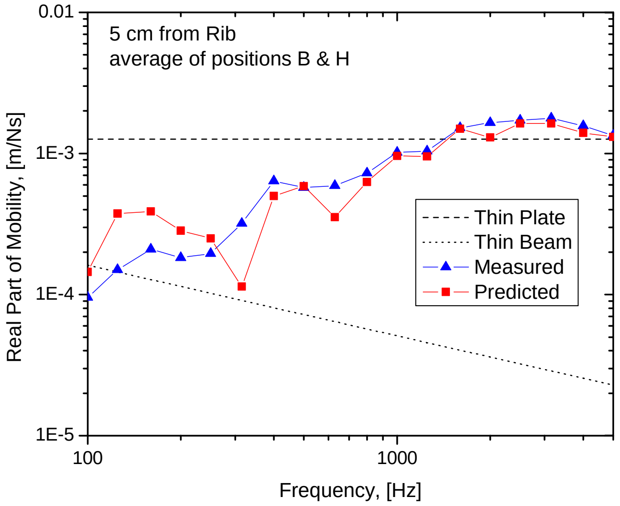

Figure 11: Measured and predicted mobility 5 cm from a 235mm deep rib. Shown for comparison are ordinary mobility estimates.

5.1 Parametric Study of Mobility

This section presents results of a preliminary study to assess the relative importance of various parameters on the power injected by a point force and the resulting space average velocity of the rib-stiffened plate. Predictions are divided into categories. First, changes made to the ribs. Second, changes made to the plate. In all cases only one parameter is changed at a time, so that results can be compared to a reference case, which is the assembly of Figure 9 without modification.

Figure 12 indicates that the real part of the drive point mobility (at position E of Figure 7) is not significantly affected by doubling of any of the following: rib width, rib depth, rib mass, or rib damping. The factor that has a significant effect is rib spacing. Halving the spacing (which is equivalent to doubling the number) reduces the distance to the nearest rib so that the ratio of source distance divided by bending wavelength of frequencies below about 400 Hz is less than 0.25 and the drive-point mobility is now controlled by the rib, (Figure 9). This causes a pronounced reduction in plate mobility and the same force injects less power.

Figure 13 indicates that changes to the plate may result in large changes. Here an 8-fold increase in stiffness of the plate (caused by an 8-fold increase in the modulus of elasticity) causes a reduction in the mobility at almost all frequencies. This 8-fold increase in the stiffness could also have been obtained by doubling the thickness of the plate, but the mass would have increased, too.

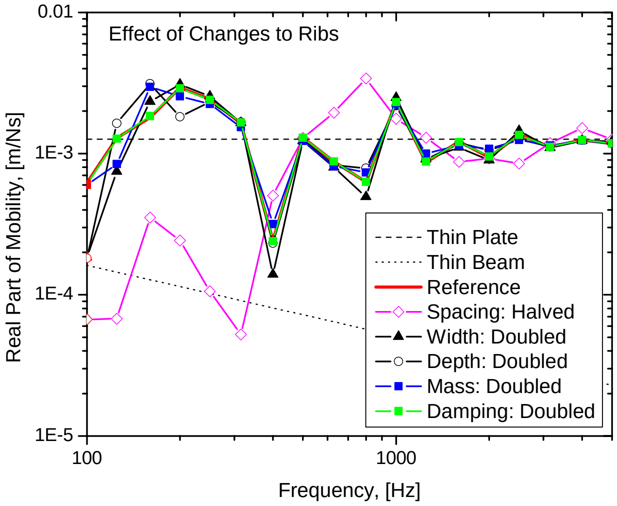

Figure 12: Effect of changes made to the ribs on the real part of the mobility.

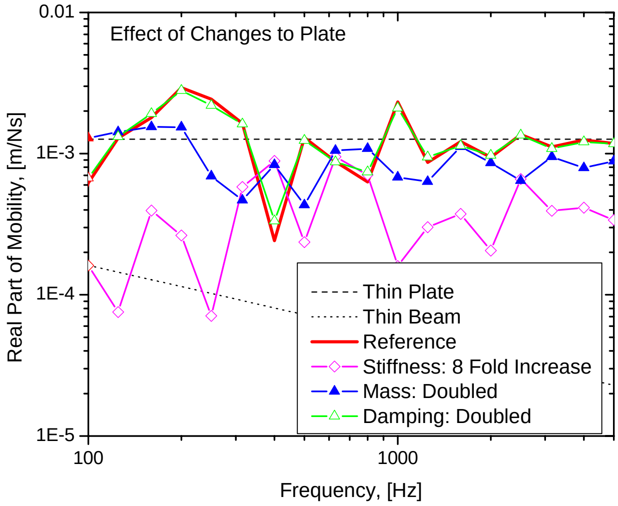

Figure 13: Effect of changes made to the plate on the real part of the mobility.

The figure shows that doubling the mass (by changing the volume density) of the plate results in a reduction of the mobility (and injected power). Thus, it would be most beneficial to increase both the mass and the stiffness, which is often done in real floors by using subfloor material that is thicker and has no slippage between the plys, if present. Increasing the damping of the plate does not appreciably affect the mobility and hence the injected power, when averaged over many frequencies that are well above the fundamental mode. To decrease the injected power one should increase the number of ribs, stiffness and mass of the plate.

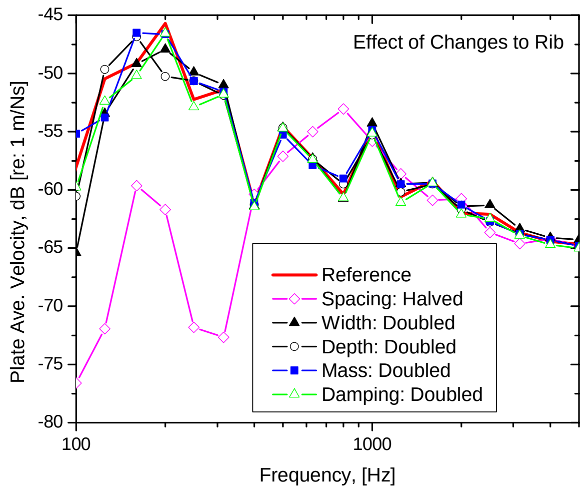

Figure 14: Effect of changes made to the ribs on the average velocity of the plate.

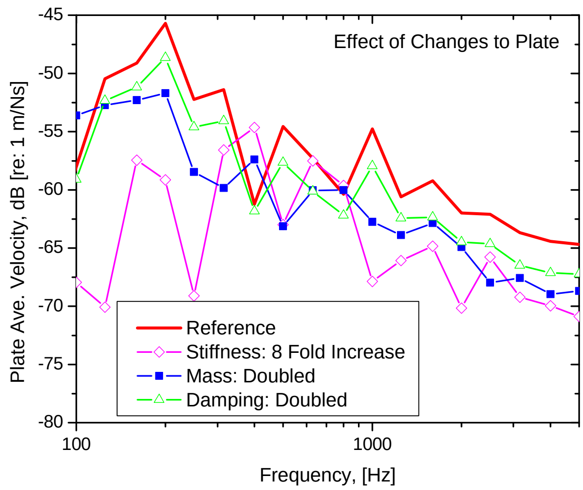

Figure 15: Effect of changes made to the plate on the average velocity of the plate.

In most cases it is the space average velocity of the plate that is of primary interest as this relates the energy available for transmission to structurally or acoustically coupled systems. Figure 14 indicates changes to the ribs have little effect, except for the spacing (or number) of the ribs. The prediction suggests that below 400 Hz doubling the number of ribs (by halving the spacing) will result in about a 20 dB reduction in the space average velocity of the plate. Above 400 Hz, the effect of joist spacing is minimal.

Figure 15 indicates increasing the stiffness, damping or mass of the plate will significantly reduce space average velocity at most frequencies. Above 400 Hz, doubling the mass is about as effective as doubling the damping, but neither is as effective as an 8-fold increase in plate stiffness, for which a 10 to 20 dB reduction in the space average velocity of the plate was indicated below 400 Hz. It should be noted that the effects shown typically will not be additive.

6. CONCLUSIONS

Measured and predicted VLD's showed typically more attenuation at the first junction than at subsequent junctions. This is because of weak bending to bending transmission at each junction while there is significant wave conversion from in-plane to bending due to the eccentric nature of the junctions. Models assuming pure bending significantly overestimate the VLD when the source and receive bays are separated by several junctions. Surprisingly, a full wave model of the junction used in an SEA model produced quite good results which suggests that stop bands, while obviously present in this periodic structure, are not overly important. This might be explained by noting that a stop band relates to the trace wavenumber so in a reasonably diffuse field there will be a limited number of waves (in any one third-octave band) that will satisfy the necessary conditions, so the effect is minimal for band-averaged results.

Measured results show the mobility of an uncoupled plate or beam provide reasonable upper and lower bounds on drive-point mobility. The actual value is a function of the bending wavelength and the distance from the drive point to the nearest rib. Both measurements and predictions show that if the ratio of the distance to the bending wavelength was less than 0.25 the rib will significantly lower the mobility. For ratios greater than 0.25 the mobility would be controlled by that of the plate and, for practical purposes, the mobility could be approximated by that of an uncoupled infinite plate of the same thickness.

Results of a preliminary parametric study suggest indicate the most obvious parameters to consider when designing floors for good low frequency impact sound insulation are joist spacing (minimize), plate stiffness (maximize), plate mass (maximize), and plate damping (maximize), probably in this order. Joist spacing and plate stiffness appear to be the most promising approaches for controlling the low frequencies in real buildings because adding substantial mass can introduce stability problems, especially in high seismic velocity zones.

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