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SYSTEM IDENTIFICATION IN STOCHASTIC DOMAIN USING SYSTEM OUTPUT ONLY

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Abstract

Most of the study on system identification has been carried out using input/output relation in physical domain. Generally relation between input and output in physical domain gives important information to identify the system. However identification concept of stochastic system has not been reported up to now. Interest is focused to identify an unknown stochastic dynamic system under random external disturbances which is not measurable. A concept to identify the system parameters in stochastic domain is proposed and implemented in terms of simulation and experiment as well. Previously developed stochastic transformation method via F-P-K (Fokker-Planck-Kolmogorov) equation is adopted to estimate the system output. Based on the system output only, it is attempted to identify the system parameters in inverse manner in stochastic domain. Both simulation and physical experiment are conducted to reveal quite noticeable expected performance of the proposed concept.

1. INTRODUCTION

Generally analytical model can be constructed via mathematical formulation based on physical laws. Reasonable modeling for system is essential in system design and evaluation of the system performance. However improper or excessive assumption in dealing with physical law upon the system under consideration and incorrect information of system parameters possibly cause undesirable system response and unexpected system performance. Thus accurate estimation of the system parameter should be made to show reliable performance.

In order to resolve those problems lot of attempt has been done to make mathematical model close to real system. Especially system identification is quite important to control the system exposed to immeasurable random external disturbance. In the study, the method for system identification in stochastic system using random output only is proposed, which is quite different from conventional method.

2. SYSTEM IDENTIFICATION

2.1 System Modeling in Stochastic Domain

A physical system defined in equation (1) is adopted as a system under random disturbance.

$$\begin{aligned} \dot{\mathbf{x}} &= A\mathbf{x} + B\mathbf{u}(t) \\ \ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x &= f(t) \end{aligned} \tag{1}$$

where A : system matrix,
B : input matrix,
$\mathbf{x}$: state vector,
$\mathbf{u}$: input vector,
ς : damping ratio,
ω_n : natural frequency,
x : displacement,
$\dot{x}$: velocity,
$\ddot{x}$: acceleration,
$f(t)$: irregular disturbance which is not possible to measure

Figure 1. Conceptual System

Figure 1. shows concept of black box-type system under consideration in the study.
The system equation (1) in physical domain is transferred to dynamic moment equation in stochastic domain via F-P-K equation [1], [2].

$$\dot{m} = A_m m + P_m D_Z + B_m D_V \tag{2}$$

$$\dot{m} = A_m m + P_m D_Z \tag{3}$$

D_Z is the PSD of disturbance and D_V is the PSD of control input. Since no control is involved in the study, system equation (3) is used mainly after worth.
System matrix in stochastic domain is derived as in equation (4).

$$A_m = A(\omega_n,\ \zeta) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\omega_n^2 & -2\zeta\omega_n & 0 & 0 & 0 \\ 0 & 0 & -2\zeta\omega_n & -\omega_n^2 & 1 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & -2\omega_n^2 & 0 & -4\zeta\omega_n \end{bmatrix} \tag{4}$$

2.2 System Identification Method

When immeasurable disturbance applied to the system with unknown parameters, natural frequency ω_n and damping ratio ς, the 2nd order moment equation for the system can be rewritten as following equation (5).

$$\begin{bmatrix} \dot{m}_{11} \\ \dot{m}_{20} \\ \dot{m}_{02} \end{bmatrix} = \begin{bmatrix} -2\zeta\omega_n & -\omega_n^2 & 1 \\ 2 & 0 & 0 \\ -2\omega_n^2 & 0 & -4\zeta\omega_n \end{bmatrix} \begin{bmatrix} m_{11} \\ m_{20} \\ m_{02} \end{bmatrix} + \begin{bmatrix} 0 \\ D_Z \\ 0 \end{bmatrix} \tag{5}$$

where $$m_{k_1,k_2,\cdots,k_n} = \int \cdots \int_{-\infty}^{+\infty} X_1^{k_1} X_2^{k_2} \cdots X_n^{k_n}\, p(X,t)\, dX_1 dX_2 \cdots dX_n \tag{6}$$

$p(\underline{x},t)$: non-stationary p.d.f.(probability density function)

Based upon previous research, relation between system random output and moment with system parameters is shown with equation (5) to gives equation (7) [3].

$$\omega_n,\ \zeta = \mathrm{A}^{-1}(m,\ \dot{m}) \tag{7}$$

Finally system parameters could be extracted based on previous analysis; following equation (8) is derived in terms of system dynamic moment response.[4]

$$\begin{aligned} \omega_n &= \sqrt{\frac{2\dot{m}_{11}m_{02} - \dot{m}_{02}m_{11} - 2m_{02}^2}{2(m_{02}^2 - m_{20}m_{02})}} \\ \zeta &= \frac{\dot{m}_{02}m_{20} - 2\dot{m}_{11}m_{11} - 2m_{11}m_{02}}{4\omega_n(m_{02}^2 - m_{20}m_{02})} \end{aligned} \tag{8}$$

3. SIMULATION AND EXPERIMENT

3.1 Numerical Simulation

As is seen in the Figure 1, system with unknown parameters is exposed to immeasurable irregular disturbance shown in Figure 2.

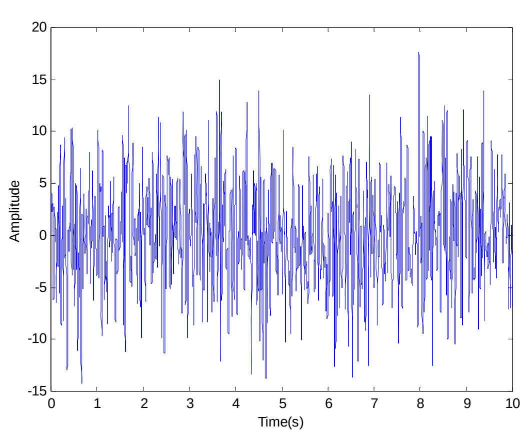

Figure 2. Unmeasurable Random Input

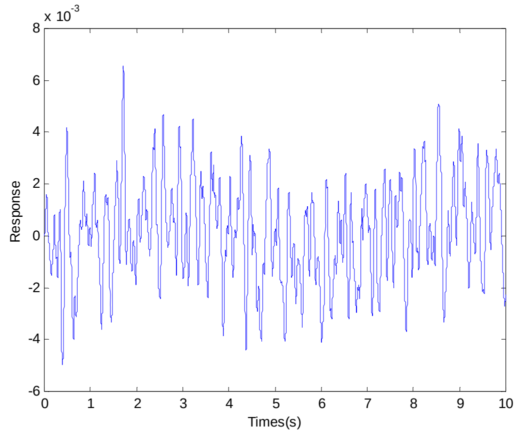

Figure 3. Random System Output

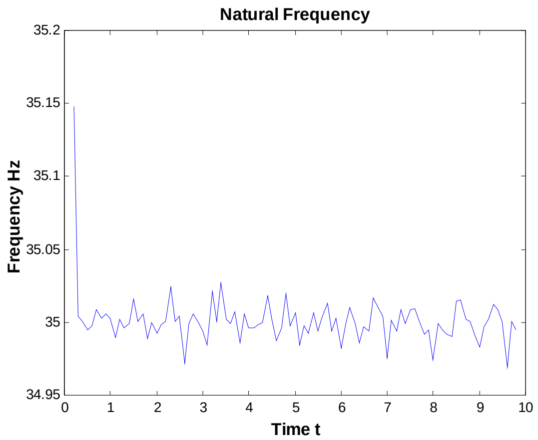

Figure 4. Natural Frequency (ω_n)

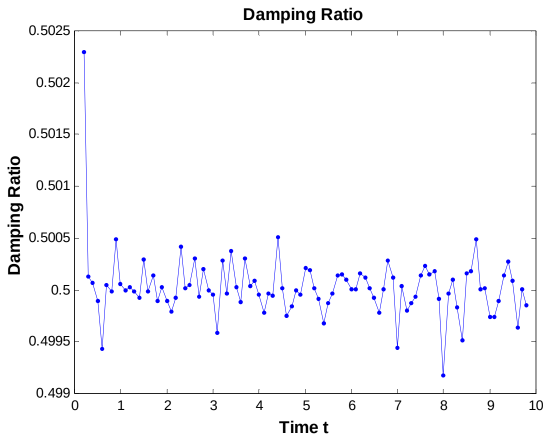

Figure 5. Damping Ratio (ζ)

In simulation, natural frequency ω_n is fluctuating around $35 \pm 0.03 H_z$ and damping ratio ς is also 0.5 ± 0.0004 under stationary response as is in Figure 5 and 6.

3.2 Experiment

Physical experiment is conducted using the system as is in Figure 7. The parameters of dynamic system are regarded as unknown ones. Then system parameters are extracted using equation (8) reveals certain value of m, c and k. System parameters used in experiment is listed in the Table 1.

Table 1. Parameters

Parameter	Value
Mass(kg)	0.45
K(Kg/m)	490
Natural frequency(Hz)	31.6

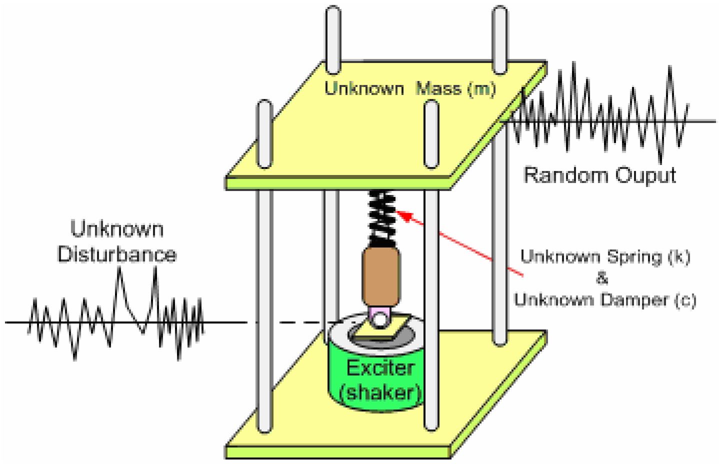

Figure 6. Experiment Model for unknown System

In experiment, the displacement and velocity is measured using laser sensor.

There are discrepancy between the mathematical result and experimental result in natural frequency ω_n and damping ratio ς as are shown in Figures (4), (5), (7), and (8). Nevertheless orders of the results are same and those two values are very close in certain range.
Experimental results are compared to those of analytical study as shown in Figures (9), (10).

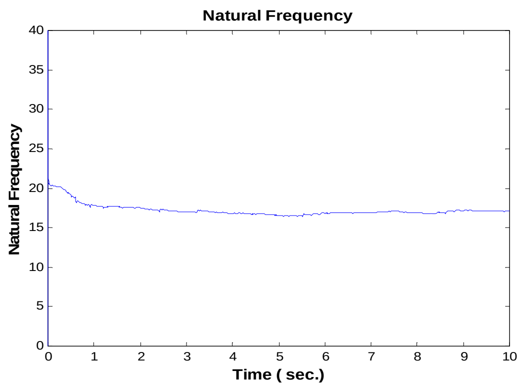

Figure 7. Natural Frequency(ω_n) from Experiment

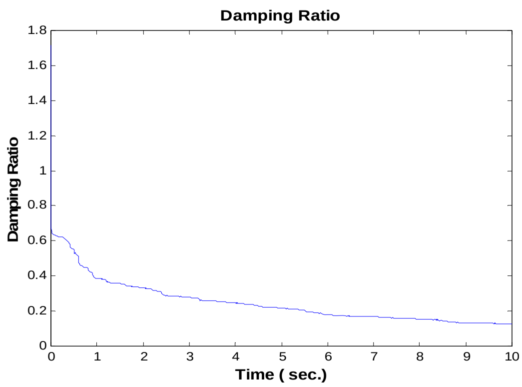

Figure 8. Damping Ratio(ζ) from Experiment

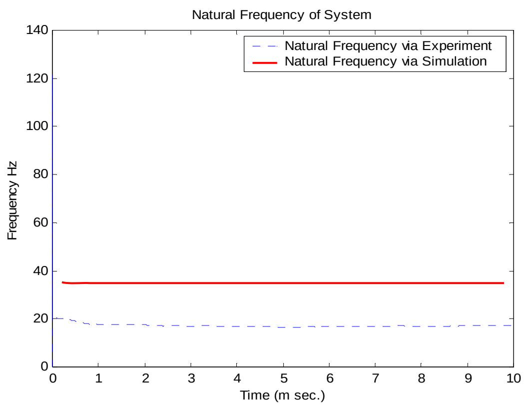

Figure 9. Comparison of Natural Frequency (ω_n)

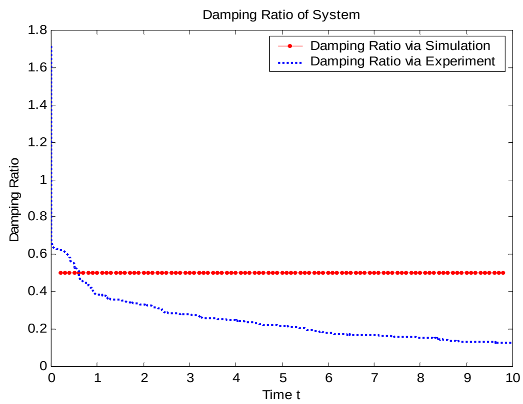

Figure 10. Comparison of Damping Ratio (ς)

4. CONCLUSIONS

New concept of system identification method is proposed for dynamic system under random disturbance whose intensity or any other information is not acquirable. Random system output is the only information available, and then the proposed stochastic system identification method shows quite promising performance. There are discrepancy between the simulation and experimental values of system parameters, natural frequency (ω_n) and damping ratio (ς), one of the reason for that may be white-noise assumption for external disturbance which is band-limited random process in actual. Intensive study is being carried out to figure out another reason for the discrepancy.

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