



MULTI-CHANNEL ADAPTIVE VIBRATION CONTROL OF SMART PLATE USING HYBRID ALGORITHM

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Abstract

The aim of this paper is simulation of multiple-channel adaptive vibration control of a clamped elastic plate with PZT sensors/actuators using hybrid algorithm. Finite element method is used for plate modelling. In the presented model, mass and stiffness of the PZT patches are considered. Transfer functions between exciting input and reference sensor, error sensors and reference sensor, actuators and reference sensor and also between actuators and error sensors have been determined by harmonic analysis via FE model. These transfer functions have been determined as IIR filters in z-domain. Off-line identification of secondary paths (from actuators to error sensors) and feedback paths (from actuators to reference sensor) have been carried out by using adaptive FXLMS identification method. Finally, for Single-Input-Multi-Output (SIMO) smart plate, the hybrid active vibration control is simulated and performance of the controller for globally vibration suppression is studied. The performance of the hybrid and feedforward controllers is compared. Obtained results indicate that the efficiency of the controller is dependent to the places of error sensors and actuators and frequency content of the exciting force.

1. INTRODUCTION

To date, the smart materials and structures have attracted much attention due to its potential advantages in a wide range of applications, such as aeronautical and aerospace engineering, civil engineering, automobile engineering, etc. Different types of sensors and actuators have been developed using piezoelectric ceramics, shape memory alloys, electrorheological fluids, magnetostrictive material, optical fibres, etc. In the field of structure shape and vibration control, piezoelectric materials received the most attention because of its certain features such as: low mass, high bandwidth, low cost, ease of bonding onto or embedding within flexible structures, etc [1]. In most cases, multiple piezoelectric material sensors and actuators are used and distributed on some specifically designed locations of the controlled structure. Combining with some types of control methods, these actuators apply control forces to the structure in a pre-defined manner and thus the structure can be deformed into a required shape or maintained in a level of vibration.

Bailey and Hubbard [2] initiated the research on the application of the smart structures in active vibration control. Utilization of discrete piezoelectric actuators has been shown to be a viable concept for vibration suppression in various works. Crawley and Luis [3] proposed an analytical solution for a static case including various actuator geometries. They stated that discrete piezoelectric actuators could be considered in vibration suppression of some modes of vibration of flexible structures. Kougen Ma [4] investigated vibration control of a clamped rectangular plate including PZT patches with using adaptive nonlinear controller. Active vibration control of an elastic plate using multiple PZT sensors/actuators has been studied by Caruso et al. [5]. They modelled an elastic plate using finite element method and compared the performance of various H_{∞} control laws. In another investigation, St-Amant and Cheng [6] simulated active vibration control of a plate with integrated piezoceramic. Their model has been coupled to a control simulator comprising both feedforward and feedback algorithms. By using ANSYS, Yaman et al. [7] worked on the finite element modelling technique for a smart beam. Based on their finite element model, they designed a controller that effectively suppressed the vibrations of the beam due to its first two modes. They demonstrated the effectiveness of H_{∞} design technique and also evaluated the robust performance.

The aim of this paper is simulation of hybrid active vibration control (feedforward plus feedback) of a clamped elastic plate with PZT sensors/actuators using filtered-x LMS algorithm. In the following sections, after presenting the finite element model of the smart plate, different transfer functions of the system is obtained via harmonic analysis. Then, following the description of the control algorithm, the numerical simulation of the hybrid control system is done. Finally, numerical results of the active control simulation are demonstrated and comparison is done with the corresponding results of the feedforward control system.

2. FE MODELING AND HARMONIC ANALYSIS OF THE SMART PLATE

The plate considered in this paper is made up of steel with cantilever boundary conditions, its physical properties are in table1. Also the damping coefficient is considered 0.005 for plate.

Thickness(mm)	Length(mm)	Width(mm)	Young	Poison	Mass (kg)	Density (kg/m3)
			modulus(Gpa)	ratio		
5	300	200	210	0.25	2.355	7850

Table 1. Physical properties of the plate.

The PZT patches that are bonded on the surface of the plate are made up of PZT4. A commercial finite element code (ANSYS) is used to model the smart plate. Theoretically, the plate elements (shell or solid) can be used in the modelling of the passive portion of the smart structure. The utilization of solid type elements in the modelling of the passive portion allows the calculation of the effects of the normal stresses and the transverse shear stresses which may be developed in the passive portion of the smart structures [8]. Yaman *et. al.* [7] investigated the influences of the element type selection on the response of the smart structures. In their work, it was shown that the use of shell elements in the modelling of the passive portion leads to the inaccurate calculation of the global stiffness matrix. The modelling of the passive portion using consistent solid elements with the actuator elements however is determined to yield accurate results. In the present study, solid elements (SOLID5) are used to model the active portion (piezoelectric actuators/sensors) and the compatible solid elements (SOLID45) are used for the passive portion (steel plate).

Before applying FXLMS algorithm, frequency response between exciting input and reference sensor, error sensors and reference sensor, actuators and reference sensor and also actuators and error sensors have been determined by harmonic analysis via FE model. FE model of the smart plate and the position of the sensors/actuators are shown in Figure 1. Modal analysis was performed before the harmonic analysis for the sake of better identifying the system. The first four natural frequencies of the smart plate are shown in table 2.

$f_1(Hz)$	f ₂ (Hz)	f ₃ (Hz)	f ₄ (Hz)
830.3	1282.8	2049.6	2056.4

Table 2. The first four natural frequencies of the smart p	late
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The following frequency responses have been determined between reference sensor (the place of exciting force) and error sensor "i" (primary path; Pi); between actuator "i" and error sensor "j" (secondary path; Sji) and between actuator "i" and reference sensor (feedback path; Fi), where i,j=1,2. Different paths between sensors and actuators are shown in Figure 1.



Figure 1. A) Position and configuration of the piezoelectric patches on the smart plate, B) Display of primary paths, secondary paths and feedback paths.

3. DERIVATION OF MATHEMATIC TRANSFER FUNCTIONS

Since most of the control and electrical devices, and electronic processors work in the digital domain, the requested transfer functions of different paths must be determined in Z-domain. The mathematical transfer functions have been determined in the following form:

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_1 + b_2 z^{-1} + \dots + b_{n+1} z^{-n}}{a_1 + a_2 z^{-1} + \dots + a_{m+1} z^{-m}}$$
(1)

The numerator and denominator coefficients can be obtained from the FE frequency responses. The best degree values of the numerator and denominator (n and m) have been selected based on try and error method and by comparison between original frequency response (obtained from FE model) and estimated mathematical frequency response. It is clear that the closer, the accommodation of these responses, the more accurate, the estimated transfer function. It should be noted that the estimated transfer function should be stable. Therefore, the zero-pole diagram is used for investigating its stability. For example, the transfer function of between concentrated force on the centre of plate and displacement at the reference sensor location has been obtained as follows:

$$\begin{cases} H_{p_1}(z) = \frac{B_p(z)}{A_p(z)} \\ B_p(z) = 99.65 + 122.8 \ z^{-1} + 112.7 \ z^{-2} + 98.55 \ z^{-3} + 77.08 \ z^{-4} + 48.11 \ z^{-5} \\ - 22.01 \ z^{-6} - 15.45 \ z^{-7} \\ A_p(z) = 1 + 1.113 \ z^{-1} + 1.004 \ z^{-2} + 0.9943 \ z^{-3} + 0.7625 \ z^{-4} + 0.4375 \ z^{-5} \\ - 0.2156 \ z^{-6} - 0.1136 \ z^{-7} + 0.01761 \ z^{-8} - 0.04519 \ z^{-9} \end{cases}$$
(2)

The above frequency response and its zero-pole diagram are shown in Figure 2. The transfer functions of the other paths can be obtained in the same way.



Figure 2. Frequency response function and zero-pole diagram of the path from concentrated force on the centre of the plate to displacement at the place of reference sensor. Original frequency response from FE model (solid) and estimated mathematical frequency response (dot).

4. HYBRID AVC SYSTEM

The feedforward Active Vibration Control (AVC) system uses two sensors: a reference sensor and an error sensor. The reference sensor measures the primary vibration to be cancelled while the error sensor monitors the performance of the AVC system. The adaptive feedback AVC system uses only an error sensor, and cancels only the predictable vibration components of the primary vibration. A combination of the feedforward AVC system using FXLMS algorithm and feedback control structures is called a hybrid AVC system, whereby the cancelling signal is generated based on the outputs of both the reference sensor and the error sensor [9].

The SISO hybrid AVC system is indicated in Figure 3. The secondary signal y(n) is generated using the outputs of both the feedforward AVC filter A(z) and the feedback AVC filter C(z). The combined controller W(z) has two reference inputs: x(n) from the reference sensor and $\hat{d}(n)$, the estimated primary signal. Filtered versions of the reference signals x'(n) and $\hat{d}'(n)$ are used to adapt the coefficients of the filters A(z) and C(z), respectively. It should be noted that, in the present study, the feedback transfer functions from secondary sources to the reference sensor is also considered, whereas it is not shown in Figure 3.

The secondary signal y(n) is generated by W(z), where W(z) is implemented by two parallel filters of length L[9]:

$$\mathbf{y}(n) = \mathbf{a}^{T}(n)\,\mathbf{x}(n) + \mathbf{c}^{T}(n)\,\hat{\mathbf{d}}(n)$$
(3)

where

$$\mathbf{a}(n) = \begin{bmatrix} a_0(n) & a_1(n) & \dots & a_{L-1}(n) \end{bmatrix}^T \quad ; \quad \mathbf{c}(n) = \begin{bmatrix} c_0(n) & c_1(n) & \dots & c_{L-1}(n) \end{bmatrix}^T \tag{4}$$

are, respectively, the weight vectors of A(z) and C(z) at time *n*, and the signal vectors:

$$\mathbf{x}(n) = \begin{bmatrix} x(n) & x(n-1) & \dots & x(n-L+1) \end{bmatrix}^T; \hat{\mathbf{d}}'(n) = \begin{bmatrix} \hat{d}(n) & \hat{d}(n-1) & \dots & \hat{d}(n-L+1) \end{bmatrix}^T$$
(5)

constitute the most recent samples of x(n) and $\hat{d}(n)$, respectively. Based on the stochastic gradient method, the FXLMS algorithm for adaptive filter A(z) and C(z) can be expressed as

$$\mathbf{a}(n+1) = \mathbf{a}(n) + \mu \,\mathbf{x}'(n)e(n) \quad ; \quad \mathbf{c}(n+1) = \mathbf{c}(n) + \mu \,\mathbf{\hat{d}}'(n)e(n) \tag{6}$$

where $\mathbf{x}'(n)$ is the filtered reference signal vector:

$$\mathbf{x}'(n) \equiv \hat{s}(n) * \mathbf{x}(n) \quad ; \quad \hat{\mathbf{d}}'(n) \equiv \hat{s}(n) * \hat{\mathbf{d}}(n)$$
(7)

In equation (7), $\hat{s}(n)$ is the estimated impulse response of the secondary path filter $\hat{S}(z)$ and the notation "*" denotes the linear convolution.

It should be noted that, as it is shown in Figure 1, in the present study the SIMO controller has been used. However, the formulation for the multi-channel control system has not been presented because of the lack of space.



Figure 3. Hybrid AVC system with combination of feedback AVC and FIR feedforward AVC [9]

In practical AVC applications, S(z) is unknown and must be estimated by an additional filter, $\hat{S}(z)$. Therefore, the filtered reference is generated by passing the reference signal through this estimate of the secondary path as indicated by equation (7) and Figure 3.Assuming that the characteristics of S(z) are time-invariant but unknown, off-line

modelling can be used to estimate S(z) during an initial training stage. Three major issues are involved in adaptive system identification: the excitation signal, the filter structure, and the adaptation mechanism. If the excitation signal x(n) is rich in frequency content (to excite all modes of the system, i.e. white noise) and if the internal plant noise is small, the adaptive filter will converge to a good model of the unknown system. The filter may be an all-zero, allpole, or pole-zero structure. An all-zero model is represented by an FIR filter. An all-pole or pole-zero models may be represented by an IIR filter. In this paper FIR filter with LMS algorithm is used as an adaptation mechanism. The results of identification for S11, the path between actuator1 and error sensor1, is shown in Figure 4. All secondary paths and feedback paths can be identified in the same way.



Figure 4. Off-line identification of the S11 path: A) plant response; B) error signal and C) the coefficients of FIR filter.

5. NUMERICAL SIMULATIONS

Numerical simulations have been performed using both "multi-channel hybrid AVC system with FXLMS algorithm" and "multi-channel feedforward FXLMS system". The location of sensors and actuators is shown in Figure 1.Three different inputs have been considered as below:

Input 1: Simple harmonic force $F(t) = F_0 Sin(\omega t + \varphi) = 10 sin(250t)$

Input 2: Summation of two harmonic force with close excitation frequencies (Beat phenomena) $F(t) = F_1 Sin(\omega_1 t + \varphi_1) + F_2 Sin(\omega_2 t + \varphi_2) = 10 sin(250t) + 15 sin(255t + \pi/2)$ Input 3: White-noise force Figure 5 indicates the simulation results of the hybrid AVC system for different inputs. The corresponding results have been obtained for the feedforward AVC system using FXLMS algorithm. Figure 6 shows the comparison between the behaviour of hybrid and feedforward AVC control systems. It can be seen that the multi-channel hybrid controller provides better performance in vibration suppression at the error sensors locations.



Figure 5. Vibration at the location of error sensor 1 for three different inputs; with hybrid AVC controller (-) and without controller (...).



Figure 6. The comparison between the multi-channel feedforward FXLMS algorithm (...) and multi-channel hybrid FXLMS algorithm (-).

CONCLUSION

This study presented a multi-channel adaptive vibration control of smart plate using hybrid algorithm, based on the finite element modelling technique. All primary, secondary and feedback transfer functions have been determined as IIR filters in z-domain via harmonic analysis. Off-line identification of secondary paths and feedback paths has been carried out by using adaptive FXLMS identification method. Computer simulations have been performed on controlling the vibration response at two sensor locations on the plate. Obtained results indicate that the efficiency of the controller is dependent to the places of error sensors and actuators and frequency content of the exciting force. In addition, the comparison between the numerical simulation results using multi-channel feedforward FXLMS algorithm and multi-channel hybrid FXLMS algorithm shows that the hybrid AVC controller provides better performance in vibration suppression at the error sensors locations. Finally, it should be noted that the proposed method can be used for complex structures i.e. vehicle's body and wings of airplane.

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