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DEVELOPMENT OF A GENERAL PERIODIC SEA SUBSYSTEM FOR MODELING THE VIBRO-ACOUSTIC RESPONSE OF COMPLEX STRUCTURES

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Abstract

Statistical Energy Analysis (SEA) has been used extensively to model the vibro-acoustic response of launch vehicles and payloads since the early 1960s. Modern SEA codes contain a large library of different subsystems that can accurately account for complicating effects such as ribs, curvature, lamination and pressurization. However, applications are sometimes encountered where a standard subsystem does not exist for a given type of construction. In some instances it is possible to estimate or update the subsystem properties from test or local FE models; however, the development of a generic subsystem that can include an arbitrarily complicated section is desirable. This paper discusses the development of a general periodic subsystem based on the use of periodic structure theory. A finite element model is created of a unit cell and analytical expressions are used to obtain the SEA properties of a larger panel comprised of a large number of such cells. The approach provides an efficient and accurate way to model arbitrarily complex sections in SEA that are difficult to model using traditional formulations. For launch vehicle applications the approach is particularly useful for modelling advanced laminate and isogrid constructions.

1. INTRODUCTION

Due to the high stiffness to mass ratio, periodically ribbed structures are commonly encountered in industrial designs and examples can be found in many industries (ship hull, ribbed fuselage, honeycomb panels, corrugated train wall, extruded train floor). At high frequency, SEA [1] is typically used to describe the dynamic response of such structures, and it can sometimes prove difficult to obtain the SEA parameters of two-dimensional complex periodic subsystems. Some formulations exist for simple and cross-wise stiffened structures, where three types of SEA representations can be adopted [2], depending on the free wavelength as compared to the rib spacing. These formulas however hardly apply to more complex geometries.

The goal of the developments presented in this paper is the use the periodic theory applied to two-dimensional structural dynamics to derive the input parameters of an SEA model of complex two-dimensional periodic structure. The periodic theory of structural vibrations initially developed by Mead [3], was studied by many authors including [4,5], and a comprehensive review is given in ref. [6]. There appears to be a large literature on the application of the periodic theory to structural dynamics, but its link with SEA has not been widely studied although some significant contributions can be found in refs. [2,7-10]. In these references, the periodicity is essentially one-dimensional and only the structural vibration is of interest. The focus in this study is on general two-dimensional systems, with interest in both the structural wave propagation and the coupling with the surrounding acoustic media.

Since a generic approach is sought, finite elements are combined with the periodic theory to allow any geometry to be considered. The periodic theory is quickly reviewed in Section 2. The derivation of the SEA parameters intrinsic to a system, i.e. modal density, damping, and equivalent mass, is then described in Section 3. In Section 4, the computation of the radiation and transmission quantities involved in the coupling with acoustics is reported. The application of the approach to an isogrid structure is presented in Section 5.

2. PERIODIC THEORY OF STRUCTURAL VIBRATION

The approach to the periodic theory developed in [4] for two-dimensional structures is adopted in what follows, and summarized here. Consider a periodic structure made of identical elements connected in a regular pattern. A periodic element (or *cell*) is extracted, and the cell's degrees of freedom are partitioned into interior (I), edge (L, R, B, T) and corner (LB, RB, LT, TR) freedoms. The internal degrees of freedom are not connected to other periodic cells. The free harmonic vibration is analyzed by specifying a phase lag between the displacement degrees of freedom at the left, right, bottom and top edges:

$$\mathbf{q}_R = \mathrm{e}^{-\mathrm{i}\varepsilon_x}\,\mathbf{q}_L\,,\quad \mathbf{q}_T = \mathrm{e}^{-\mathrm{i}\varepsilon_y}\,\mathbf{q}_B\,. \tag{1}$$

The corner degrees of freedoms may be similarly expressed in terms of left-bottom corner freedoms. The phase constants ε_x and ε_y, range from $-\pi$ to π.

If the cell is described with finite elements, the degrees of freedom refer to the nodal displacement and rotations. In order to apply Eqs. (1), it is then necessary that the nodes locations on opposite edges and corners are identical. The complete vector of local degrees of freedom of the cell is ordered as $\mathbf{q}={}^{\mathrm{T}}[\mathbf{q}_I\ \mathbf{q}_B\ \mathbf{q}_T\ \mathbf{q}_L\ \mathbf{q}_R\ \mathbf{q}_{LB}\ \mathbf{q}_{RB}\ \mathbf{q}_{LT}\ \mathbf{q}_{RT}]$. The equation of the undamped harmonic response of the cell at frequency ω can be written in terms of the corresponding mass and stiffness matrices $\mathbf{M}$, $\mathbf{K}$, and the generalized force vector $\mathbf{F}$: $\{-\omega^2\mathbf{M}+\mathbf{K}\}\mathbf{q}=\mathbf{F}$. Equation (1) allows to reduce the description to the vector $\mathbf{q}'={}^{\mathrm{T}}[\mathbf{q}_I\ \mathbf{q}_B\ \mathbf{q}_L\ \mathbf{q}_{LB}]$, via the relationship $\mathbf{q} = \mathbf{R}\,\mathbf{q}'$, where $\mathbf{R}$ is a rectangular matrix. Introducing this relation into the homogeneous version of the equation of motion yields the equation describing the free wave propagation:

$$\mathbf{R}^{\mathrm{H}}(\varepsilon_x,\varepsilon_y)\left\{-\omega^2\mathbf{M}+\mathbf{K}\right\}\mathbf{R}(\varepsilon_x,\varepsilon_y)\mathbf{q}' = \mathbf{0}\,, \tag{2}$$

where $\mathbf{R}^{\mathrm{H}}$ denote the complex conjugate transpose of $\mathbf{R}$. Given a set of real values of phase constants (ε_x, ε_y), solving the eigenvalue problem of Eq. (2) yields the real solutions for ω complying with the propagation conditions. Many solutions arise for ω since i) several waves can propagate under a given periodic condition, ii) the periodicity condition is defined modulo 2π. In what follows, a number of approaches are described for using the periodic eigensolutions to compute some SEA parameters.

3. SUBSYSTEM INTRINSIC PROPERTIES

In what follows, the multiple eigensolutions of Eq. (2) associated with each pair of phase constant are used to derive the averaged modal density, damping and velocity response of the periodic structure. In order to find all the periodic eigensolutions, the range to be covered by

ε_x and ε_y is generally $-\pi<\varepsilon_x<\pi$ and $0<\varepsilon_y<\pi$ as the eigensolution for the pair $(-\varepsilon_x,-\varepsilon_y)$ equals the one for $(\varepsilon_x,\varepsilon_y)$ thanks to the reversal property of the propagation direction. In the case of orthotropic structures, additional symmetry allows to further reduce the range to $0<\varepsilon_x<\pi$. For each pair of phase constants, many eigensolutions can be extracted. In the current analysis, only the solutions with eigenvalue below a given circular frequency (typically the maximum frequency of interest time a multiplicative factor) are extracted, similarly with what would be done for a standard modal analysis.

The common assumption in SEA is made that the vibration field is diffuse: waves propagating in a frequency band are uncorrelated, and equally contribute to the total energy.

3.1 Modal density

The modal density can be computed as the derivative with respect to frequency of the mode count (number of mode below a given frequency ω). According to ref. [5], the mode count of a periodic structure with $N_x \times N_y$ cells is obtained by computing the total area of phase constant surfaces lying below the frequency ω:

$$n(\omega)=\frac{\partial N}{\partial \omega}, \qquad \text{with} \qquad N(\omega)=\frac{N_x N_y}{\pi^2}\sum_n\left(\iint_{\omega_n(\varepsilon_x,\varepsilon_y)<\omega} d\varepsilon_x d\varepsilon_y\right). \tag{3}$$

In the integral, the values of ε_x and ε_y range from 0 to π.

3.2 Damping loss factor

If a structure has a spatially uniform damping, then the damping loss factor to be used in the SEA equations is the structural damping. Alternatively, if the structure exhibits an inhomogeneous damping distribution like typically encountered in laminates or ribbed panels, the damping loss factor becomes dependent of frequency as the structure undergoes different deformation field in each frequency band.

For each wave propagating in a given frequency band, the damping can estimated by $P_{\text{diss}}/\omega(T+U)$, where U and T are the strain and kinetic energies associated with the wave motion, and P_{diss} is the power dissipated. If damping is introduced through the structural damping mechanism, the power dissipated is proportional to the strain energy. If the components of a structure have different damping levels, the power dissipated is expressed in terms of the energies of each component, which are obtained from the corresponding finite element mass and stiffness matrices and the wave shape (i.e., the eigenvector $\boldsymbol{\varphi}_n$ of Eq. (2)). Considering the fact that only the resonant motion is of interest, the strain and kinetic energies are equal and the wave damping loss factor is written

$$\eta^{(n)}(\omega_n)=\frac{\sum_s \eta_s \boldsymbol{\varphi}_n^{\mathrm{H}}\left(\mathbf{R}^{\mathrm{H}}\mathbf{K}_s\mathbf{R}\right)\boldsymbol{\varphi}_n}{\boldsymbol{\varphi}_n^{\mathrm{H}}\left(\mathbf{R}^{\mathrm{H}}\mathbf{K}\mathbf{R}\right)\boldsymbol{\varphi}_n}. \tag{4}$$

where $\mathbf{K}_s$ is the contribution of the s^{th} component to the total stiffness matrix of the cell. Assuming that the waves propagating in a frequency band are uncorrelated, and carry the same energy, the damping loss factor of the periodic structure is found by averaging the numerator and denominator of Eq. (4) over the waves with propagating frequency within the frequency band of interest.

3.3 Velocity distribution per unit energy

The primary outputs from an SEA calculation are the powers exchanged between subsystems and the subsystems vibrational energy. From the energy, the modulus square velocity can sometimes be post-processed. Typically, for simple structural subsystems with spatially homogenous mass density, the spatial averaged of the modulus squared velocity is given by $<|v|^2> = E/M$, where M is the mass of the subsystem and E is the energy.

The value of velocity squared will depart from the spatial average in the vicinity of discontinuities such as edges, driving points, or coupling points. For a periodic structure like a ribbed plate, one expects the velocity distribution to be very inhomogeneous as the velocity response on a rib may be very different from the one on the skin. It is also expected that the distribution is frequency dependent because the structure undergoes different deformation in each frequency band.

For the displacement field $\boldsymbol{\varphi}_n$ associated with a wave, the time-averaged energy of a cell is given by $E_n(\omega) = \boldsymbol{\varphi}_n^{\mathrm{H}}(\mathbf{R}^{\mathrm{H}}\{\omega^2\mathbf{M}+\mathbf{K}\}\mathbf{R})\boldsymbol{\varphi}_n/2$, which can be reduced to two times the mass term at the wave propagating frequency. The RMS velocity squared at a point $\mathbf{x}$ of the cell is $\omega^2(|\varphi_{n,x}(\mathbf{x})|^2+|\varphi_{n,y}(\mathbf{x})|^2+|\varphi_{n,z}(\mathbf{x})|^2)/2$, where the translation in the three directions contribute. The velocity distribution per unit energy of the n^{th} wave at the wave propagating frequency is then given by

$$v_{RMS,n}^2(\mathbf{x},\omega_n) = \frac{\left|\boldsymbol{\varphi}_{n,x}(\mathbf{x})\right|^2 + \left|\boldsymbol{\varphi}_{n,y}(\mathbf{x})\right|^2 + \left|\boldsymbol{\varphi}_{n,z}(\mathbf{x})\right|^2}{\boldsymbol{\varphi}_n^{\mathrm{H}}\left(\mathbf{R}^{\mathrm{H}}\mathbf{M}\mathbf{R}\right)\boldsymbol{\varphi}_n}. \tag{5}$$

By virtue of the diffuse field assumption (energy equipartition and uncorrelation of waves), the total energy and modulus square velocity in a frequency band are the sums of the contribution of each wave propagating in the band. The RMS velocity squared per unit energy is then obtained by averaging the numerator and denominator of Eq. (5) over the waves with propagating frequency within the frequency band of interest.

4. COUPLING WITH ACOUSTICS

In this section, the acoustic radiation and transmission properties of a periodic structure are computed from the phase constant surfaces, solutions of Eq. (2). In all cases, the calculation makes use of the spatial Fourier transform of some vibration fields, and this is introduced in the first subsection below.

4.1 Wavenumber transform and periodicity

Consider the finite periodic structure made of $N_x \times N_y$ cells of dimensions $l_x \times l_y$. For any displacement field with periodic properties related to the phase constants $(\varepsilon_x,\varepsilon_y)$, the displacement field $\tilde{u}$ of the cell at (n_x,n_y) cells from the reference cell is written in terms of the reference displacement field u as [9]: $\tilde{u} = u\exp(-in_x\varepsilon_x - in_y\varepsilon_y)$. If the displacement is assumed to be zero outside of the finite periodic structure (rigid baffle), the spatial Fourier transform of the field is written as a surface integral confined to the finite area A_N of the structure. Using the above form of the displacement field, the integral over the whole structure area can be split into N_xN_y integrals over the cells area A, and the integration variables changed from global to local. The modulus square of the Fourier transform is then obtained in terms of the Fourier transform of the field over the reference cell by

$$|\tilde{U}(k_x,k_y)|^2 = |U(k_x,k_y)|^2\,\kappa(N_x,\varepsilon_x,k_xl_x)\kappa(N_y,\varepsilon_y,k_yl_y) \quad \text{with} \quad \kappa(N,\varepsilon,kl) = \frac{1-\cos\left(N(\varepsilon+kl)\right)}{1-\cos(\varepsilon+kl)}. \tag{6}$$

The function κ describes the effect of the finite size and of the periodicity on the wavenumber content of the unit cell's vibration field. Its peaks at $kl=-\varepsilon+2\pi n$ with n an integer, become more pronounced as N increases.

4.2 Acoustic radiation

This section describes the computation of the averaged radiation efficiency used in SEA to describe the coupling of structures with surrounding fluids. Consider a finite periodic structure surrounded by a semi-infinite fluid along the surface area A_N. The structure is assumed to be baffled by an infinite rigid plan.

The motion of a unit periodic cell due to a wave is described by the eigenvector $\boldsymbol{\varphi}_n$ solution of Eq. (2). The out-of-plane degrees of freedom are extracted to form the vector φ_n. Let $\tilde{\varphi}_n$ denote the corresponding out-of-plane displacement field of the complete periodic structure made of $N_x \times N_y$ cells. The radiation properties can be obtained from the self-radiation dynamic stiffness of the wave deformation of the complete surface looking into the fluid,

$$D_{rad,n} = \iint_{A_N} \tilde{\varphi}_n^*(X,Y)\, \tilde{p}_n(X,Y)\, dXdY \,, \tag{7}$$

where $\tilde{p}_n(x,y)$ is the pressure field on the structure surface, due to the n^{th} wave of unit amplitude. Thanks to the rigid baffle condition, the finite area integral can be extended to infinity, allowing the use of Fourier transform. The fluid wave dynamic stiffness relates the displacement wavenumber transform to the pressure wavenumber transforms by $\tilde{P}_n(k_x,k_y) = D(k_x,k_y)\tilde{\phi}_n(k_x,k_y)$. If the radiating area is bare (no noise control treatment), then $D(k_x,k_y,\omega) = i\rho\omega^2(k^2 - k_x^2 - k_y^2)^{-1/2}$, where $k=\omega/c$ is the acoustic wavenumber, and ρ and c are the fluid density and sound velocity. The effect of a noise control treatment could be included in two ways in this analysis: i) the trim can be described by finite elements and thus be part of the wave eigensolution, the radiating surface being the outer surface of the trim, or ii) the trim can be included in the wave dynamic stiffness $D(k_x,k_y)$ by using available wavenumber descriptions of the multilayered trims [11].

Applying the inverse Fourier transform to the pressure wavenumber transform, substituting the resulting pressure field into Eq. (7), and inverting the summation order yields the radiation dynamic stiffness of the wave

$$D_{rad,n} = \frac{1}{4\pi^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |\tilde{\phi}_n(k_x,k_y)|^2\, D(k_x,k_y)\, dk_x dk_y \,, \tag{8}$$

where only the wave radiation stiffness and the wavenumber transform of the wave field over the whole structure are involved.

The use of the Fourier transform makes the current computation strictly valid for flat radiating surfaces. For curved structures, the assumption is made that the acoustic impedance of the plane surface is a good approximation if the curvature is *not too small* compared to the acoustic wavelength. The structure is *unwrapped* to its flat equivalent by conserving the area.

The radiation efficiency of the wave is defined in terms of the time-averaged acoustic power radiated by the structure with the wave deformation field and the averaged mean square velocity of the structure undergoing the wave motion: $\sigma_{rad,n} = \Pi_{rad,n} / (\rho c A < |v_n|^2 >)$. The power radiated can be derived from Eq. (8) by $\Pi_{rad,n} = \mathrm{Re}\{i\omega D_{rad,n}\}/2$, and the mean square velocity is $< |v_n|^2 >= \omega^2 \iint_{A_N} |\tilde{\varphi}_n(X,Y)|^2\, dXdY / 2A_N$. Using Eq. (6) in conjunction with Eq. (8), the radiation efficiency of a wave propagating along a finite $N_x \times N_y$ periodic structure can be obtained from the Fourier transform of the wave displacement field on the reference cell.

By virtue of the diffuse field assumption (energy equipartition and uncorrelation of waves), the radiation efficiency used in SEA can be obtained by averaging the radiated power and mean square velocity over the waves.

4.3 Acoustic transmission

In this section, the acoustic transmission loss of a periodic structure is derived. A diffuse acoustic field is applied on one side of the structure, and the power radiated on the other side is computed. The diffuse acoustic field can be described i) as a set of uncorrelated acoustic plane waves with equal intensity, and incident on the structure at all angles; ii) using a recent diffuse-field reciprocity result [12] giving the blocked pressure from the radiation impedance computed by Eq. (8). Only the first approach is demonstrated here.

Consider a structure with two faces (not necessarily coincident) with one face excited by a diffuse acoustic field. For a given incidence defined by the angles θ and ϕ in spherical coordinates, the incident pressure on the face of the structure is $p(x,y)=p_0 \exp(-jk_{px}x-jk_{py}y)$ where $k_p=k \sin\theta$ is the acoustic wavenumber trace on the structure and $k_{px}=k_p\sin\phi$ and $k_{py}=k_p\cos\phi$. Since both the loading pressure and the structure are periodic, the dynamic response is spatially periodic and can be computed using Eq. (2) with a forcing term. The force vector on the right-hand side of the equation is the projection of the blocked pressure (two times p) onto the FE shape functions associated with the out-of-plane degree of freedom at each node. Given the frequency ω and the real wavenumbers k_{px} and k_{py}, solving the equation yields the response of the degrees of freedom of a unit cell to the pressure field.

From the response of the periodic cell, the power radiated can be estimated by substituting in Eq. (8) the wavenumber transform of the out-of-plane displacement field of the cell's response (this accounts for the finite-size effect of the structure). The power transmission coefficient for a given incidence is the ratio of radiated and incident powers. The diffuse-field transmission coefficient is obtained by averaging over all incidences:

$$\tau = \int_0^{\pi}\int_0^{\theta_{lim}} \Pi_{rad}(\theta,\phi)\sin\theta d\theta d\phi \Bigg/ A_N \frac{|p_0|^2}{2\rho c}\pi \int_0^{\theta_{lim}} \cos\theta \sin\theta d\theta \,. \tag{9}$$

For the *diffuse-field* transmission coefficient, the limit incidence is $\theta_{lim} = \pi/2$, whereas for the *field-incidence* often used in practice, the angular range is confined below $\theta_{lim} = 78^o$.

5. APPLICATION CASE: ISOGRID STRUCTURE

The formulae derived above are used to characterize the SEA properties of an isogrid panel. A picture of the panel is shown in Fig. 1, together with the FE mesh of a single periodic cell comprising CQUAD and CTRIA elements and 690 nodes. At this stage, isotropic aluminium has been considered for simplicity, and ribs and skin have different thicknesses (different colors in Fig. 1). The ribs have 0.1% damping and the skin 2%.

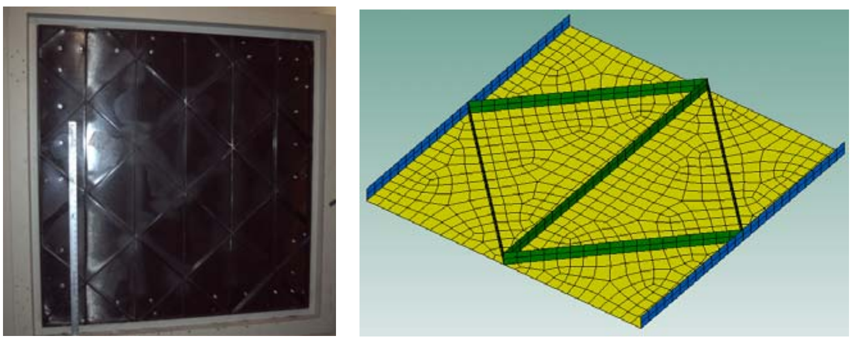

Fig. 1: *Left: picture of an isogrid panel. Right: finite element meshes of a single cell.*

For a panel made of 12×12 cells, the damping loss factor, number of modes in frequency band, radiation efficiency and transmission loss were computed using the development above. Results are shown in Fig. 2. For the damping, the ribs have an impact at low frequency because the motion if fairly uniform. When the frequency increases, the motion tends to localise and the more energetic skin region dominates the damping. The modes in band show a band with very few modes (at 250 Hz), which is typical of periodic behaviour (this is almost a “stop band”). The transmission is seen to be dominated by non-resonant path over most of the frequency range. The radiation efficiency was computed with and without a noise control treatment made of a soft wool layer and a heavy rubber layer. The first resonance of the layup is 300 Hz, and this is where the effect on radiation starts to be significant.

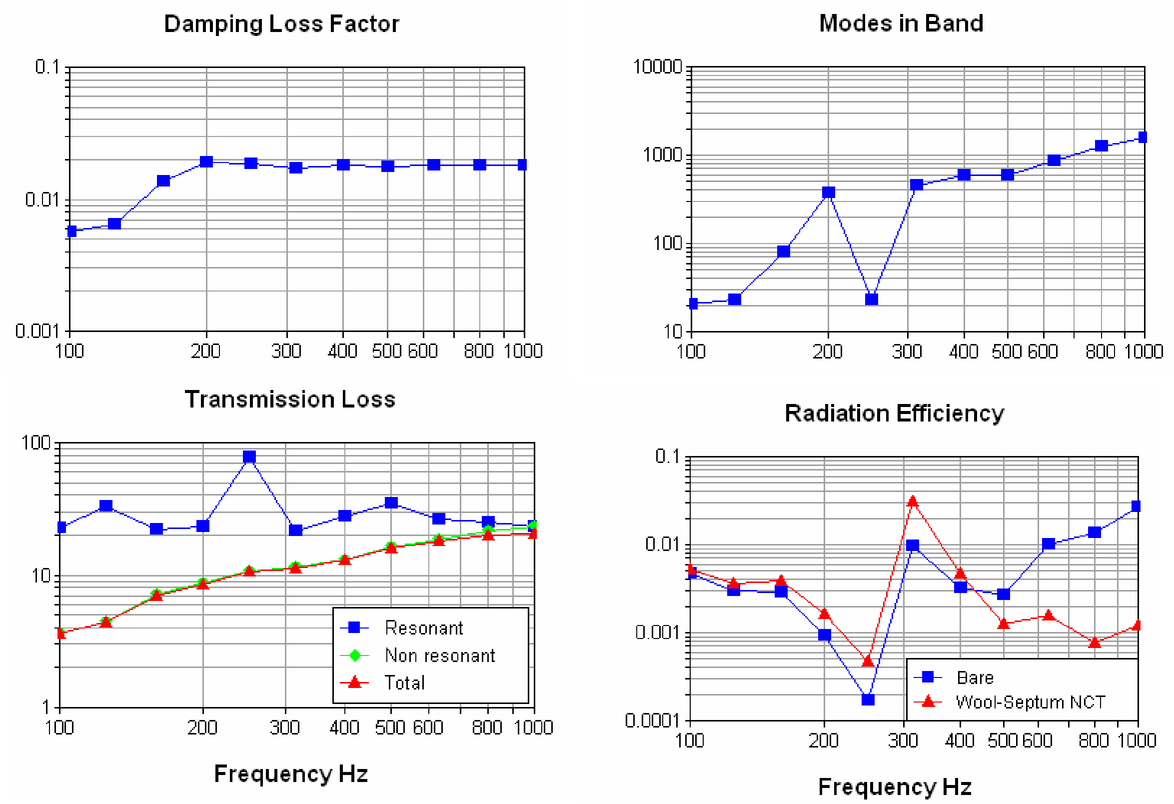

Fig. 2: *Some SEA properties of the isogrid panel, as computed using the periodic formulation.*

The distribution of RMS velocity in the third octave band centred at 100 Hz is shown in Fig. 3, where it can be seen that the skin responds more than the ribs.

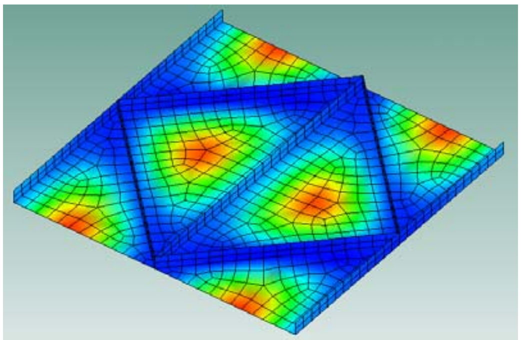

Fig. 3: *Distribution of RMS velocity per unit energy of the structure at 100 Hz.*

6. CONCLUSIONS

By combining finite element and periodic theory, an approach has been developed to compute some input parameters to the SEA description of two-dimensional periodic structures. Starting from the finite element description of a single periodic cell, some phase constant surfaces are computed by specifying some periodic boundary conditions and solving an eigensolution problem. The eigensolutions are then used to derive some intrinsic properties of the structure as well as properties describing its coupling with acoustics.

The subsystem intrinsic properties that were derived are the modal density, the damping loss factor (from the distribution of damping within the periodic cell), and the distribution of modulus square velocity per unit energy. Regarding the coupling with acoustics, the acoustic radiation and transmission have been derived using the wavenumber descriptions of the vibration fields. The finite-size effect is accounted for. The contribution of resonant and non resonant modes in the transmission can be computed. Noise control treatment can be added by adequately changing the wave radiation impedance.

Recently, the point and line impedances of a periodic structure have been derived, allowing the calculation of the coupling loss factors with connected structural subsystems. Also, the power input due to point forces, diffuse acoustic field and turbulent boundary layer have been derived using the wavenumber description presented here. The effect of pressurization has been introduced through the use of “differential stiffness matrix” in the FE description of the cell.

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