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STABILITY OF SOUND SPEED AND ATTENUATION IDENTIFIED IN MATERIAL SAMPLES

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Abstract

Within an experimental framework, in porous saturated media, it remains a delicate problem to identify sound speed and attenuation, two related characteristics. Using a temporal method is problematic as samples are short and, moreover, the Fourier transform is unable to extract information from the frequency domain.

Wavenumbers can be identified by stationary methods, but the ratio of displacements at the sample exit to displacements at the entry does not depend linearly on the wavenumber. A non-convex minimization is quite unreliable, since it depends on the initial value it has to start from. A genetic algorithm could certainly be of some help here. However, with a rather more physical insight, when dealing with samples of different lengths, averaging the results could already improve reliability. Here we suggest going further in the same vein, using a different approach to reach a robust result.

Indeed, as said previously, the function available is not linear with regard to the parameters. But starting from a first identified value, the perturbation method makes it possible to linearize the function with regard not to the parameter itself but to its variation. Such an approach provides not only information about the stability of the function with the parameter variation but also, within the limits of stability, an analytical solution, thanks to the now convex optimization of the variation which will refine the initial value.

This presentation provides an analysis of the approach. It is the first step towards the study of how the model influences the stability of the identification, a problem which is definitely of interest today.

1. INTRODUCTION

Knowing the acoustic properties of sediments is a key element in the interpretation of measurements performed by geophysical instruments such as sub-bottom profilers. Various theoretical approaches allow these acoustic properties to be determined. However, surprisingly, experimental approaches are extremely rare. Stevenson et al. [1] present a laboratory work combined with in situ experiments on silty clays. Wilson et al. [2] worked on laboratory experiments on water-saturated granular sediments. These two recent publications give an idea of how delicate the identification of acoustic properties through laboratory measurements can be today.

In our case, a laboratory experimental set-up is developed and an acoustic wave propagation model is deployed in high-water-content soils, in order to the measure the sound speed and attenuation. The experimental set-up features a shaker which delivers acoustic waves (frequencies between 1 and 15 kHz) to the sample. The sample is placed in a rigid envelope and behaves like a waveguide. Two accelerometers measure the emitted and the transmitted signals.

A spectral study on displacements enables an inverse analysis to be performed to extract sound speed and attenuation from the measurements. It appears that results for the sound speed are accurate enough for the final aim, but results for the attenuation are not. Therefore, measurements are performed on three samples containing identical material but of different lengths, and these measurements are used simultaneously in an attempt to improve reliability.

The study is divided into two steps. First, the identification method, followed by the improvement method, is performed on numerically simulated signals, which are obtained with a direct problem analysis. Then, the signal analysis is performed on real measurements. This paper deals with the first step.

2. DIRECT PROBLEM

2.1. Introduction

For the time being, the acoustic model is based on the linear propagation of a single wave. The cylindrical sample diameter is 5 cm and the expected sound speed is around 1500m/s. According to the modal theory [3], the guided acoustic wave propagating along the axial direction is plane (i.e. all the points on a cross section vibrate in phase) till around 17 kHz. The material is supposed to be homogeneous and isotropic (behaving like a one-phase-material), and also viscoelastic.

The study of wave propagation in viscoelastic materials, the constitutive law (stress-strain relationship $\sigma(\varepsilon)$) of which is $\sigma = E\varepsilon + E'\dot{\varepsilon}$, yields the harmonic wave equation:

$$\frac{d^2u}{dx^2} + \frac{\rho\omega^2}{E + i\omega E'}u = 0 \tag{1}$$

where u is the longitudinal acoustic displacement (along the cylinder axis), ω the angular frequency in rd/s, E and E' the elastic and the viscoelastic coefficients which appear in the constitutive law. Identification with the well-known Helmholtz equation leads to the complex wavenumber k such that

$$k^2 = \frac{\rho\omega^2}{E + i\omega E'} \tag{2}$$

For $x = 0$, the measured emitted displacement u_0 constitutes the first data. For $x = L$, a rigid plate coupled to an accelerometer (the mass of both the plate and the accelerometer is noted m) lies on the sample. The analysis of that specific boundary leads to the following condition, in the spectral domain:

$$\frac{du}{dx}(L) + \frac{\omega^2 m}{(E + i\omega E')S} u(L) = 0 \tag{3}$$

The previous paragraphs lead to the operator:

$$\begin{cases} d^2u/dx^2 + k^2 u = 0 \\ u(0) = u_0 \\ du/dx\,(L) + ik\beta u(L) = 0 \end{cases} \tag{4}$$

with the acoustic reduced admittance $\beta = -imk/S\rho$. Its solution is easily obtained by hand:

$$u(x) = u_0 \frac{e^{ik(L-x)} + \mathrm{R}e^{-ik(L-x)}}{e^{ikL} + \mathrm{R}e^{-ikL}} \tag{5}$$

with the acoustic reflection coefficient at L defined by $R = \dfrac{1-\beta}{1+\beta} = \dfrac{S\rho + imk}{S\rho - imk}$. In particular, the solution at L is

$$u(L) = u_0 \frac{1+R}{e^{ikL} + \mathrm{R}e^{-ikL}} \tag{6}$$

For the sake of simplicity, the ratio $u(L)/u_0$ will now be written $u_{L/0}$, and is in fact the main data.

For a viscoelastic material, the complex wavenumber k is given by the following expression

$$k = \frac{\omega}{c} - i\alpha \tag{7}$$

due to the choice of $\cos(\omega t) = \mathrm{Re}\left(e^{+i\omega t}\right)$ for the frequency domain calculations, c being the sound speed in m/s, α the attenuation in Np/m. Therefore, the acoustic properties that are dealt with are the sound speed and the attenuation, through the complex wavenumber.

3. IDENTIFICATION PROCEDURE USING STATIONARY METHODS

The identification procedure consists in determining the material properties through the evaluation of the complex wavenumber, starting from the ratio (and values for m, S and ρ).

Considering Eq.(5), let $F(k)$ be the following function:

$$F(k) = R + \frac{1 - u_{L/0} e^{ikL}}{1 - u_{L/0} e^{-ikL}} \tag{8}$$

There are two ways of dealing with the identification of k. The first consists in finding the zeros of $F(k)$. The second consists in seeking the wavenumber which minimizes $|F(k)|$, starting from an initial value k_{ini}. The latter method is chosen in the present study.

The wavenumber is expected to depend on the frequency, according to its expression, but the manner in which it depends on the frequency is not yet known. Therefore, there is no point in looking for an average wavenumber over the whole frequency range, and each inverse analysis is carried out for one particular frequency.

Complex wavenumbers are obtained with this method, but as it is a non-convex problem, it may often lead to erroneous results, first because it depends on k_{ini}, second in case of error in the data $u_{L/0}$. Obviously, the closer the initial value k_{ini} to the solution, the better the chance of obtaining a good estimation of the wavenumber. And it has been observed that the greater the amplitude of $u_{L/0}$, the better the chance of obtaining a good estimation of the wavenumber also.

To go further in terms of precision, the perturbation method is presented in the next paragraph. It makes it possible to linearize the function with regard not to the parameter itself but to its variation. Several simulations will be carried out in order to estimate the performances of the method, and compare it to the more straightforward average method, in which the three wavenumbers, obtained from the identification with the three samples of different lengths, are averaged.

4. LINEARIZATION OF THE ACOUSTIC MODEL WITH REGARD TO THE WAVENUMBER VARIATION, AND ITS OPTIMIZATION IN L_2

Let k be the solution to the problem. A close value to that wavenumber, k_0, is supposed to be known. The solution can now be written $k = k_0 + \varepsilon$. It then becomes possible to linearize Eq.(5) with regard to the perturbation ε, yielding the following expression:

$$u_{L/0} \approx f + \varepsilon g \tag{9}$$

with $f = \dfrac{1 + R_0}{c}$ where $R_0 = \dfrac{S\rho + imk_0}{S\rho - imk_0}$, and $c = e^{ik_0L} + R_0 e^{-ik_0L}$

and $g = \dfrac{b}{c} - \dfrac{(1 + R_0)d}{c^2}$ where $b = \dfrac{im}{S\rho - imk_0}(1 + R_0)$, and $d = e^{ik_0L} iL + e^{-ik_0L}(b - R_0 iL)$ under the assumptions: $im\varepsilon/(S\rho - imk_0) << 1$, $\varepsilon L << 1$ and $\varepsilon d/c << 1$. Later, it will be seen that these hypotheses may be the same as those which allow local and global minima to be merged in the non-convex identification problem. Therefore,

$$g(k_0, S, \rho, m, L)\varepsilon \approx u_{L/0} - f(k_0, S, \rho, m, L) \tag{10}$$

It should be noted that the physical propagation model exists in g as well as in f, though from the computational point of view, the model is only formed by g, while the objective is formed by both the $u_{L/0}$ measurement and f. Therefore, the situation is one where the physical model also plays a role in the objective.

Eq. (10) is established for each sample length. However, three measurements for $u_{L/0}$ are available, and the perturbation ε ought not to depend on the sample length. For one length L_j $(j = 1,2,3)$, the equation can be written $g_j\varepsilon \approx (u_{L/0})_j - f_j$ (for the sake of simplicity, $(u_{L/0})_j$ will now be written $u_{L_j/0}$). Given these considerations, the simple matrix equation can be written $\overline{\overline{G}}\overline{\varepsilon} = \overline{F}$, the solution of which is given by the following expression:

$$\overline{\varepsilon} = \left(\overline{\overline{G}}^T . \overline{\overline{G}}\right)^{-1} . \overline{\overline{G}}^T . \overline{F} \tag{11}$$

A proper conditioning of $\overline{\overline{G}}^T . \overline{\overline{G}}$ ensures that the above expression will provide an acceptable result. That conditioning is related to the independence of terms in $\overline{\overline{G}}$ associated to various sample lengths and can only be observed ex post facto. In the proposed tests, no indication of poor conditioning emerged during the computations.

5. NUMERICAL SIMULATIONS AND COMPARISON BETWEEN TWO METHODS FOR IMPROVING THE WAVENUMBER OBTAINED

5.1 Efficiency

The displacements which stand for data are simulated thanks to the direct problem. The present study deals with the influence of the initial value for the wavenumber, at a given frequency (here f =2.3kHz). In the identification procedure, the initial value is k_{ini}, and in the perturbation method the initial value is k_0.

5.1.1 First case : $k_{ini} = k_0 = 1.7k$

DATA	$k = 9.63 - 0.41i$ and $k_{ini} = 16.38 - 0.70i$
NON-CONVEX METHOD:	$k_0^{L_1} = 23.39 + 0.41i$, $k_0^{L_2} = 9.63 - 0.41i$, $k_0^{L_3} = 25.80 - 0.39i$
	$k_{average}(L_2, L_3) = 17.72 - 0.40i$
CONVEX METHOD:	$k_{perturb} = 16.38 - 1.47i$

One of the values obtained using non-convex optimization ($k_0^{L_1}$) is not acceptable, as the imaginary part of the wavenumber is expected to be negative, since it characterizes attenuation and not amplification. One acceptable result is erroneous ($k_0^{L_2}$), and the third value obtained is the solution. With the average method the acceptable results lead to an erroneous result. Finally, the perturbation method gives an acceptable result which is quite far from the expected solution, and it seems that iterations do not lead to the solution either.

5.1.2 Second case: $k_{ini} = k_0 = 1.5k$

DATA	$k = 9.63 - 0.41i$ and $k_{ini} = 14.45 - 0.62i$
NON-CONVEX METHOD:	$k_0^{L_1} = 9.63 - 0.41i$, $k_0^{L_2} = 12.03 + 0.41i$, $k_0^{L_3} = 9.63 + 0.41i$
	$k_{average}(L_1, L_3) = 9.63 - 0.41i$
CONVEX METHOD:	$k_{perturb} = 14.00 - 1.15i$

In this case, as previously, one of the obtained values is not acceptable ($k_0^{L_2}$). The two other results are the expected solution, and, therefore, the average method yields the solution. The perturbation method yields an erroneous result, but nine iterations are enough to make it converge to the solution.

5.1.3 Third case: $k_{ini} = k_0 = 1.2k$

DATA	$k = 9.63 - 0.41i$ and $k_{ini} = 11.56 - 0.50i$
NON-CONVEX METHOD:	$k_0^{L_1} = 9.63 - 0.41i$, $k_0^{L_2} = 12.03 + 0.41i$, $k_0^{L_3} = 9.63 - 0.41i$
	$k_{average}(L_1, L_3) = 9.63 - 0.41i$
CONVEX METHOD:	$k_{perturb} = 11.06 - 0.77i$

Results obtained for $k_{ini} = k_0 = 1.2k$ are very similar to those obtained for $k_{ini} = k_0 = 1.5k$, except that only 5 iterations are required to obtain the solution with the perturbation method.

5.1.4 Conclusion

These few tests show that performances of both methods (average method after identification procedure, and perturbation method) are globally equal. If results could be generalized, the conclusion could be that hypotheses for the convex method do correspond to the equivalence of local and global minima in the non-convex procedure.

5.2 Robustness

The displacements which stand for data are simulated thanks to the direct problem, and undergo random noise, with a Gaussian distribution around the simulated value $u_{L/0}$, so that they are always in the $0.8u_{L/0}$ to $1.2u_{L/0}$ range. The initial values proposed in the identification process are written $\left(k_{ini}^{L_1}, k_{ini}^{L_2}, k_{ini}^{L_3}\right)$. The inverse analysis on the three different lengths yields $\left(k_0^{L_1}, k_0^{L_2}, k_0^{L_3}\right)$. The result obtained for the first length L_1 cannot be improved by iteration in a new inverse analysis, but it is possible to improve it by using it as the initial value k_0 in the perturbation method. In the previous paragraph, $k_0 = k_{ini}^{L_1}$, whereas now $k_0 = k_0^{L_1}$. This approach allows us to compare the non-convex method alone to the non-convex method followed by the convex one (perturbation method). The average method yields $k_{average} = \left(k_0^{L_1} + k_0^{L_2} + k_0^{L_3}\right)/3$. The perturbation method yields $k_{perturbation} = k_0^{L_1} + \varepsilon$.

In order to compare performances of the two procedures, two series of tests are run. In the first case, $k_{ini} = 0.8k$, whatever the length, which helps to determine the robustness of the two methods. The second case is supposed to approach reality with $0.8k < k_{ini} < 1.2k$. It should be underlined that when considering $0.8k < k_{ini} < 1.2k$ in the identification procedure, these initial values for the wavenumber are different for the three different lengths, and are different for the real and the imaginary parts of these wavenumbers: $\mathrm{Re}(k_{ini}^{L_1}) \neq \mathrm{Im}(k_{ini}^{L_1}) \neq \mathrm{Re}(k_{ini}^{L_2}) \neq \mathrm{Im}(k_{ini}^{L_2}) \neq \mathrm{Re}(k_{ini}^{L_3}) \neq \mathrm{Im}(k_{ini}^{L_3})$.

In each case, a graph is plotted showing the percentages of error obtained by the average method (in green) and by the perturbation method (in red). The results presented were computed for a frequency of $f = 2.3kHz$, which corresponds to a resonance peak for $L_1 = 0.2m$, and to a normal regime for $L_2 = 0.4m$ and $L_3 = 0.6m$, according to the direct problem, (for $S = 1.59.10^{-3}m^2, \rho = 1300kg.m^{-3}, m = 0.02kg$), as shown on Fig 1.

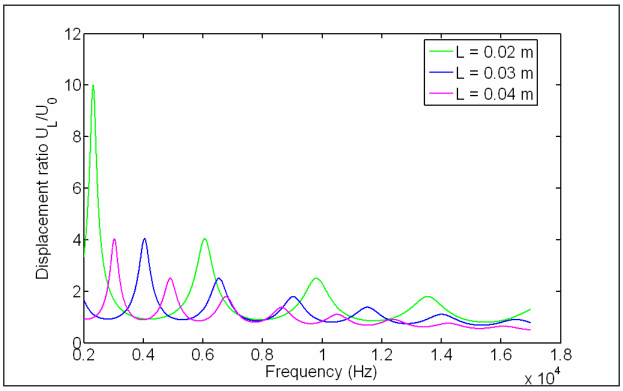

Fig. 1. – Spectrum obtained with the direct problem
($S = 1.59.10^{-3}m^2, \rho = 1300kg.m^{-3}, m = 0.02kg$).

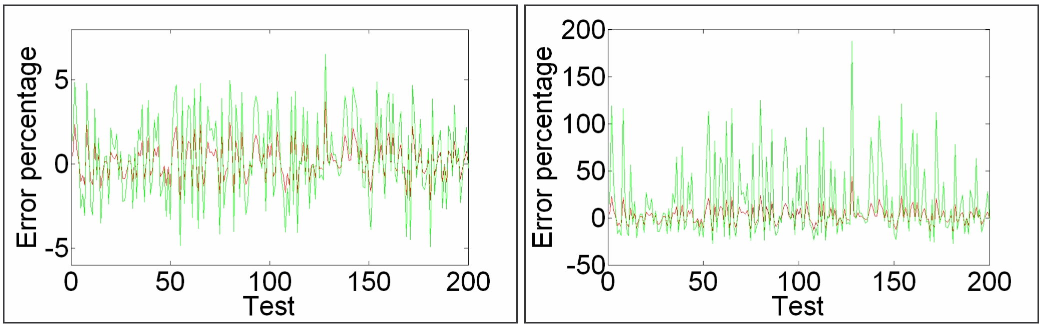

Fig. 2. – Error percentages on the real part (left) and on the imaginary part (right) of the wavenumber obtained with signals which have undergone random noise, and for $k_{ini} = 0.8k$, and $k_0 = k_0^{L_1}$
($S = 1.59.10^{-3}m^2, \rho = 1300kg.m^{-3}, m = 0.02kg$).

Fig.2 shows that the perturbation method is more robust than the average. This observation can be explained by the fact that the initial value in the perturbation method is $k_0^{L_1}$, which is already close to the solution as it derives from the inverse analysis performed on the signal which shows a resonance peak at $f = 2.3kHz$. Error percentages, in the perturbation method, reach 3% at most on the real part of the wavenumber (and therefore on the sound speed) and 30 % at most on the imaginary part (and therefore on the attenuation).

Fig. 3 shows results obtained in the case where signals have undergone random noise, and $0.8k < k_{ini} < 1.2k$ in the identification procedure. The reason for presenting this case is that it can reflect the computation carried out using laboratory measurements. When $0.8k < k_{ini} < 1.2k$, the perturbation method is still better than the average method, just as for

$0.8k = k_{ini}$, and the error percentages are smaller than in the previous case, an outcome which was expected intuitively.

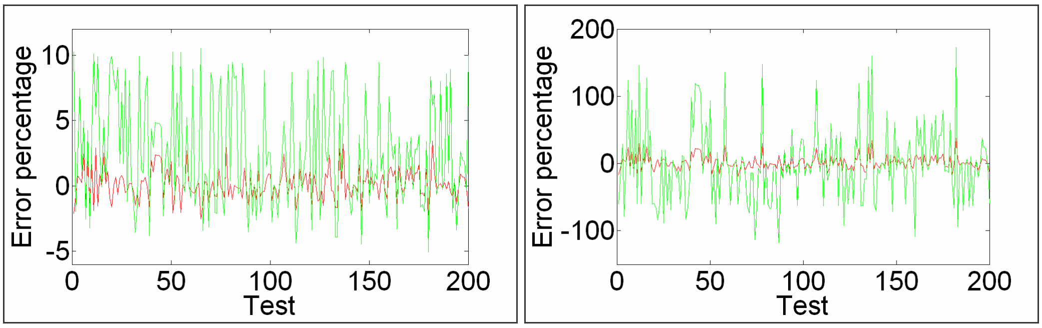

Fig. 3 - Error percentages on the real part (left) and on the imaginary part (right) of the wavenumber obtained with signals which have undergone random noise, and for $0.8k < k_{ini} < 1.2k$, and $k_0 = k_0^{L_1}$ ($S = 1.59.10^{-3} m^2, \rho = 1300 kg.m^{-3}, m = 0.02 kg$).

6. CONCLUSION

When seeking the acoustic properties of high-water-content material, the difficulty lies in finding simultaneously sound speed and attenuation. Modeling the sediment and the container (especially boundary conditions) in a laboratory set-up, and performing an identification procedure on measurements using that model does lead to a complex wavenumber, and therefore to the acoustic properties of interest here. To improve these results, two methods were envisaged, both exploring the fact that measurements can be performed on samples with different lengths. The present study consisted in comparing them, and it appeared that, as far as efficiency was concerned, performances of the perturbation method and of the average method on results obtained from the identification procedure seem to be similar. But, as far as robustness was concerned, the perturbation method performed using a primary result obtained with the identification method gave far better results than the simple average. The procedure will soon help to obtain the acoustic properties of material tested in the laboratory.

The approach could be extended to more complex models. Indeed, nothing guarantees today that high-water-content clayey soils behave as one-phase-material, and are perfectly viscoelastic. With a two-phase material, the model would be extended to coupled linear waves. Considering increasingly complex behaviours, cases of weak and strong non-linearity ought also to be envisaged.

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