



# STUDY ON ACOUSTIC SCATTERING BY HARD PLANE AND SOFT SURFACE MULTI-CRACKED PLATE

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## Abstract

The acoustic scattering characteristics of finite plates with multi-cracks and different surface conditions are studied .The boundary element method is used to evaluate the exterior acoustic field. Numerical examples show that the cracks have an influence on scattering acoustic pressure in that the sound pressure decrease as the cracks increase. Both hard plane and soft surface with cracks can be examined by the diversification of sound pressure and directivity of pattern. Further research of acoustic scattering by damage structure is discussed.

# **1. INTRODUCTION**

Acoustic scattering by objects has been a topic of interest over the past few decades, it has great importance in wave propagation and structural analysis in helping us understand the dynamic response of the structures and acoustic field distribution. This work is particularly useful in the sound wave nondestructive evaluation field, which is based on elastic waves. If one is to interpret correctly a signal from a scattering, we can get the most important information of the structures. The scattering sound from a prolate spheroid was first investigated by Spence<sup>[1]</sup>. The earlier works predominated concentrated on far-field scattering or the determination of the acoustic pressure at the surface of the spheroid. Furthermore, Goodman and Stern<sup>[2]</sup> got the sphere acoustic scattering solution under plane wave by using 3-dimensional elastic mechanics method. Gaunaurd and Werby<sup>[3]</sup> studied the relationship between inner shell resonance and lamb wave. Rozhdestvenskii and Tolokonnikov<sup>[4]</sup> explored the acoustic scattering by an elastic spheroid, they gave reliable result only for low aspect ratio prolate spheroids. Maciulaitis et al are only the few studies for the rigid oblate spheroid scattering problem. They investigated the frequency limit below that sound scattering by a microphone body is sufficiently small to permit accurate pressure gradients measurements. The boundary element method (BEM) has been an efficient numerical technique to model acoustic

radiation and scattering from bodies in homogeneous free space or half-space<sup>[5, 6]</sup> (Seybert et al, 1985; Wu, 1994). The major advantage of the BEM is that only the surface of the bodies has to be modeled. The Sommerfeld radiation condition at infinity is automatically satisfied. Furthermore, Wu<sup>[7]</sup> and Marinez<sup>[8]</sup> discussed the using condition of BEM that made it more improved to calculate the radiation and scattering problems.

The plates are used as components in various engineering structures, such as in ocean engineering and civil engineering. In a plate, surface crack is the familiar defects that maybe make the structures insecurity. In previously studies, most of researchers focused on intact structure acoustic scattering and few studies investigated the defects structures acoustic scattering. In this paper, we study the acoustic scattering field of hard plane and soft surface thick plate with multi-cracks by using BEM, acquire some important information that can be used in sound wave underwater nondestructive evaluation.

# **2. BASIC THEORY**

Consider a known acoustic field is incident on a motionless rigid object B, and the boundary surface S in an acoustic medium B' that is assumed to be half-space, homogeneous, and non-absorbing the mean density is  $\rho$  and the speed of sound is c.  $p_i$  and  $p_s$  are incident wave and scattering wave. The total field is the superposition of the two waves:

$$p = p_i + p_s \tag{1}$$

The governing differential equation in the steady—state linear acoustics is the well-known Helmholtz equation:

$$\nabla^2 p + k^2 p = 0 \tag{2}$$

Where p is the acoustic pressure,  $k = \frac{\omega}{c}$  is the wave number and  $\omega$  is the angular frequency. By the weighted residual formulation, Eq. (1) is reformulated into the boundary integral equation:

$$C(P)p(P) = \int_{s} (p(Q)\frac{\partial G}{\partial n}(P,Q) - G(P,Q)\frac{\partial p}{\partial n})dS(Q) + 4\pi p_{i}(P)$$
(3)

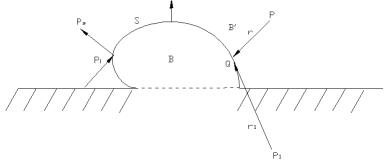


Fig.1 Scattering sketch for a half -space problem

$$C(P) = \begin{cases} 4\pi & P \in B' \\ 4\pi + \int_{S+S_c} \frac{\partial}{\partial n} (\frac{1}{r}) dS(Q) & P \in S_0, z_P > 0 \\ (1+R_p)[2\pi + \int_{S+S_c} \frac{\partial}{\partial n} (\frac{1}{r}) dS(Q)] & P \in S_0, z_P = 0 \\ 0 & P \notin S_0, B' \end{cases}$$

$$G(P,Q) = \frac{1}{r} e^{-ikr} + R_p \frac{1}{r_1} e^{-ikr_1}$$
(5)

where

Where r = |Q - P|,  $r_1 = |Q - P_1|$ . Q is any point on S and P may be in B, B' or on S.  $P_1$  is the image of P. Furthermore, for a scattering problem the scattering pressure  $p_s$  should satisfy the Sommerfeld radiation condition:

$$\lim_{r \to \infty} \left[ \sqrt{r} \left( \frac{\partial p_s}{\partial r} - ikp_s \right) \right] = 0 \tag{6}$$

To solve the Helmholtz integral equation numerically, we discretize the boundary into a number of quadratic elements and the corresponding method to that used in Ref. [9].

## 2.1 Improvement on Numerical Non-Unique Problem

Different types of methods aiming to improve the non-unique problem, most of which are based on the Burton and Miller method or the CHIEF <sup>[9]</sup> method, have been proposed. These methods based on a high computational cost and cannot always work successfully, especially in cases where the surfaces have a complex profile and the plates with multi-cracks also belongs to it. Herein, another method, a boundary that defines a barrier surface is modified to reduce the bounded area while keeping the barrier configuration <sup>[10]</sup>.

Fig.2 shows a modified boundary S that maintains the barrier shape defined by S in Fig.1. The modified barrier contains a small opening that the original does not have. Therefore, it is important that the dimension d is small enough so that the opening will not affect the sound field around the barrier. A perfectly absorbing interior surface suppresses re-radiation from the opening. It should be noted that boundary modifications added to produce a more accurate numerical analysis that cannot be included on practical barrier construction models. This improved technique is easier to apply to analyses than other methods that require a number of formulation and calculation changes.

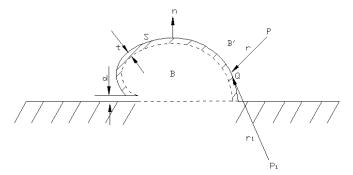


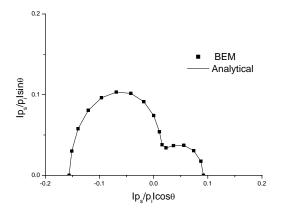
Fig.2 Modified boundary to improve BEM non-unique problem

## 2.2 Verification of the BEM Program

We consider the scattering problem is the scattering of a plane wave from a rigid sphere. The incoming plane wave is the negative x direction and is described by  $p_i = p_a \exp(ikx)$ . For the scattering from a sphere of radius a, the scattering potential at a distance r from the centre of the sphere and at an angle  $\theta$  from the direction of the incoming wave is given analytically:

$$p_s = p_a \sum_{m=0}^{\infty} i^{m=1} (2m+1) P_m(\cos\theta) \sin\delta_m(kr) \times \exp[i\delta_m(ka)] h_m^2(kr)$$
(7)

Where the terminology corresponds to that used in Ref. [11]. When the improved technique was applied, the dimension d was set to 0.001m. All Figs.3~5 show good agreement between BEM program and the analytical solutions.



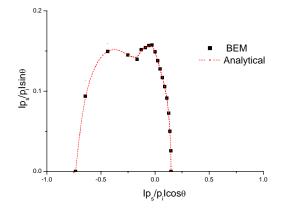


Fig.3  $p_s / p_i$  versus polar angle for a rigid sphere when ka=1 and r=3a

Fig. 4  $p_s / p_i$  versus for a soft sphere when ka=5 and r=4a

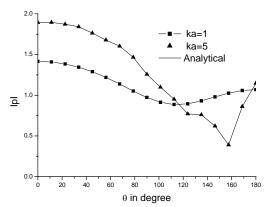


Fig.5 Sound pressure with polar angle for a rigid sphere surface

# **3. NUMERICAL RESULTS**

#### 3.1 Plate with Single Crack

Consider a three-dimensional thin rigid plate in half space with length 1m, width 0.5m and the thickness h=0.04m. The water density is  $1000 \text{kg/m}^3$  and the acoustic speed in water c=1500m/s. Assumed a crack like letter v appeared in the centre of the thin plate and crack

depth is 0.02m. The incident plane wave is  $0^{\circ}$  in the direction, as shown in Fig.6.Reference pressure:  $2x10^{-5}$ Pa.

Fig.6 compares the scattering pressure of the single cracked plate on two different surface conditions in near field. For both conditions, the results show that the scattering pressure on a plate with a single crack is lower than that of an intact plate, and the pressure made by the soft surface condition is lower than that of the rigid surface and it presented that the soft surface performs more effectively on reducing scattering pressure than the rigid one does.

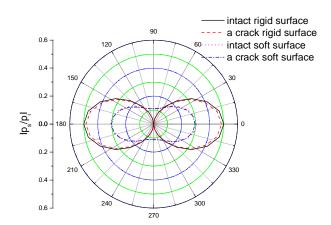


Fig.6 Comparison of scattering pressure by a crack with two different surface conditions when vertical incidence ka=5 and r=1.5m

Another comparison is presented in Fig.7, which shows the sound pressure effect of frequencies on a plate with a single crack and on different surface conditions in near field (r=0.1). The solid and dashed lines represent the results for the rigid surface and the soft surface, respectively. It reveals that the rigid surface condition with a single crack is more sensitive to the changing of frequency than the soft surface plate. For example, when the acoustic frequency is 100 Hz, the discrepancy of sound pressure between an intact plate and a single crack plate is 3dB, but the pressure will decrease with the frequency increasing. It can be explained the scattering effect made by high frequencies is intensely and it is difficult to distinguish the scattering effect between the cracked plate and intact plate. For the soft case, the results are similar in cracked plate and intact plate since the sound pressure on the soft surface is zero.

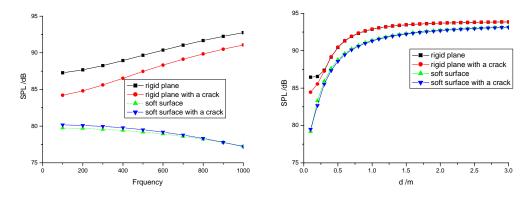


Fig.7 Effect of frequencies on sound pressure.

Fig.8 Distribution of sound pressure along the finite plate centre

Fig.8 shows the distribution of sound pressure along the finite plate centre, it is indicated that pressure made by rigid surface condition is higher than that of soft plate in near field, but in far field, the pressure are almost equal. So it can be concluded that soft surface condition may perform large sound pressure in far field where the scattering problem can be computed simply as rigid surface condition in infinite far field.

#### **3.2 Plate with Multi-Cracks**

Plates with multi-cracks are more common in modern structure engineering. For computing convenience, we assumed that all cracks have the same characteristics, such as width and depth. Fig.10 shows the sound pressure-frequency effect of a multi-cracked in near field (r=0.1). Generally speaking, the sound pressure will decrease as the cracks increasing. The discrepancy of sound pressure between rigid surface intact plate and a rigid surface with three cracks plate is 4dB when the frequency is 100Hz, while the discrepancy of sound pressure between intact plate and one crack plate is 3dB which indicates that multi-cracks make scattering intensely and the pressure decline indistinctly with multi-cracks. The similar result happens in the soft condition.

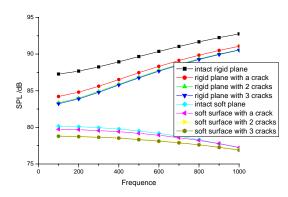


Fig.10 Effect of frequencies on sound pressure(r=0.1m) with multi-cracks

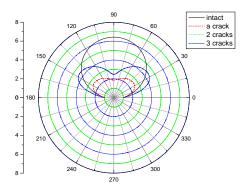


Fig.11 Directivity of pattern for multi-cracks soft plate when f=500Hz and r=0.1m

Fig.11 displays directivity of pattern for soft plate with multi-cracks when f=500Hz and r=0.1m. The results for the plate under soft surface conditions reveal that the pole angles of the

cracked and intact plate are different. The directivity of pattern for intact plate is not obvious and pole angles of cracked plate does not appear at zero degree, the discrepancy of sound pressure at pole angle about intact plate and cracked plate is more than 4dB that can help to estimate the existence of the cracks.

# 4. CONCLUSIONS

The acoustic scattering characteristic of finite plate with cracks and different surface conditions were computed by using BEM and the numerical non-unique problem has been improved by using a simple technique. The results show that scattering pressure of the plate with cracks will be lower than that of intact plate and it is lower in the soft surface condition than that of rigid surface. The soft surface performs more effective in reducing scattering than the rigid one does in near field. In far field, the soft surface condition may perform large sound pressure that almost equals to that of the rigid surface generates. So the scattering problem can be computed simply as rigid surface plate with cracks. This present research is useful for nondestructive evaluation field.

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