



SPECIAL ISSUES RELATED TO DETECTION OF CIRCUMFERENTIAL CRACK AT DIFFERENT ORIENTATIONS IN PIPES BY VIBRATION METHOD

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Abstract

A vibration based crack detection technique is applied to circumferential cracks in empty straight horizontal pipes with straight front in different orientations with the vertical. The crack is modelled through rotational spring stiffness. The stiffness has been determined experimentally by the deflection and vibration methods for various crack orientations and crack sizes/depths. The rotational spring stiffness for any arbitrary orientation has been theoretically related to the stiffness for the corresponding crack in the vertical orientation. This relation has been compared with experimental results. Some finite element investigations have been done to check the transferability of data from one pipe material and dimensional combination to other materials and dimensional combinations. Further, sensitivity of the vibration method for prediction of crack location on variations in experimental data has been examined. Sensitivity study has been carried out by changing the difference between the frequencies of pipes with and without crack by $\pm 10\%$. The method is found to be very robust; the absolute maximum variation in location is only 1.45%.

1. INTRODUCTION

Presence of a crack alters physical behaviour of a component, e.g. natural frequency, mode shapes, damping characteristics, static deflection, strain at a point, etc. Changes in natural frequency, mode shape, damping characteristics are global in nature and hence offer potential for developing NDE. Transverse vibration based detection of cracks has been in the focus of development for a very long time [1-18]. Some investigators have used the modal analysis; others have employed the frequency based approach. The frequency based approach offers the advantage that it can be easily measured from a single accessible point on a component. This also makes it suitable even for inaccessible components. The method requires an accurate measurement of frequencies of components with and without crack. The accuracy of detection is dependent on the accuracy of measurement of the change in frequencies. In the past no attention has been paid to the study of sensitivity of the method to measurement of this change. Further, most studies involving pipes with and without fluids deal with circumferential crack with a straight front in the 12 o'clock position (i.e. 0^0 orientation) in

horizontal pipes [5, 10, 11]. Existence of cracks in other orientations is possible due to manufacturing defect, corrosion, corrosion-fatigue, etc. Such problems are important and require attention. Whether the rotational spring stiffness for any arbitrary crack orientation is related to that for 0^0 orientation is not known. This rotational spring stiffness can be made dimensionless for a particular set of pipe dimensions. Whether, these data can be transferred to other pipe dimensions and materials have not yet been examined. This requires studies with more than one material, cracks at different inclinations, and different dimensional combinations. Experimental and theoretical results concerning some of these issues are presented.

2. THEORY

2.1 Transverse Free Vibration Analysis of Pipe with Crack

The modelling for the detection of crack in slender pipes is based on the assumption that crack introduces only local discontinuity in the slope at the crack location and a very small difference exists between the mode shapes of the pipes with and without a crack. Therefore, a pipe containing a crack, for example in $\theta = 0^{\circ}$ orientation, can be conveniently represented by two pipe segments connected by a rotational spring [4-6, 8, 10] of stiffness K_t at the crack position (Figure 1).



Figure 1 Crack in vertical orientation and its representation by rotational spring.

From the characteristic equation, the rotational spring stiffness K_t can be written as a nonlinear function of circular frequency ω and crack location β . Using a measured value of ω and assuming β , K_t can be determined [10, 11, 19, 20]. Thereby a variation of K_t with β for a particular mode is obtained. Intersection of three such curves, corresponding to three natural frequencies, gives the crack location and rotational spring stiffness for the unknown crack.

2.2 Rotational Spring Stiffness (K)

When the rotational spring is used to represent a crack, the spring acts as sink for the energy released due to the crack. This energy is equal to the difference in energies of pipes with and without crack. The spring stiffness K_t can be determined experimentally using deflection method or vibration method. For a cantilever beam with load acting at the free end,

$$K_t = \frac{M_{empty \ pipe}^2}{P(\delta_c - \delta_{nc})}$$
(1)

where P is load acting at the free end, $M_{empty\ pipe}$ is the bending moment due to P at the crack section at a distance L_2 from the support and, δ_c and δ_{nc} are the deflections along the load line for pipes with and without crack respectively. This relation provides the basis for determination of variation of K_t with crack size by the deflection method. Thus, if K_t is known, crack size a/t can be obtained using these plots.

While the above approach is for pipe with cracks at $\theta = 0^0$, it can also be applied to cases with orientations other than 0^0 assuming that the crack can still be represented by a rotational spring. A crack gives rise to an asymmetry in the cross-sectional area. Therefore, for a crack orientation other than $\theta = 0^\circ$ (Figure 2), excitation of the pipe in vertical direction may lead to triply coupled vibrations, i.e. coupling of bending vibrations in the two orthogonal directions (θ and normal to θ) and torsional vibration [21]. Nevertheless, there will be a distinct transverse vibration frequency in the vertical plane [21]. This offers the basis for prediction for crack in any orientation. The spring stiffness can as well be measured through static deflection measurements with similar orientation and loading in the vertical plane $|\gamma|$



Figure 2 Crack in $\theta = 0^0$ and arbitrary θ orientations.

2.3 Relationship between K_{θ} and K_{θ}

If a bending moment M is applied about x-axis (Figure 2) to a pipe with a crack in orientation θ , the component of moment along x' axis is $M \cos \theta$. This gives rise to a jump in slope $(M \cos \theta/K_0)$ about the x' axis. K_0 corresponds to the rotational spring stiffness for the same crack depth and zero degree orientation. This manifests as a local rotation $(M \cos^2 \theta/K_0)$ about the axis x. This local rotation/jump is also given by M/K_{θ} , where K_{θ} is the stiffness of the spring for a crack in orientation θ and corresponds to rotation about axis x. Therefore, equating the two relations, $K_{\theta}/K_0 = 2/(1 + \cos 2\theta)$. This relation is verified by experiments.

3. EXPERIMENTAL AND NUMERICAL STUDIES

3.1 Experiments:

Experiments were performed [22, 23] to determine the rotational spring stiffness by the static deflection and vibration methods. Pipes of mild steel and aluminium were employed. The specimen details and dimensional combinations are as given in Table 1A to 1C.

13 specimens each of mild steel and aluminium were tested. Each lot included one crack-free specimen. Thus, a total of 84 cases for mild steel (12 specimen with crack in 7 orientations) and 13 cases for aluminium (12 specimens with only 0^0 orientation) pipes were

examined. Cracks were made by wire-cut machining, using a wire of diameter 0.20 mm, in sizes 1mm, 2mm, 3mm and 4mm. More experimental details are given in [22, 23].

Specimen material	Specimen	Outer	Inner	Mass	Modulus of	Orientations
	length	diameter	diameter	density ρ	elasticity	considered
	(m)	$D_o(\mathbf{m})$	Di (m)	(kg/m^3)	E	heta
Mild Steel	0.95	0.0378	0.0278	7860	173.81	$0^{\circ}, 10^{\circ}, 20^{\circ}, 30^{\circ},$
					1/3.81	$45^{\circ}, 50^{\circ} \text{ and } 60^{\circ}$
Aluminium	0.95	0.04	0.0298	2645	60.3478	0^0

Table 1A: Specimen dimensions and other details.

Table 1B: Dimensional combinations considered for experiments with steel pipes.

Boundary Conditions	Crack location β			a/t			
Cantilever	0.218	0.305	0.389	0.2	0.4	0.6	0.8
Simply Supported	0.3975	0.5036	0.6061	0.2			

Table 1C: Dimensional combinations considered for experiments with aluminium pipes.

Boundary Conditions	β			a/t			
Cantilever	0.218	0.305	0.389	0 1061	0.3922	0.5882	0.7843
Simply Supported	0.3975	0.5036	0.6061	0.1901			

3.2 FE Study:

FE study was done using ANSYS (Version 9.0) to calculate K_t through the deflection method. Empty steel (*E*=173.81 GPa and Poisson's ratio = 0.3) and aluminium (E=603478 GPa and Poisson's ratio = 0.33) pipes of various dimensions were considered. For the discretisation, 10 noded tetrahedron (SOLID 92) with three translational degrees of freedom per node was employed. The crack was modelled as a narrow slit of 0.25 mm width. Three pipe thicknesses, 0.004 m, 0.005 m, and 0.006 m each with five crack depths, 30%, 40%, 60%, 80% and 100%, and one orientation ($\theta = 0^0$) only, were studied. Inner diameter and length for all pipes were 0.04m and 0.75m respectively. Totally 30 combinations were analysed.

4. RESULTS AND DISCUSSION

4.1 Crack Location and Spring Stiffness:

Using the characteristic equation of vibration, variation of K_t with β was obtained for the first three modes of vibration using the experimentally measured frequencies. Intersection of the three curves gives the required crack location and the rotational spring stiffness. There are two possible crack locations because of the geometric symmetry in the case of a simply supported beam. Only the value close to actual β is selected for the comparison (Table 2). Predicted crack location for a few cases are compared with the actual values in Table 2 for a/t = 0.6. More results are included in [23, 24]. The maximum absolute error in prediction is observed to be 7.29% for a/t = 0.4 and crack orientation of 30^0 .

Irrespective of the method of determination of the spring stiffness K_t by the deflection and vibration methods, for each crack orientation and a particular crack size, three crack positions were considered and three stiffnesses were obtained. Since the rotational spring stiffness does not depend on crack positions, the average of the three in each case was taken as the required stiffness. Variations of K_t with a/t and θ obtained based on the experimental data (in the range $\theta = 0^0$ to 60^0 and a/t = 0.2 to 0.8) are shown in Figures 3 (A) and (B).



Table 2: Comparison crack location β by vibration method for a/t = 0.6 (t = 5mm).

Figure 3 Variation of K_t with a/t and θ by (A) deflection method and (B) vibration method [23].

The rotational spring stiffness reduces, as expected, with an increase in crack size; it increases with an increase in the crack orientation angle. The stiffness obtained by the

frequency method differs from that obtained through the deflection method by a maximum of -4.64%. Thus, there is a good agreement between the results obtained by the two methods.

In Figure 4 fitted variation of K_{θ}/K_0 is plotted against θ using the experimental data (for various a/t and β values) and FE results for steel and aluminium pipes (with a/t=0.8). The trend of variation is similar to $2/(1+\cos 2\theta)$ theoretically predicted. The fitted finite element based variation tally exactly with the theoretical results. The fitted experimental variation is based on the vibration method and it differs more from the theoretical results for $\theta > 30^0$. The reason for this difference requires further investigations. The relationship between K_{θ} and K_{θ} can be exploited to determine K_{θ} provided K_{θ} for various crack depths are only known.



Figure 4 Theoretical, experimental and FE variations of K_{θ}/K_0 with crack orientation.

4.2 Non-dimensional Stiffness:

Variation of non-dimensional rotational spring stiffness $K = K_t L/(EI)$ for $\theta = 0^0$ has been plotted with crack depth (Figure 5) using the finite element results for aluminium and steel pipes. This shows that for a given pipe thickness *t*, *K* is independent of the material. This is an important result. This may help transfer of data on K_0 from one material to another. Further, the variation of *K* with a/t, as expected, is found to be asymptotic (Figure 5). It is observed from the same plot that *K* converges to a single value (\approx 93), for a leaking crack, i.e. a/t = 1. This constant value is very important in judging the extent of crack depth.

4.3 Sensitivity/Robustness Study:

In order to assess the influence of errors in measurements of natural frequencies, keeping the measured frequency of a crack-free pipe unchanged, the measured frequency of the corresponding pipe with a crack was perturbed by $\pm 10\%$ of the measured difference between the two. This was repeated for three combinations, by changing all three frequencies of a pipe with crack, or any two frequencies or any one frequency. Thus there were seven possible combinations. The location of crack was determined following the usual procedure for aluminium pipes. The maximum change in β is -1.45% only. This is very small in comparison with the perturbation $\pm 10\%$ in $\Delta \omega_i$. The vibration method is therefore quite robust. Figure 8

shows variation of % change in β with β for various a/t corresponding to $\pm 10\%$ variation in ω_3 .



Figure 5 Variation of non-dimensional rotational spring stiffness with crack depth.



Figure 6 Percentage change in β for ± 10 % variation in $\Delta \omega$ of mode 3 only.

5. CONCLUSIONS

The method of prediction of crack based on the changes in natural frequency can handle crack in various orientations in straight pipes. The maximum error in prediction of location is -7.29%. For a given crack size, the rotational spring stiffness increases with an increase in the crack orientation angle. Also, for a particular orientation of crack, the rotational spring stiffness decreases with an increase in crack size. The predictions for rotational spring stiffness as obtained by the deflection and vibration methods do not differ significantly; the maximum difference is -4.636%. The ratio of rotational spring stiffnesses corresponding to orientations θ and 0^0 , for a particular crack size, are shown to be related by a geometric relationship. The experimental results show a trend, which is very similar; the difference between the two increases as θ increases above 30⁰. The dimensionless rotational spring stiffness $K = K_{L}/(EI)$ is independent of material. This may help in transfer of data from one material to another. Further, this can be exploited to determine the rotational spring stiffness corresponding to any arbitrary crack orientation and crack size provided the stiffness for one material in the zero degree orientation and the same depth is known. The non-dimensional rotational spring stiffness decreases as a/t increases and it converges asymptotically to a constant value when the crack depth is equal to the thickness and pipe thickness and length are the same. The vibration method for prediction of crack location is quite robust; for a $\pm 10\%$ change in the difference between frequencies of pipe without and with crack the maximum absolute change in prediction of crack location is 1.45%.

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