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GENERALISATION OF VESELAGO'S NEGATIVE REFRACTION THEORY USING SYMMETRIES

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Abstract

In this paper, we generalize Veselago's negative refraction theory to anisotropic solids using symmetry. Veselago's theory is applicable only to electromagnetic wave and is limited only to isoptropic solids using linear dispersion. Most solids are anisotropic and we also extend his theory to acoustic waves. Veselago's theory depends on negative permeability and negative permittivity to obtain negative refractive index. We derive the expression for the transmission/refraction coefficient for acoustic wave at the interface between two anisotropic materials in terms of the angle of refraction in the second medium. We find that this expression is invariant whether we use the positive angle of refraction or the negative angle of refraction. This shows reflection/mirror symmetry and the existence of negative refraction. We next apply righthanded and lefthanded symmetry to polarization and refraction. In acoustic fields theory and electromagnetic theory there is righthand or clockwise polarization and lefthand or anticlockwise polarization. The direction of polarization is in turn related to the angle of refraction. For instance transverse electric polarization will give rise to negative refraction and transverse magnetic polarization will give rise to positive refraction. This shows there is a relation between the direction of polarization and the angle of refraction. Next we look at the possibility of obtaining negative refraction even with materials with positive permeability and positive permittivity. This will ease the difficulty of fabrication of materials yielding negative refraction and widen the scope of application of such materials.

1. INTRODUCTION

In this paper we generalise Veselago's[1] negative refraction theory to anisotropic solids using symmetry. Veselago's theory is applicable to electromagnetic wave and is limited only to isotropic solids using negative permeability and negative permittivity. Most solids are anisotropic. Our treatments of using symmetries to interpret negative refraction is a more sophisticated approach.

2. LEFT-RIGHT OR MIRROR SYMMETRY

It is well known that in nature, all forces except the weak nuclear force obey the left-right or mirror symmetry, and hence the parity conservation. This will include also elastic forces. In nonpiezoelectric anisotropic media, the Fresnel relations are further complicated by the fact that the plane wave solutions are not always pure shear and pure longitudinal. Consequently, all three polarizations may be coupled at the boundary. Numerical computation is generally required for solving such problems. In performing these calculations, it is most convenient to use only the particle velocity field $\vec{v}$. Consider propagation in the YZ plane of a hexagonal crystal, there exists a pure shear wave polarized normal to this plane, but the propagation characteristics are anisotropic with respect to different directions in the plane. For generality, the boundary will be assumed to be tilted with respect to the Z axis, which is assumed to have a different orientation in the upper and lower media. This leads to a difference between the angles of incidence and reflection. B. A. Auld [2] shows that the transmission coefficient is given by

$$\frac{B}{A} = \frac{2Z_S(\theta_{IS})\cos\theta_{IS}}{Z_S(\theta_{RS})\cos\theta_{RS} + Z'_S(\theta'_{TS})\cos\theta'_{TS}} \tag{1}$$

where $Z_S =$ characteristic shear wave impedance for the upper media, $Z'_S =$ characteristic shear wave impedance for the lower media, $\theta_{IS} =$ angle of incidence for the shear wave, $\theta_{RS} =$ angle of refraction for the shear wave, $\theta'_{TS} =$ angle of transmission or refraction for the shear wave.

Eqn. (1) shows mirror symmetry for the angle of refraction θ'_{TS} as both positive angle of refraction and negative angle of refraction give the same value for the transmission coefficient. This shows that negative refraction also obeys the same physical law or negative refraction can exist.

3. RELATIONSHIP BETWEEN POLARIZATION AND REFRACTION

There is righthanded or clockwise polarization and lefthanded or anticlockwise polarization. The direction of polarization in turn is related to the angle of refraction. For instance transverse electric polarization will give rise to negative refraction and transverse magnetic polarization will give rise to positive refraction. Thus this approach has a more general application, applicable also to the anisotropic solids. There is analogy between the electromagnetic fields and the acoustic fields. The stress field is equivalent to the electric field and the velocity field is equivalent to the magnetic field. There is no dependence on the permittivity and the permeability for the electromagnetic case which are equivalent to the stiffness and the mass density for the acoustic case. The stiffness matrix in acoustics in turn has rotational symmetry. This in turn can strengthen the left-right symmetry of the angle of refraction.

4. POSSIBILITY OF OBTAINING NEGATIVE REFRACTION WITH MATERIALS WITH POSITIVE PERMITTIVITY AND PERMEABILITY

It is well known that negative refraction happens at an interface between a usual isotropic medium (vacuum) and a material with negative parameters. However, recent studies have shown that backward waves can propagate also in materials with positive parameters provided one of the material is chiral. Anisotropic materials exhibit a richer manifold of anomalous behaviour and offer more flexibility in application. More recently it was found that negative refraction can occur in anisotropic material where all the susceptibilities are positive[3] This will make lefthanded materials easier to fabricate and also provide a wider application range.

5. CONCLUSION

Symmetry principle provides a more sophisticated approach to negative refraction phenomenon. It also enables the extension of negative refraction to anisotropic materials and to acoustic fields.

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