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EXTRACTION OF WAVENUMBERS AND AMPLITUDES FROM PLATE VIBRATION MEASUREMENTS

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Abstract

This paper introduces a technique for extracting the wavenumbers and displacement amplitudes from experimental vibration measurements of a plate simply supported along two parallel edges. The displacement field at any location on the plate can be predicted from the extracted wavenumber and amplitudes used in conjunction with an analytical waveguide-based model. Using simply supported boundary conditions along two parallel edges of the plate, the structural response can be described by a combination of a modal and travelling wave solution. A technique is described whereby the measured response is separated into modal components using spatial Fourier transforms. The travelling wave solution is then extracted from each of these modal components using an iterative least-mean-square technique to identify the wavenumbers and wave amplitudes. The technique is applied to vibration measurements obtained experimentally for a single plate under broadband excitation. The waveguide properties from the experimental data are successfully extracted and the errors involved in applying this technique to experimental results are discussed. This method can be used to experimentally determine plate properties, vibrational responses such as energy levels and in the calculation of transmission coefficients for finite coupled structures.

1. INTRODUCTION

The analytical waveguide method has been extensively used to model the dynamic responses of beam [1], plate [2-4] and cylindrical [5] structures. Two important parameters in this method are the wavenumbers and wave amplitudes from which the displacement at any location on the structure can be calculated. Wave-based models have also been used to determine other quantities of built-up structures such as energy flow [2] and transmission coefficients [6].

This paper focuses on determining structural wave amplitudes and wavenumbers from response measurements. Experimental data has been used with waveguide based models to determine the Young's modulus and damping in beams [7-11]. Grosh and Williams [7] used a method originally developed for radar and sonar applications to determine the flexural wavenumber of a beam. A least-mean-squares (LMS) technique was then used to determine the wave amplitudes. McDaniel *et al.* [8] highlighted that the method employed by Grosh and

Williams [7] was sensitive to noise and requires a large number of measurement locations. They demonstrated a method of extracting the wave amplitudes of a beam from experimental data collected under impact loading. This method uses a wave-based analytical expression for the beam's displacement to determine the wave amplitudes from a set of measured displacements based on an estimate of the wavenumber. The wavenumber was varied such that the difference between the displacements obtained using the extracted wave amplitudes and the measured data was minimised using an iterative LMS approach. Using this technique, the structural damping could be estimated at any frequency, whereas in other methods, damping in structures had been calculated at natural frequencies using a bandwidth approach. McDaniel and Shepard [9] further investigated the above method using spatially sparse data points and found that the method was still robust. Interestingly, they found that better results could be achieved using random measurement spacing rather than the traditional fixed width spacing required by Fourier analysis. Liao and Wells [11] presented a variation on the aforementioned technique in order to minimise the variation in the wave amplitudes predicted across the plate. This was achieved by collecting the data using a laser vibrometer whereas the previously mentioned work used either accelerometers or strain gauges.

In this paper, the wave amplitude and wavenumber extraction techniques presented in refs. [8] and [9] are extended to plates that are simply supported along two parallel edges. The dynamic response of the plate is modelled by a modal solution across the width of the plate and a travelling wave solution along the length of the plate. A spatial Fourier transform can be used to decompose the displacement field measured across the width of the plate into modal components. The parameters of the travelling wave solution are then obtained using the wave extraction techniques. The technique is applied to experimental response data measured from a rectangular plate. The capability to determine the wave amplitudes and wavenumbers from experimental data allows the vibratory field to be extrapolated to any location on the plate. The wave amplitudes determined using this method enables the transmission coefficients between coupled structures to be experimentally obtained by using the wave amplitudes to evaluate the directional energy flow [6]. In addition, using this method to develop an interpolated model of the displacement field would allow the average energy levels in a plate to be calculated from sparse data measurements.

2. THEORY

2.1 Analytical waveguide method

The equation of motion for the flexural motion of a plate, w, under a point force excitation at a location (x_o, y_o) is given by [12]:

$$D\nabla^4 w + \rho h \frac{\partial^2 w}{\partial t^2} = F_o \delta(x - x_o)\delta(y - y_o)e^{j\omega t} \quad (1)$$

where $\nabla^4 = \nabla^2\nabla^2$ and $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the Laplace operator. $D = Eh^3/12(1-\upsilon^2)$ is the plate flexural rigidity, where E is the Young's modulus, υ is Poisson's ratio, ρ is the density, h is the plate thickness. Damping is introduced into the system using a complex Young's modulus given by $\hat{E} = E(1 + j\eta)$ where η is the structural loss factor. The flexural displacement of a plate with two parallel simply supported edges results in a modal solution in the $y-$direction and a travelling wave solution in the $x-$direction. A general solution for the flexural displacement is given by [3]:

$$
\begin{aligned}
w(x,y,t) &= \sum_{m=1}^{\infty} (A_{1,m} e^{-jk_x x} + A_{2,m} e^{jk_x x} + A_{3,m} e^{-k_n x} + A_{4,m} e^{k_n x}) \sin(k_y y) e^{j\omega t} \\
&= \sum_{m=1}^{\infty} \phi_{x,m}(x) \phi_{y,m}(y) e^{j\omega t}
\end{aligned} \tag{2}
$$

where $A_{q,m}$ are the wave displacement amplitudes. The subscripts q and m respectively refer to the wave index and mode number. The first two waves of amplitude $A_{1,m}$ and $A_{2,m}$ represent travelling waves in the positive and negative $x-$directions, respectively. The last two waves are respectively evanescent waves in the positive and negative $x-$directions. $k_y = m\pi / L_y$ is the wavenumber in the $y-$direction. $k_x = \sqrt{k_p^{\ 2} - k_y^{\ 2}}$ and $k_n = \sqrt{k_p^{\ 2} + k_y^{\ 2}}$ are the wavenumbers along the $x-$direction for the propagating and evanescent waves, where the flexural wave number of the plate is given by $k_p = \sqrt[4]{\omega^2 \rho h / D}$. Since the plate flexural displacement is time harmonic with radian frequency ω, the time dependent term $e^{j\omega t}$ is omitted in the proceeding analysis.

2.2 Wave extraction technique

A total of $R \times S$ response measurements are taken in a grid pattern on the plate. For each set of S evenly spaced measurements across the width of the plate ($y-$direction), the discrete Fourier transform is applied to the displacement given by Eq. (2), thereby removing the contribution of the function $\phi_{y,m}(y)$ and separating the response along the $x-$direction into its individual modal components, $\phi_{x,m}(x_r)$. The discrete Fourier transform is given by [13]:

$$
\phi_{x,m}(x_r) = -\frac{j}{S} \sum_{s=0}^{S-1} w(x_r, y_s)\, e^{-j2\pi s m/S} \tag{3}
$$

where x_r is the r^{th} measurement from R locations in the $x-$direction and y_s is the s^{th} measurement from S locations in the $y-$direction. The discrete Fourier transform produces a solution of the form $z_m(s) = Z_{real,m} \cos(m\pi\, s/S) + jZ_{imag,m} \sin(m\pi\, s/S)$. The $-j$ term in Eq. (3) converts the result into a real sine rather than cosine form as is required by the form of Eq. (2). In this paper, the discrete Fourier transform is performed using the MATLAB FFT function and the result is then multiplied by $-j$ to obtain $\phi_{x,m}(x_r)$.

Once $\phi_{x,m}(x_r)$ has been determined using Eq. (3) for each mode m, a combination of a LMS technique and iterative process is used to estimate the wavenumbers and amplitudes for the plate. The error function used for the LMS fit is given by:

$$
\begin{aligned}
\varepsilon_r &= \phi_{x,m}(x_r) - \phi^{e}_{x,m}(x_r) \\
&= \phi_{x,m}(x_r) - A_{1,m} e^{-jk_x x_r} - A_{2,m} e^{jk_x x_r} - A_{3,m} e^{-k_n x_r} - A_{4,m} e^{k_n x_r}
\end{aligned} \tag{4}
$$

where $\phi^{e}_{x,m}(x_r)$ is the LMS estimate of the measured $\phi_{x.m}(x_r)$ at the r^{th} measurement location. The square of this function needs to be minimised for all the measurement points such that

$T = \sum_r \varepsilon_r^{\ 2}$, where T is the total error function. The total error function is differentiated with respect to each of the wave amplitudes and is then set to zero to find the minimum error. It can be shown that the LMS wave amplitudes are given by:

$$\begin{Bmatrix} A_{1,m} \\ A_{2,m} \\ A_{3,m} \\ A_{4,m} \end{Bmatrix} = \begin{bmatrix} \sum_r e^{-2jk_x x_r} & R & \sum_r e^{-jk_x x_r} e^{-k_n x_r} & \sum_r e^{-jk_x x_r} e^{k_n x_r} \\ R & \sum_r e^{2jk_x x_r} & \sum_r e^{jk_x x_r} e^{-k_n x_r} & \sum_r e^{jk_x x_r} e^{k_n x_r} \\ \sum_r e^{-k_n x_r} e^{-jk_x x_r} & \sum_r e^{-k_n x_r} e^{jk_x x_r} & \sum_r e^{-2k_n x_r} & R \\ \sum_r e^{k_n x_r} e^{-jk_x x_r} & \sum_r e^{k_n x_r} e^{jk_x x_r} & R & \sum_r e^{2k_n x_r} \end{bmatrix}^{-1} \begin{Bmatrix} \sum_r \phi_{x,m}(x_r) e^{-jk_x x_r} \\ \sum_r \phi_{x,m}(x_r) e^{jk_x x_r} \\ \sum_r \phi_{x,m}(x_r) e^{-k_n x_r} \\ \sum_r \phi_{x,m}(x_r) e^{k_n x_r} \end{Bmatrix} \tag{5}$$

The LMS function in Eq. (5) is dependent on the wavenumbers k_x and k_n of the plate but can be described by a single unknown corresponding to the plate flexural wavenumber, k_p. An iterative approach is used to find k_p such that the error is again minimised using the total error function. In this study, the MATLAB function FMINSEARCH was used to iteratively search for the optimal value of the wavenumber. Fig. 1 gives details the iterative process used to determine the wave number and amplitudes for the plate.

Once the plate flexural wavenumber is determined and assuming that the density and Poisson's ratio are known, the properties of the plate can be found using the complex Young's modulus, which is given by $\hat{E} = 12(1-\upsilon^2)\omega^2\rho / k_p^{\ 4}h^2$. The Young's modulus and damping are respectively given by $E = \mathrm{Re}(\hat{E})$ and $\eta = \mathrm{Im}(\hat{E}) / \mathrm{Re}(\hat{E})$ [9].

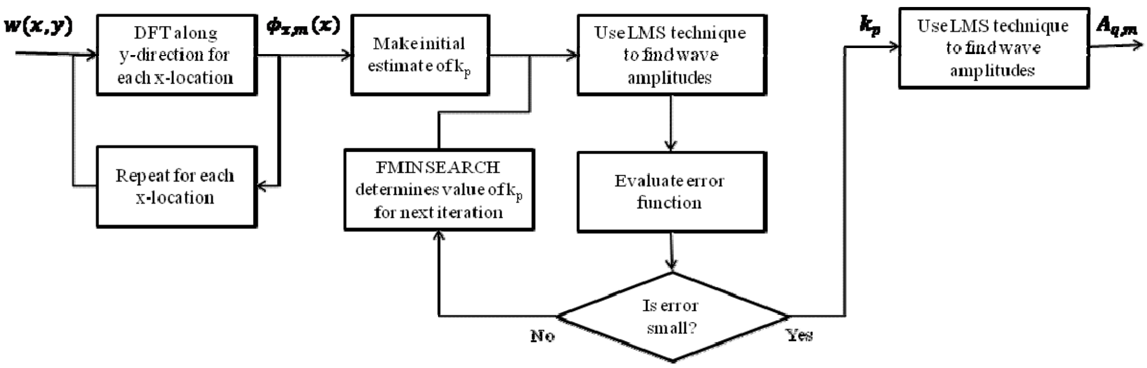


Figure 1. Process for extracting wavenumbers and amplitudes from experimental data.

3. RESULTS

Experimental results are presented for a flat plate with dimensions of length L_x=480 mm, width L_y=1100 mm and with a thickness of h=2 mm. The plate is of aluminium with an estimated Young's modulus of E=71 MPa, density ρ=2800 kg/m^3 and Poisson's ratio υ=0.3. The damping loss factor is estimated to be η=0.001. All coordinate locations are in mm. The aluminium plate is supported between two 'V-shaped' pieces of aluminium (see Fig. 2) to imitate simply supported boundary conditions. The plate was excited at

$(x_o, y_o) = (300,192)$ using pseudo random noise. An LDS V203 shaker used to drive the plate via a thin wire stinger was vertically mounted on the plate as shown in Fig. 2 to simulate point force excitation. An Endevco 2312 force transducer was used to measure the input force. The responses were measured using a B&K Type 4393 accelerometer attached to the plate with a thin layer of wax. The signals detected by the accelerometer and force transducer were conditioned by the B&K Type 2635 charge amplifiers and the results were processed by a B&K Pulse signal analyser.

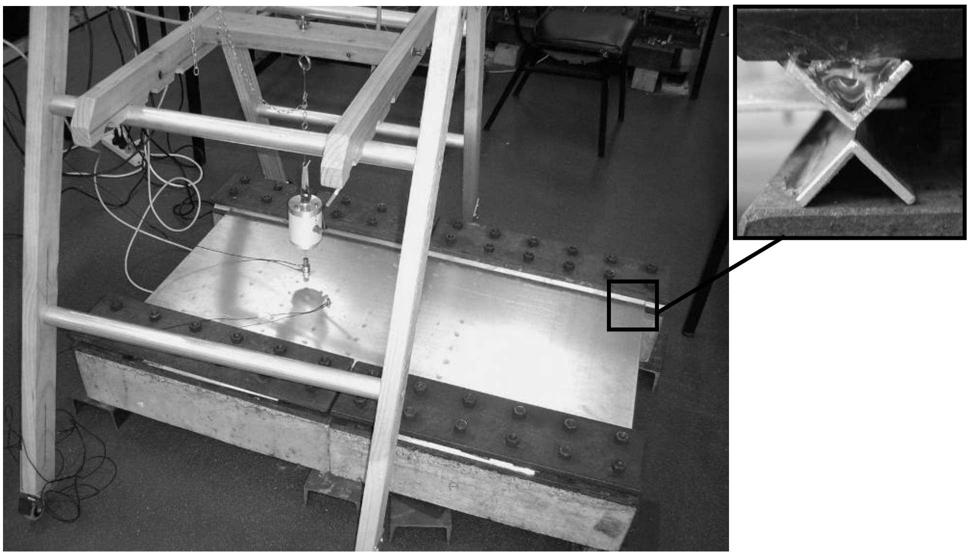
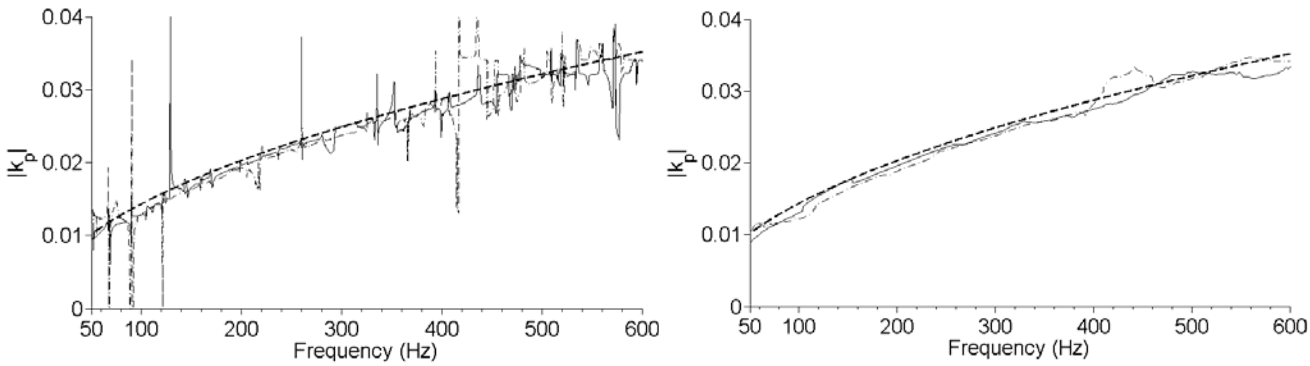


Figure 2. Photograph of the experimental rig showing details of the simply supported boundaries.



(a) (b)

Figure 3. Wavenumbers extracted from experimental data: predicted wavenumber (--------) and experimentally extracted wavenumbers for mode *m*=1 (———) and mode *m*=2 (— · —). (a) Raw data; (b) 50 Hz moving frequency band averaged.

The plate flexural wavenumbers extracted from the experimental results are shown in Fig. 3. Data is presented above 50 Hz as below this frequency the coherence was poor. Figure 3(a) shows the predicted wavenumber based on the estimated Young's modulus and damping compared to those extracted from the experimental result. As can be seen from this figure, the general trend of the wavenumbers extracted for both the *m*=1 and *m*=2 modes follows the predicted values reasonably well. In Fig. 3(b), the extracted wavenumbers have been averaged using a 50 Hz moving frequency averaging band and the resulting wavenumbers are shown to very closely match the predicted wavenumber.

There are distinct peaks in the experimentally extracted wavenumbers that significantly deviate from the predicted values. The reason for these discrepancies is attributed to the mass loading introduced by the accelerometers at each measurement location. Due to this mass loading the natural frequencies for some measurement locations have been shifted such that some of the measurements on the plate have been taken prior to resonance and others after resonance. There is a significant phase change in lightly damped structures associated with resonance, thereby altering the dynamics of the plate for each measurement location. This introduces significant error when considering the shape of the plate at a particular frequency that is close to resonance. This phenomenon is shown in Fig. 4, where the acceleration field is shown for two frequencies. Figure 4(a) shows the acceleration field measured just prior to resonance, while Fig. 4(b) shows the acceleration field measured very close to resonance. In theory Fig. 4(a) and 4(b) should have very similar shapes as the same mode is dominating the response at both frequencies (see Fig. 4), but the measured shapes are very different. This is because the phases of some points in Fig. 4(b) are 180º out of phase with the surrounding measurements which is shown by the highlighted section of Fig. 4(b). In fact it can be shown that reversing the phase of the affected region produces an acceleration field very similar to that shown in Fig. 4(a) and would also match results predicted by an analytical waveguide model. This mass loading error introduces significant error into the spatial Fourier transform used to separate the modal components and causes this transform to detect higher order modes that are not actually significantly contributing to the response. The mass loading problem could be reduced or eliminated by using lighter accelerometers or alternatively a laser vibrometer system [11]. Unfortunately neither of these solutions were readily available for the present study.

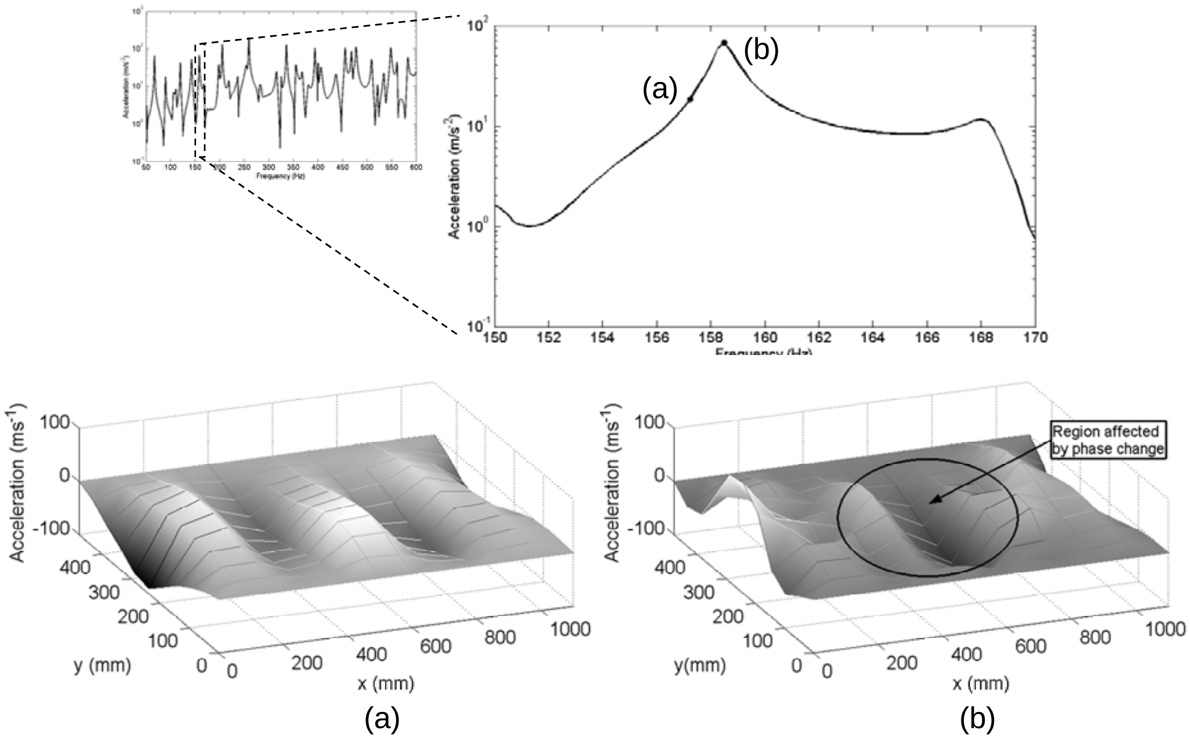
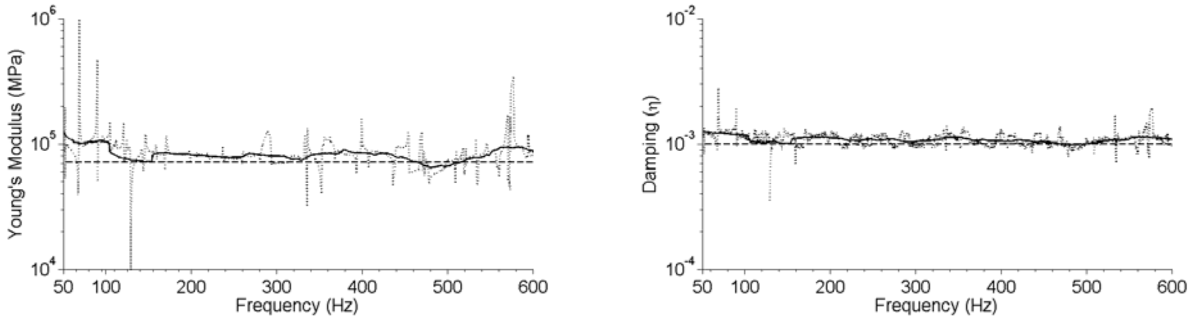


Figure 4. Acceleration spectrum measured at (x,y)=(700,240). Also shown is the acceleration field over the whole plate at (a) 157.25 Hz which is prior to resonance and (b) 158.5 Hz which is close to resonance showing the localised effect of mass loading changing the phase of the measurement relative to the surrounding points.

Even though the results are subject to experimental errors, the wavenumber can still be used to estimate the Young's modulus and damping of the plate. Figure 5(a) presents the Young's modulus calculated from the experimental wavenumber data and shows reasonable agreement with the initial estimate. Similarly, the damping shown in Fig. 5(b) is also close to the initial estimates. It should be noted that the initial estimates are purely based on typical properties of aluminium so deviation of the experimental values from this estimate does not necessarily indicate an error. In order to remove the distinct peaks, a 50 Hz moving frequency averaging band has been used on the values for the Young's modulus and damping and the results are also shown in Fig. 5. This averaging shows that the extracted values are in very agreement with the typical values once the error introduced by the mass loading is removed.



(a) (b)

Figure 5. Comparison of Young's modulus (a) and damping (b) extracted from the raw data (·········), experimental data (———) with a 50 Hz moving frequency averaging band and values used in the analytical model (----------).

The wave amplitudes have also been extracted. In order to generate an analytical model that more closely represents the experimental model, the moving frequency averaged Young's modulus and damping extracted from experimental results were used in an updated analytical model. Figure 6 shows the amplitude $A_{1,1}$ for the fundamental mode (m=1) for travelling waves along the positive x-direction extracted from the experimental data. The wave amplitude generated by the waveguide model using the updated material properties is also shown. This figure shows a relatively good match between the experimental and analytical results considering the errors that have been found associated with the extraction of the wavenumbers. The peaks present in the experimental data that are not present in the analytical results are thought to be due to the mass loading and thus introduced resonances from higher order modes. Above 500 Hz the modelling also appears to breakdown and the reason for this requires further investigation. It is suggested that all of the extracted parameters could be enhanced using an improved measurement method with either lighter accelerometers or a laser vibrometer to minimise the impact of mass loading associated with the instrumentation.

4. CONCLUSIONS

The structural wavenumbers, Young's modulus and damping have successfully been extracted using a wave extraction technique for experimental data. It was found that the instrumentation altered the natural frequencies associated with some measurement locations, leading to errors in the experimental measurements. It is suggested that using lighter accelerometers or a laser vibrometer system would reduce the error associated with the added mass of the accelerometers. The wave amplitudes were also extracted from the experimental data and these compared well with analytical results. Future work using this method includes using the experimentally extracted wave amplitudes to predict the average energy levels and transmission coefficients in coupled structures.

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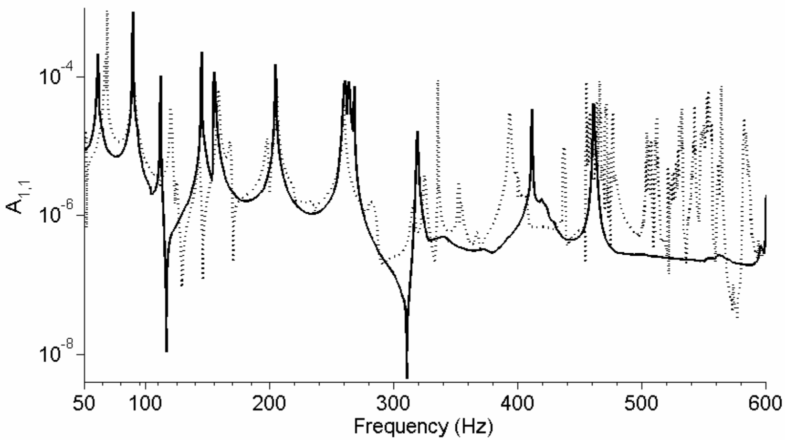

Figure 6. Wave amplitude $A_{1,1}$ calculated analytically (———) and extracted from experimental measurements (··········). Note that in the analytical waveguide model, the Young's modulus and damping were updated with the frequency averaged wavenumber extracted experimentally.

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