ICSV14
Cairns • Australia
9-12 July, 2007
14th International Congress on Sound & Vibration

STATISTICAL ENERGY ANALYSIS USING TRANSMISSION COEFFICIENTS DERIVED FOR FINITE COUPLED PLATES

Michael Skeen and Nicole Kessissoglou

School of Mechanical and Manufacturing Engineering, University of New South Wales
Sydney, NSW 2052, Australia
m.skeen@student.unsw.edu.au

Abstract

Prediction of vibration transmission in built up structures such as buildings or ship hulls is an important consideration in modern engineering design. As the excitation frequencies increase and hence the number of modes increase, it is more practical to consider average vibrational responses and their distribution over the structure using energy techniques such as Statistical Energy Analysis (SEA). The successful use of SEA strongly depends on the accurate estimation of the SEA parameters, namely the coupling loss factors (CLFs). This paper investigates the dynamic response of coupled plates using an SEA model with CLFs obtained from transmission coefficients derived for finite coupled plate structures. Traditionally, the CLFs in an SEA model have been obtained from transmission coefficients derived for infinite and semi-infinite structures, due to the ease with which they can be derived. Conversely, transmission coefficients for finite structures are difficult to obtain due to the reverberant field. In this paper, transmission coefficients for finite coupled structures are derived using an analytical waveguide method which is used to determine the wave amplitudes on each side of a junction. A scattering matrix is then used to separate the reverberant waves leaving the junction into reflected and transmitted components. The energy flow due to each of these waves is obtained using a wave impedance method, which is subsequently used to determine the transmission coefficients. CLFs are obtained from the transmission coefficients for a finite L-shaped plate under multiple point force excitation. The SEA subsystem energy levels using these CLFs are compared to results obtained from traditional SEA theory as well as frequency and spatially averaged energy levels obtained from the analytical waveguide method. Results for both the CLFs and SEA energy levels for semi-infinite and finite structures are in very good agreement with each other and with the exact average energy levels obtained from the analytical model.

1. INTRODUCTION

Statistical Energy Analysis (SEA) is an established predictive technique for high frequency vibration analysis and is based on the transfer of energy between subsystems [1,2]. SEA relies on the accurate determination of a number of parameters including the coupling loss factor (CLF), damping loss factor and the input power in order to make a meaningful prediction of the vibrational energy of each subsystem. SEA is based on coupling power proportionality which implies that the power transfer between two connected subsystems is proportional to the modal energy difference between them [2]; the CLF is essentially the proportionality constant for this relationship. Due to the importance of the CLFs as an SEA parameter, it has received much research attention. There are several different approaches for deriving CLFs but the most common is the wave approach to determine transmission coefficients from which the CLFs can be obtained [2-5].

To determine the transmission coefficient of a junction, it is generally required to calculate the energy flow incident on and through the junction and this has been the focus of various energy methods. Energy flow in a plate structure using the Poynting vector method has been presented by Romano *et al.* [3] and accounts for energy transmission due to flexural and in-plane motion. Transmission coefficients for finite structures have not been widely studied due to the general assumption that, at least in the frequency average, the transmission coefficients for semi-infinite and finite structures are equivalent [4]. It has been common practice to use the transmission coefficient derived for a semi-infinite system as an approximation for that of a finite system [2,4-6]. Park *et al.* [7] examined 'semi-finite' structures where either the source or receiver plate was finite. They demonstrated that the finite boundary conditions significantly altered the transmission coefficients from those predicted for the semi-infinite structure, though there were similarities in the transmission coefficients predicted for both systems. Another reason for using the transmission coefficient derived from a semi-infinite structure as an approximation to that of a finite system is due to the difficulty involved in separating the transmitted and reflected wave components from the outgoing waves leaving the junction in a reverberant structure. Wester and Mace [8] used a scattering matrix derived from a wave approach to determine the outgoing waves produced by incoming waves incident on a junction of a coupled plate structure and used a combination of these coupled junctions to model an entire plate structure.

In this paper, CLFs are obtained from transmission coefficients derived for finite coupled plates. The scattering matrix method has been employed as a means of separating the incoming and outgoing waves at a coupled finite plate junction. A wave impedance method is then used to determine the energy due to each wave component, from which the energy flow associated with incident, reflected and transmitted waves can be obtained. The transmission coefficients derived for finite structures are used to determine the SEA CLFs and are compared to similar results obtained using transmission coefficients derived for infinite and semi-infinite coupled plates. In addition, SEA energy levels for semi-infinite and finite systems are compared with the exact average energy levels obtained from the analytical waveguide model.

2. THEORY

2.1 Analytical waveguide method

An L-shaped plate system as shown in Fig. 1 is examined, which is simply supported along two parallel edges corresponding to $y = 0$ and L_y and free at the other two edges corresponding to $x_1 = 0$ and $x_2 = L_{x2}$. The junction of the two plates corresponds to $x_1 = L_{x1}$ and $x_2 = 0$. The two plates are of the same material and thickness. An external point force excitation of amplitude F_{in} at a location $(x_{in},\ y_{in})$ is applied to plate 1. The equation of motion for the flexural motion of a plate w is given by [9]:

$$D\nabla^4 w + \rho h \frac{\partial^2 w}{\partial t^2} = F_{in}\delta(x - x_{in})\delta(y - y_{in})e^{j\omega t} \tag{1}$$

where $\nabla^4 = \nabla^2\nabla^2$ and $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the Laplace operator. D is the plate flexural rigidity, ρ is the density and h is the plate thickness. Damping has been included using a complex Young's modulus, $\hat{E} = E(1 + j\eta)$, where E is the Young's modulus and η is the structural loss factor.

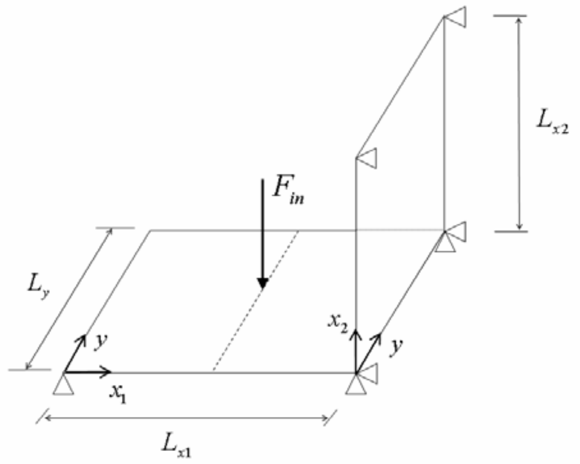

Figure 1. L-shaped plate dimensions and coordinate system.

Using the analytical waveguide method, the flexural displacement of the plate with two parallel simply supported edges results in a modal solution in the $y-$direction and a travelling wave solution in the $x-$direction. A general solution for the flexural displacement is given by [10]:

$$w_p(x, y, t) = \sum_{m=1}^{\infty}(A_{p,1}e^{-jk_x x} + A_{p,2}e^{jk_x x} + A_{p,3}e^{-k_n x} + A_{p,4}e^{k_n x})\sin(k_y y)e^{j\omega t} \tag{2}$$

where $A_{p,q}$ are the wave displacement amplitudes, where the subscripts p and q refer to the plate number and wave index, respectively. The first two waves of amplitude $A_{p,1}$ and $A_{p,2}$ represent travelling waves in the positive and negative $x-$directions respectively. The last two waves are evanescent waves in the positive and negative $x-$directions. $k_y = m\pi/L_y$ is the wavenumber in the $y-$direction, where m is the mode number. $k_x = \sqrt{k_p{}^2 - k_y{}^2}$ and $k_n = \sqrt{k_p{}^2 + k_y{}^2}$ are the wavenumbers along the $x-$direction for the propagating and evanescent waves, where $k_p = \sqrt[4]{\omega^2 \rho h / D}$ is the plate flexural wavenumber. The wave displacement amplitudes in each section of the L-shaped plate are evaluated using the boundary conditions at the free edges and continuity equations at the external force location and L-junction. The boundary conditions at the free edges corresponding to $x_1 = 0$ and $x_2 = L_{x2}$ result in zero bending moment and net vertical shear force [9]. Due to the external force, there are four coupling equations at $x = x_{in}$, corresponding to continuity of displacement and slope and equilibrium of the moments and shear forces [10]. The coupling equations at the junction of an L-shaped plate are well established [6,11]. The internal forces and moments acting on the plate are given by [9]:

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \upsilon\frac{\partial^2 w}{\partial y^2}\right), \quad Q_x = -D\left(\frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x \partial y^2}\right), \quad M_{xy} = -D(1-\upsilon)\frac{\partial^2 w}{\partial x \partial y} \qquad (3\text{-}5)$$

where M_x is the bending moment, Q_x is the shear force due to bending, M_{xy} is the twisting moment and υ is Poisson's ratio. The unknown wave coefficients can be determined by arranging the various equations into a matrix of the form $\boldsymbol{\alpha}\mathbf{A} = \mathbf{F}$, where the matrix $\boldsymbol{\alpha}$ contains details of the boundary and continuity equations, the vector **A** contains the unknown wave displacement amplitudes, and the vector **F** contains details of the external force applied to plate 1. The wave displacement amplitudes can then be found by $\mathbf{A} = \boldsymbol{\alpha}^{-1}\mathbf{F}$.

2.2 Transmission Coefficients

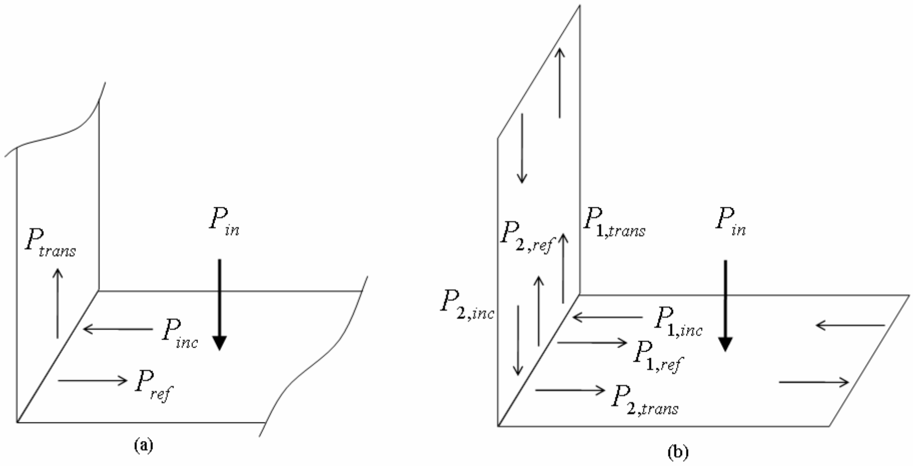

Figure 2. Power incident, transmitted and reflected on the junction of (a) semi-infinite and (b) finite plates.

The transmission coefficient τ for a junction is defined as the ratio of transmitted power P_{trans} to the incident power P_{inc} and is given by [6]:

$$\tau = \frac{P_{trans}}{P_{inc}} \qquad (6)$$

A simplified method of calculating the transmission coefficient for infinite L-shaped beam and plate structures was presented by Cremer *et al.* [6]. An approximation for the transmission coefficient for an infinite L-shaped plate with homogeneous properties is given by:

$$\tau_{12} = \frac{2}{\left(\sigma^{5/2} + \sigma^{-5/2}\right)^2} \qquad (7)$$

where $\sigma = h_1 / h_2$ is the ratio of the thickness of the two plates. For plates of the same thickness ($\sigma = 1$), the transmission coefficient is $\tau_{12} = 0.5$. To evaluate the transmission coefficient for finite structures, it is necessary to separate the energy flow leaving the junction into components due to reflection and transmission.

Figure 2(a) and 2(b) shows the waves generated by an external force for a semi-infinite and finite L-shaped plate, respectively. P_{in} is the input power due to the external force and P_{ref} is the reflected power. With a single excitation, semi-infinite structures only generate one set of active incident, transmitted and reflected waves, as shown in Fig. 2(a). In the case of a finite structure (Fig. 2(b)), a reverberant field is generated by reflections at the finite plate edges.

There are a number of ways that energy flow can be evaluated from a waveguide solution including the Poynting [3] and wave impedance [8] methods. Using the Poynting method, the time averaged net energy flow in the $x-$direction per unit width of plate is given by [3]:

$$P_x = \frac{1}{L_y}\int_0^{L_y} \mathrm{Re}\left(\dot{w}^* Q_x - \frac{\partial \dot{w}^*}{\partial x} M_x - \frac{\partial \dot{w}^*}{\partial y} M_{xy} \right) \partial y \tag{8}$$

where $\dot{w}$ denotes derivative of w with respect to time. When Eq. (8) is fully expanded, there are cross power terms involving interactions between the positive and negative travelling and evanescent waves. These cross power terms are commonly neglected in wave impedance based methods on the assumption that travelling waves alone are the dominant mode of energy transmission [8]. Using this assumption, the energy flow that is associated with a particular wave can be determined from Eq. (8) by substituting only the component of the displacement associated with that wave. Referring to Eq. (2), the displacement $w_{p,q}$ associated with a travelling wave of amplitude $A_{p,q}$ ($q = 1, 2$) is given by:

$$w_{p,q} = \sum_{m=1}^{\infty} A_{p,q} \sin(k_y y) e^{\pm j k_x x} e^{j\omega t} \tag{9}$$

Using this approach, the power flow is separated into positive and negative components associated with the positive and negative travelling waves. This method can be used to obtain the energy flow associated with the incoming and outgoing waves at a junction. The energy flow due to reflected and transmitted waves at a junction can now be determined using a scattering wave matrix which is described in what follows.

2.3 Scattering Matrix Method

The scattering matrix is used to separate the outgoing waves generated at a junction in a reverberant system into components due to reflected and transmitted waves. The scattering matrix is developed by rearranging the coupling equations for the plate junction such that the outgoing wave amplitudes are calculated for a given set of incoming wave amplitudes. Hence, the coupling equations can be expressed as $\mathbf{a}\mathbf{A}_{out} = \mathbf{b}\mathbf{A}_{in}$, where $\mathbf{A}_{in} = \{A_{1,1} \;\; A_{1,3} \;\; A_{2,2} \;\; A_{2,4}\}^T$ is a vector containing the amplitudes of waves incoming to the junction from each plate, and $\mathbf{A}_{out} = \{A_{1,2} \;\; A_{1,4} \;\; A_{2,1} \;\; A_{2,3}\}^T$ contains the amplitudes of waves produced at and leaving the junction. Both $\mathbf{a}$ and $\mathbf{b}$ are matrices simply derived by rearranging the coupling equations in the matrix $\boldsymbol{\alpha}$. The scattering matrix $\mathbf{T}$ is given by $\mathbf{T} = \mathbf{a}^{-1}\mathbf{b}$ and hence $\mathbf{A}_{out} = \mathbf{T}\mathbf{A}_{in}$.

The first step in separating the reflected and transmitted wave components in a finite structure is to evaluate the wave displacement amplitudes using the waveguide method. The incoming waves (both travelling and evanescent) that are incident on one side of the junction are then used as the input to the scattering matrix with the incoming wave from the opposite direction being set to zero. The outgoing wave amplitudes are then calculated and these represent the transmitted and reflected waves. The process is repeated for incoming waves on the opposite side of the junction. By separating the wave displacement amplitudes, the energy flow associated with each wave can be determined using Eqs. (8) and (9). The transmission coefficient is then evaluated using Eq. (6).

2.4 Coupling Loss Factors

SEA is based on coupling power proportionality, which theorises that the energy flow between two sub-systems is proportional to the modal energy difference between the two sub-systems. This has been proven analytically for two coupled oscillators [12] and generic structures such as plates and beams for which CLF expressions have been derived in terms of the frequency and space averaged Green functions of the coupled system [13]. The CLFs are an important SEA parameter and have a large impact on the accuracy of subsystem energy level predictions. There are a number of ways that CLFs can be derived, for example, the general expression used to determine the CLF for two plates joined along a line in terms of a wave-based transmission coefficient is given by [2]:

$$\eta_{12} = \frac{2c_p L\langle\tau_{12}\rangle}{\pi\omega S_1} \tag{10}$$

where L is the length of the junction, ω is the centre frequency of the band of interest, S_1 is the surface area of the plate 1, $c_p = \omega / k_p$ is the bending wave velocity of flexural waves in the first plate and $\langle\tau_{12}\rangle$ is the diffuse wave transmission coefficient from plates 1 to 2. Using a wave-based approach, the transmission coefficient is generally evaluated for semi-infinite plates by determining the transmission coefficient with respect to the incident wave angle and then taking the diffuse wave transmission coefficient to be that integrated over all incident angles [2]. In this paper, the diffuse wave transmission coefficient is evaluated by averaging the transmission coefficients over an ensemble of random input force locations [7]. Details of SEA equations used to find subsystem energy levels are well established and can be found in texts such as Lyon and DeJong [2].

3. RESULTS

Results are presented for an L-shaped plate as shown in Fig. 2 with dimensions of $L_{x1} = 1200\,\text{mm}$, $L_{x2} = 500\,\text{mm}$ and $L_y = 450\,\text{mm}$. Both plates have a thickness of $h = 2\,\text{mm}$. The plates are of aluminium with a Young's modulus of E=71 MPa, density ρ=2800 kgm^{-3}, Poisson's ratio υ=0.3 and structural loss factor of η=0.03. A high value of damping was used to ensure high modal overlap and a broad frequency range over which the SEA analysis is valid.

Figure 3 presents the frequency averaged (1/3 octave band) CLFs using Eq. (10). In Fig. 3, CLFs are presented for transmission coefficients which were obtained for infinite, semi-infinite and finite L-shaped coupled plates. The CLF using the transmission coefficient τ_{12} for an infinite L-plate using Eq. (7) predicts a much higher CLF than that predicted using τ_{12} for the semi-infinite and finite plates. As there are waves travelling in both directions in a finite plate, the transmission coefficient can be evaluated in both directions and hence there are two CLFs for the finite system; τ_{12} (plates 1 to 2) and τ_{21} (plates 2 to 1). For the semi-infinite plate, reciprocity was used to determine the CLF from plates 2 to 1 [2]. The CLFs for energy transmission from plates 1 to 2 are nearly identical for both the finite and semi-infinite plate systems. However, for the semi-infinite system, the CLF from plates 2 to 1 is considerably higher than that predicted for the finite case.

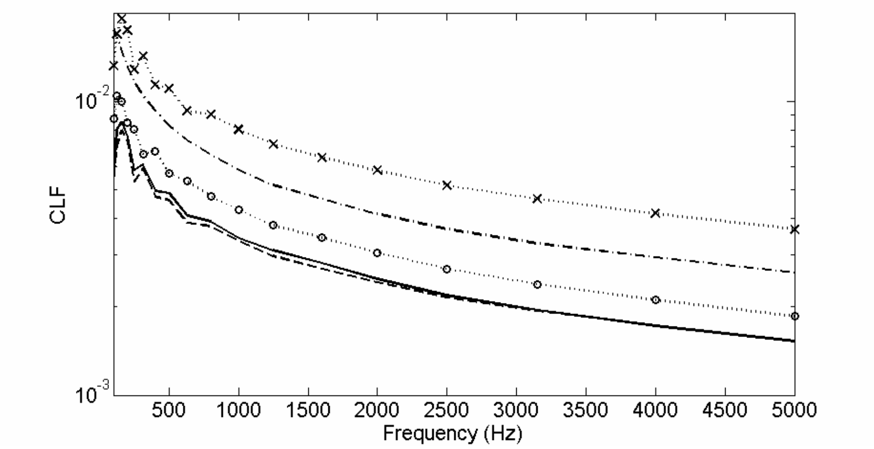

Figure 3. CLFs predicted using transmission coefficients for infinite plates (1 to 2 – · –), semi-infinite plates (1 to 2 – – –, 2 to 1 ···×···) and finite plates (1 to 2 ——, 2 to 1 ···○···).

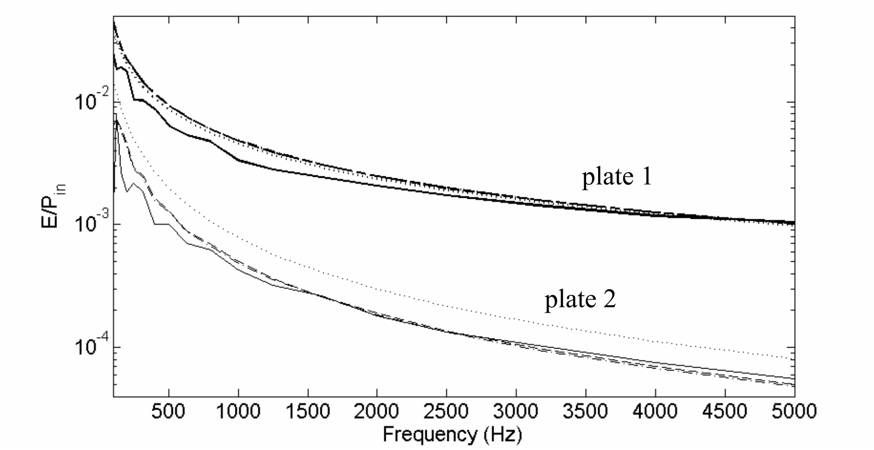

Figure 4. Average (1/3 octave) normalised energy levels predicted using CLFs for infinite (·······), semi-infinite (– – –) and finite (– · –) plates compared with average normalised energy levels predicted by an analytical waveguide method (——) for an ensemble of 100 random excitation locations on plate 1. Plate 1 is shown as thick lines and plate 2 is shown as thin lines.

Figure 4 shows the frequency averaged plate energy levels normalised with respect to the input power predicted by the SEA models for the three different CLF models. These results are also compared to the normalised frequency and spatially averaged plate energy levels evaluated using the analytical waveguide model of the structure. The plate flexural displacement from the analytical waveguide model was found using 'rain-on-the-roof' excitation at 100 different excitation locations. The total vibrational energy at each frequency ω for each plate was evaluated by [2]:

$$E_p = \rho h \omega^2 \int_A w_p^* w_p \, dA \qquad (11)$$

where A is the area of each plate and the superscript $*$ denotes the complex conjugate.

Figure 4 shows that using the transmission coefficient for infinite plates in an SEA model over-predicts the mean energy levels. The SEA energy levels predicted using both the finite and semi-infinite plates are nearly identical. Results show that an acceptable SEA model can be produced using semi-infinite transmission coefficients and that the differences between the CLFs observed in Fig. 3 make little difference to the end result in this case. The frequency and spatially averaged plate energy levels evaluated from the analytical model also followed a similar trend to those obtained from the SEA models.

4. CONCLUSIONS

This work presents a method to obtain transmission coefficients for finite coupled structures. The transmission coefficients were used to obtain the coupling loss factors in an SEA model. The CLFs obtained for the semi-infinite and finite plates were significantly lower than that for an infinite structure. It was shown that the transmission coefficients and subsequent CLFs derived for the finite and semi-infinite structures resulted in SEA energy levels that were in very good agreement with each other.

This work was carried out by funding provided by the Australian Research Council under an ARC Discovery grant DP0451313.

REFERENCES

[1] F.J. Fahy, "Statistical energy analysis: A critical overview", *Philosophical Transactions of the Royal Society of London, Series A* **346**, 431-447 (1994).

[2] R.H. Lyon and R.G. DeJong, *Theory and Application of Statistical Energy Analysis*, Butterworth-Heinemann, Boston, 1995.

[3] A. Romano, P. Abraham and E. Williams, "A Poynting vector formulation for thin shells and plates, and its application to structural intensity analysis and source localization. Part I: Theory", *Journal of the Acoustical Society of America* **87**, 1166-1175 (1990).

[4] T.D. Scharton and R.H. Lyon, "Power flow and energy sharing in random vibration", *Journal of the Acoustical Society of America* **43**, 1332-1343 (1968).

[5] R.J.M. Craik, *Sound transmission through buildings using Statistical Energy Analysis*, Gower, London, 1996.

[6] L. Cremer, M. Heckl, and E.E. Ungar, *Structure-borne sound*, 2nd ed., Springer-Verlag, Berlin, 1988.

[7] W.H. Park, D.J. Thompson and N.S. Ferguson, "The influence of modal behaviour on the energy transmission between two coupled plates", *Journal of Sound and Vibration* **276**, 1019-1041 (2004).

[8] E.C.N. Wester and B.R. Mace, "Wave component analysis of energy flow in complex structures - Part I: a deterministic model", *Journal of Sound and Vibration* **285**, 209-227 (2005).

[9] S. Timoshenko and S. Woinowsky-Krieger, *Theory of plates and shells*, 2nd ed., McGraw-Hill, New York, 1959.

[10] X. Pan and C.H. Hansen, "Active control of vibratory power transmission along a semi infinite plate", *Journal of Sound and Vibration* **184**, 585-610 (1995).

[11] N.J. Kessissoglou, "Power transmission in L-shaped plates including flexural and in-plane vibration", *Journal of the Acoustical Society of America* **115**, 1157-1169 (2004).

[12] R.H. Lyon and G. Maidanik, "Power flow between linearly coupled oscillators", *Journal of the Acoustical Society of America* **34**, 623-639 (1962).

[13] R.S. Langley, "A general derivation of the statistical energy analysis equations for coupled dynamic systems", *Journal of Sound and Vibration* **135**, 499-508 (1989).