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AN ANALYTICAL MODEL FOR DRILLSTRING AXIAL VIBRATION

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Abstract

The axial vibrational states experienced by the active components of a drilling assembly such as that found in the oil or gas industry are discussed in the context of an analytical model. This paper is purposed by the need to understand the axial vibrational states that such a system can demonstrate in order to better control their constructive and destructive potential. The objective is to present more reliable analytical models of drillstring axial vibration. In the proposed model, the drillstring is modeled as an equivalent piecewise uniform bar and particular attention is given to boundary conditions, the excitation mechanism of a bit, the effect of mud viscosity, and the effect of tooljoints. Field experiments were performed in Ahwaz well (in the South of Iran). The results are used to validate the proposed model and to better understand drillstring axial vibration. The good agreement between the measured and predicted result give considerable confidence to the use of the analytical model developed.

1. INTRODUCTION

A drilling assembly consist essentially of a series of hollow cylindrical steel pipes connected to form a long flexible drillstring to which is attached a short heavier segment containing a cutting device at the free end (the drillbit). This segment may contain stabilizing fins designed to minimize lateral motion during drilling and together with the drill-bit constitutes the bottom-hole assembly (BHA). A schematic drawing of drillstring system, which includes all the basic components of a conventional drillstring (drill collar section, pipe section, Kelly, swivel, and traveling block), and an elastic wire cable and derrick is shown in Fig. 1. The drillstring is used to transmit energy from the surface to the drillbit. The drillstring is driven in a rotary fashion from the top end, often by means of an electric motor and gearbox, the top drive, and constrained to pass at a controlled rate through a rotating mass (the rotary) near the surface. Such a drilling system is designed to construct a borehole linking the earth's surface to a reservoir of oil or gas. The bore-hole is lined and the excess in the diameter of this cavity over the diameter of the drill-pipe is referred to below. This annular gap is necessary for the conduction of fluids. These are a source of external interaction along the drill-string in addition to gravity and the bore-liner. During the process of drilling pressurized mud is continuously circulated down the centre of the drill-string, out of holes in the drill-bit and back to the surface via the space between the rotating drill-string and the surface of the bore-hole. Its primary purpose is to cool and lubricate the drillbit as well as to remove cuttings produced by the bit. Such a system is prone to dynamic instabilities that are not fully understood. Field experience provides ample testament to the destructive consequences of such instabilities.

The dynamic behavior of a drillstring has long been of interest because of the damage it might cause to surface drilling equipment and to the drillstring itself and because of its potential influence of penetration rate. Drillstring vibrations are an important cause of premature failure of drillstring components, Washout, Twist-off and drilling inefficiency. For a vertical well, the application of drillstring dynamics is mostly to prevent fatigue failure of drillstring. Drillstring vibration is also an important issue for mud pulse elementary because of coupling between the mud acoustics and drillstring vibration. The dynamic behavior of a drillstring includes axial, torsional, bending, and whirling motion [1-3]. Among these, axial vibration has received the most attention in literature [1, 4-8], because the governing equations are the effects of the axial vibration can be observed at the surface in the form of Kelly bouncing and whipping of the draw works cables. Axial vibration cause bit bounce and rough drilling, behavior that destroys bits, damages bottom hole assemblies (BHA), increases total drilling time and may be detected at surface. The objective of this paper is to present more reliable analytical models of drillstring axial vibration. Axial vibration and torsional vibration are analogous, and many of the equations and result that follow may be easily applied to torsional vibration problems.

2. DEVELOPMENT OF AXIAL VIBRATORY THEORY

In [9] a theory of wave propagation in drilling borehole, which is a cylindrically multi-layered waveguide, was presented. The results revealed that the confining effects of the internal and external mud on drillstring axial vibration are largely negligible. The damping due to radiation into the formation was theoretically derived, and was found to negligible at low frequencies.

The prediction of drillstring axial vibration requires the development of better analytical models. Improvements to the existing models are to be found in modeling of the effects of mud, tooljoints, boundary condition, and excitation mechanism of a bit.

An analytical model of drillstring axial vibration is proposed. The theory assumes that each type of vibrations along the drillstring is independent of the other two. Since the different equations of motion and boundary conditions for the longitudinal and torsional motions are similar, one general solution can be used for both cases. The theory is developed only in terms of longitudinal parameters, however, the final results can be used to predict both longitudinal and torsional along the drillstring. In the proposed model, the drillstring is modeled as an equivalent stepwise uniform bar, and topside and bottom boundary conditions are modeled with equivalent masses, springs, and dampers as shown in Fig.1. Computer codes have been developed to predict the response.

2.1 Boundary conditions

Forces act at top of the drillstring and are therefore considered part of drill string boundary conditions. These forces are internal force of swivel, traveling block, Kelly, forces associated with pipe stress directly below Kelly, and forces acting through the stranded wire cable. The topside boundary condition of drillstring depends on the mechanical properties of the derrick and the power swivel or rotary table. In Fig. 1, is modeled with equivalent masses, springs, and dampers. Summation of these forces gives an equilibrium equation that serves as the boundary condition for the top end of the drillstring. Continuing down the drillstring, the drillpipe and drill collars are connected rigidity.

The bottom boundary condition depends on the nature of the contact between the bit and the formation, bit type, and the properties of the formation which are investigated experientially. The relative bit displacement model is used. The bottom boundary condition of drillstring is modeled as an equivalent linear spring and damper, assuming that bit stays in contact with the bottom formation. As long as the bit force is compressive, this assumption is valid. It is possible to predict the bit bounce for given excitation. If the bit bounces and loses the contact, this boundary condition would be free at that instant.

When a drillstring vibrates in the axial direction in a borehole filled with mud, the effect of the mud can be treated as an added mass and damping distributed along the drillstring. The effect of mud can be treated as an added mass and damping distributed along the drillstring.

The effect of mud viscosity, which was neglected in multi-layer analysis of the [9] is estimated as an added mass and damping, for purpose of predicting axial vibration of drillstring.

2.2 Equation of motion

The equation of motion is obtained by summing the forces that act on a differential drillstring element, as shown in Fig. 1. These forces include an internal force associated with material stress: $EA\frac{\partial^2 u}{\partial x^2}$, a viscous friction force:
$\left\{\left[2\pi(a+b)\left(\frac{\rho_m \mu\omega}{2}\right)\right]+c_r+c_o\right\}\frac{\partial u}{\partial t}$ and internal forces: $\left\{\rho A+\left[2\pi(a+b)\left(\frac{\rho_m \mu}{2\omega}\right)\right]\right\}\frac{\partial^2 u}{\partial t^2}$
where u, x, t are axial displacement, axial coordinate, time coordinate respectively, ρ, E, A are density, Young's modulus, cross sectional area of drillstring respectively, a, b are inner, outer radius of drillstring respectively, μ, ρ_m are density viscosity of mud respectively, ω is circular frequency in radians, c_r, c_o are the radiation losses into the surrounding formation, the damping due to internal hysteretic losses in the drillstring material and frictional losses due to rubbing against the wall. Therefore, the equation governinig the axial motion of the drillstring become:

$$EA\frac{\partial^2 u}{\partial x^2}=\left\{\rho A+\left[2\pi(a+b)\left(\frac{\rho_m \mu}{2\omega}\right)\right]\right\}\frac{\partial^2 u}{\partial t^2}+\left\{\left[2\pi(a+b)\left(\frac{\rho_m \mu\omega}{2}\right)\right]+c_r+c_o\right\}\frac{\partial u}{\partial t} \qquad (1)$$

Computer codes have been implemented to obtain transfer functions, describing the response per unit harmonic input excitation at any location on the drillstring. The solution technique is used in the transfer matrix method [10]. The whole system is subdivided into subsystems which are either uniform bar or mass-spring-damper. For n^{th} the subsystem, the displacement and force at the lower end $(u_{1,n}, F_{1,n})$ can be obtained in terms of those at the upper end $(u_{u,n}, F_{u,n})$ by introducing a transfer matrix.

The displacement and force at lower end can be expressed in terms of those at the upper end as shown below:

$$\begin{Bmatrix} u_{lower} \\ F_{lower} \end{Bmatrix} = T \begin{Bmatrix} u_{upper} \\ F_{upper} \end{Bmatrix} \tag{2}$$

where T is the transfer matrix. The transfer matrix of uniform bar, T_{bar} is:

$$T_{bar} = \begin{bmatrix} \cos kl & \sin kl/EAk \\ -EAk\sin kl & \cos kl \end{bmatrix} \tag{3}$$

where l, E, and A are length, Young's modulus and cross sectional area, respectively. $k^2 = \frac{\rho\omega^2}{E}\left(1 - i\frac{c}{\rho A\omega}\right)$ where c is damping coefficient per unit length and ω is circular frequency. A drillstring consists of drillpipe and bottom hole assembly (BHA). The similar sections of drillcollars in BHA are modelled as one equivalent uniform bar. The drillpipe body is modelled as an equivalent uniform bar and the tooljoint is modelled as another equivalent uniform bar.

The transfer matrix of a mass-spring-damper system, T_{m-s-d} as shown below:

$$T_{m-s-d} = \begin{bmatrix} 1 & 1/(k + i\omega C) \\ \omega^2 M & 1 - \omega^2 M/(k + i\omega C) \end{bmatrix} \tag{4}$$

where M is mass, C is damping coefficient and k is spring constant.

The topside boundary condition of drillstring is modelled as mass-spring-damper subsystems. The blocks and power swivel are modelled as masses of subsystem. The wire rope and the supporting structure are modelled as spring and damper of subsystem. The bottom boundary condition of the drillstring is modeled as an equivalent linear spring and damper.

The interaction between the bit and the bottom formation depends on the bit type. For a roller cone bit, previous measurements [6] have shown that the main frequency of the drillstring axial excitation is usually at $3 \times RPM$ (three times the rotational speed).

In the proposed model, a unit hormonic relative displacement is imposed at the bit, as described in the equation below:

$$R_b = D_b - D_f = e^{i\omega t} \tag{5}$$

where D_b, D_f and R_b are bit displacement, displacement of bottom formation and relative displacement between bit and bottom formation, respectively. The transfer functions due to this relative displacement can be obtained from those due to the absolute bit displacement.

This model is a simple beginning at extending the understanding of the effect of the actual bottom boundary conditions on the dynamics of the drillstring.

3. EXPERIMENTAL TESTS

For each point along a drillstring, the vibration theory discussed can be employed to predict bit displacement caused by bit in the formation. Field experiments were performed in Ahwaz well. Most of measurements of drillstring forces and motions in this well were of low amplitude. Simultaneous measurements of variation in longitudinal motion were made at the bit with downhole recorder and at the surface with an accelerometer mounted on the nonrotating section of the swivel. Sensors measured axial acceleration, axial force, rotational speed and pressure. Using the source of topside excitation, it was possible to excite the drillstring at the top with a known axial force. The input force and resulting acceleration were measured at swivel during the topside excitation at different rotational speeds. An RPM increasing test was performed while drilling at the constant weight on bit (WOB) with an insert bit, without the topside excitation. Four kinds of tests were performed and the results are compared with theoretical results. More field tests should be conducted as a further check on the vibration theory presented in this paper.

4. DISCUSSION OF THEORETICAL PREDICTIONS

Some of the dynamical properties of the active components of a drilling assembly are used in terms of a simple model. The equations of motion arise from the geometrically equations together with a prescription of constitutive relations and external forces. Many of the approximations can be readily derived from such a model.

4.1 Effect of rotary speed on drillstring vibration

The angular speed is typically between 30 and 150 RPM. For a drill-string with Young's modulus of elasticity E and shear modulus G with (dimensions $[ML^{-1}T^{-2}]$) a natural unit of time is chosen to be $T_0 = L_0/c$ where $c^2 = E/\rho$ and the value of L_0 is the physical length of the drill-string. This suggests the introduction of the dimensionless evolution parameter $\eta = t/T_0$.

For the off bottom test with topside excitation, the measurements are taken at various RPM's (30, 60, and 90) to investigate the effect of RPM on drillstring axial vibration. In these measured transfer functions, it can be seen that the frequencies of the peaks and zeros are invariant respect to the RPM change. The principal features of the magnitudes and phase of the measured acceleration per unit force transfer functions ($\dfrac{Acceleration\ at\ swivel}{Force\ at\ swivel}$) are 4 peaks at 2.95, 8.75, 12.5 and 15.5 to 16.5 Hz as well as 3 zeros at around 7, 12 and 14.65 Hz.

The three mode shapes at 3.05, 8.65 and 15.65 hertz are called pipe modes and the mode at 12.6 hertz is called a BHA mode. These tests revealed that the dynamic properties of the drillstring axial vibration are RPM independent. However, the excitation at the bit would be RPM dependent when drilling. These tests also good agreement between the measured and predicted result gave considerable confidence to the use of the analytical model developed.

The on bottom tests without rotation showed that the bottom boundary condition strongly depends on static WOB. The RPM increasing test while drilling showed that the effective bottom boundary condition is quit different than under weighted but nonrotating conditions.

The difference is that while rotating, the bit is fracturing the rock. This appears to introduce a lower equivalent stiffness at the rock-bit boundary and introduces additional damping.

4.2 Effect of mud on drillstring vibration

The effect of mud viscosity on the drillstring axial vibration is estimated as an added mass $2\pi(a+b)\sqrt{\dfrac{\rho_m \mu}{2\omega}}$ and damping as $2\pi(a+b)\sqrt{\dfrac{\rho_m \mu \omega}{2}}$ distributed along the drillstring.

4.3 Effect of damping on drillstring vibration

The response of the drillstring to excitation is quite sensitive to damping. Anagona [11] and Dareing [6] determined the damping by measuring the transient axial vibration. Squire [12] presented theoretical methods to estimate the damping due to radiation and mud viscosity, but his expression for radiation damping is incorrect.

The damping of the drillstring comes from sources: viscous losses due to the motion of the drillstring in the viscous drilling fluid, $2\pi(a+b)\sqrt{\frac{\rho_m \mu}{2\omega}}$ was obtained theoretically, radiation losses in to the surrounding formation, c_r was obtained from analysis of the formation, and internal hysteretic losses in the drillstring material and frictional losses due to rubbing against the wall. The last two of sources are modeled damping coefficient, c_o as being dependent on "weight on bit" and have been determined experimentally from forced vibration tests with topside excitation.

The increase in axial vibration is induced by a change of rock lithology as shown in Fig. 2. The increase in axial vibration is also induced by a change of rock lithology as shown in Fig. 3. In Fig. 4, the reaction force on the drillbit WOB relative to the top-tension at the rotary during the period is shown. The dynamic history of the axial vibration speed in the drill-string during the period is shown in Fig. 5.

4.4 Effect of tooljoint on drillstring vibration

A drillstring is assembled from sections of pipe approximately 9.144 m (30 ft) in length. the ends of each section have tooljoints with cross sectional areas that are several times larger than the pipe body. Barnes [13] has studied the effects of the tooljoints on wave propagation in the infinitely long drillstring.

The drillstring consists of drillpipe and bottom hole assembly (BHA). The drillpipe consisted of 35 sections and the length of the BHA is 201.473 m (661 ft). the stop band around 250 and 500 Hz can be seen in this predicted transfer function.

5. CONCLUSIONS

1- A relative bit displacement model was proposed by including the effect of flexibility of rock being drilled.
2- The flexibility of the rock should be included to correctly predict the resonant frequencies of the drillstring axial vibration.
3- The on bottom tests without rotation showed that the bottom boundary condition strongly depends on static WOB.
4- The RPM increasing test while drilling showed that the effective bottom boundary conditions quit different than under weighted but nonrotating conditions.
5- The good agreement between the measured and predicted result gave considerable confidence to the use of the analytical model developed.
6- The new model discussed above has a wide domain of applicability. The model can be extended in a number ways to include more realistic interactions of the drillstring and BHA with their environments. It could also be useful for investigating the vortex induced vibration of marine risers in shear flow that are responsible for instabilities that occur in offshore drilling operations. It offers practical guides to engineering problems under consideration and provides an efficient means of gaining both detailed information and broad understands into the dynamical behavior associated with "strings" with attachments.

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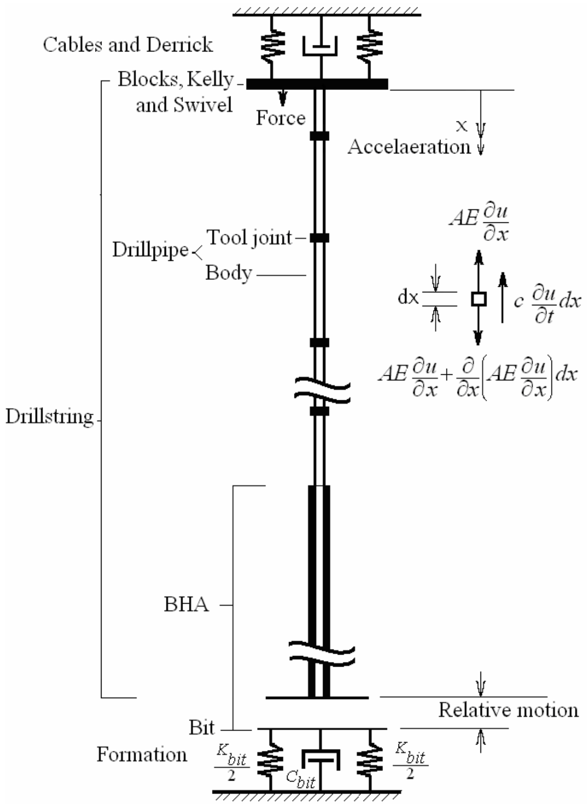

Figure 1. Analytical model of drillstring.

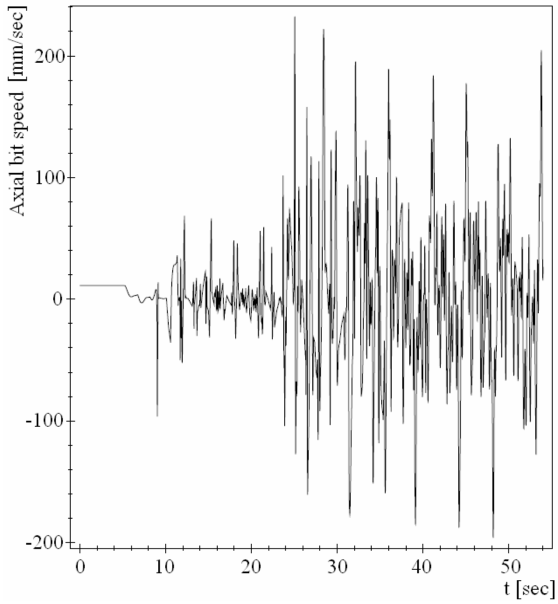

Figure 2. Variation of the vertical oscillatory component of the speed of the drill-bit as a function of time (Secs).

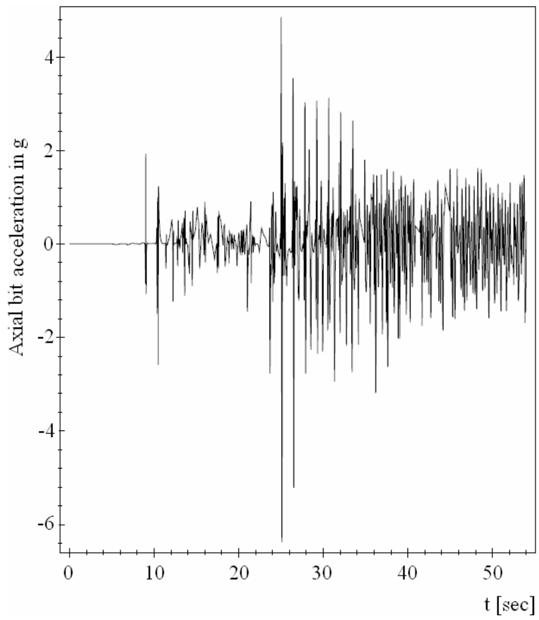

Figure 3. Variation of the vertical acceleration of the drill-bit as a function of time.

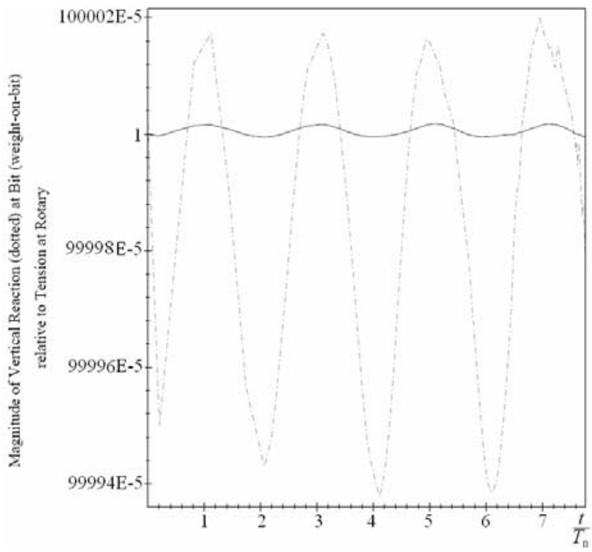

Figure 4. The reaction force on the drill-bit (weight-on-bit) relative to the top-tension at the rotary during the period.

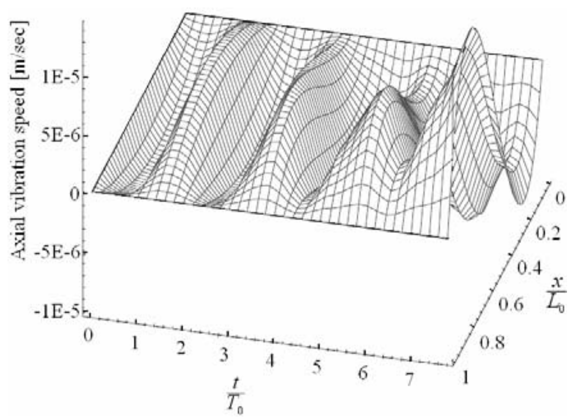

Figure 5. The dynamic history of the axial vibration speed in the drill-string.