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VIBRATION BASED MODELLING OF THREE POINT BEND SPECIMEN FOR EVALUATION OF DYNAMIC STRESS INTENSITY FACTOR

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Abstract

A vibration based modelling of impact loaded three point bend (TPB) specimen is presented for evaluation of dynamic stress intensity factor (DSIF). As a first approximation, neglecting the shear deformation and rotational inertia, the specimen has been analyzed through Euler-Bernoulli beam theory. In the present approach a multiple degree of freedom (MDOF) model has been proposed by representing the discontinuity due to crack through a mass-less rotational spring to obtain temporal motion of the specimen under impact loading. This approach offers scope for an evaluation of the DSIF and even dynamic fracture toughness through the motion related data. Sample results on the DSIF have been presented for the case of step and ramp loading. These results compare closely with results computed through two-dimensional transient finite element (FE) analysis. Possibility of an application of present method of analysis to instrumented impact tests is shown through a comparison of results with those published in the literature.

1. INTRODUCTION

Fracture toughness of engineering materials is a function of rate of loading and temperature [1]. The safety assessment of critical structures subjected to such loadings is based on dynamic fracture toughness data. Under such loadings, the inertia of material coupled with inelastic deformation at the crack tip, may increase resistance to crack initiation and further growth [2]. However, for certain materials, especially in ferritic steels, the mode transition from ductile to brittle fracture at higher rate of loading leads to a remarkable reduction in the fracture toughness [3]. It has been reported that the fracture toughness of ferritic steels, when tested at constant temperature decrease with increasing strain-rate [4]. Due to all these, it is important to evaluate the dynamic fracture toughness of engineering materials. This makes the dynamic modelling of test specimens very important. The present paper addresses this issue.

Dynamic fracture toughness evaluation over a wide range of loading rates is extensively done through impact testing of pre-cracked TPB specimens. Typically, instrumented impact testing is carried out using pendulum, drop-weight or Hopkinson bar setup. The dynamic fracture toughness K_{Id} is then referred to the value of DSIF at instant of crack initiation, i.e. $K_{Id} = K_I(t_i)$, where t_i is time of crack initiation. This makes measurement of DSIF, $K_I(t)$, and accurate recording of the instant of crack initiation, t_i, very important. A dynamic modelling of the specimen is very helpful in collection of data and interpretation of the test results.

Direct measurement of the DSIF has been possible through the method of caustics, which is based on the crack-tip out-of-plane deformations [5]. However, these methods require very complex and expensive equipments for collection of data. Usually the estimation of the DSIF is done through a remote measurement of force-time history in an instrumented impact hammer [6]. In such an approach the fundamental assumption is that the spatial variation of field variables over the entire specimen including the boundaries is related. It is known that the inertia effects become significant when the specimens are subjected to impact loading and must be taken into account in the analysis. Hence the analysis of test results is proposed on simple inertial models of the pre-cracked TPB specimens. Various researchers have considered spring-mass models for simplified one-dimensional modelling of the pre-cracked specimen where the approximate stiffness of the specimen is evaluated by means of static deflection of pre-cracked beam [6]. Among different approaches used for modelling of the pre-cracked specimen, the solution based on local change of moment of inertia has been extensively used [7].

Modelling of crack like discontinuity using mass-less rotational-spring located at crack location has been widely employed in vibration based methods for crack detection [8]. In the present work a model based on vibration of the pre-cracked TPB specimen is given for analysis of impact tests and to evaluate the history of variation of DSIF with time. The discontinuity due to crack is modelled using the mass-less rotational spring [8]. Possibility of an application of the modelling to the dynamic fracture testing is discussed. The results obtained through present approach compare well with those obtained through two-dimensional Finite Element (FE) analysis.

2. FORMULATION

These In the case of transverse vibration of slender beams with a crack, it is generally assumed that there is an extra angular rotation at the crack location proportional to the bending moment. Hence the crack can be modelled as a rotational spring of infinitesimal length (Figure 1). The stiffness of the spring can be related to the beam geometry and the crack length [8].

$$K_t = \frac{E\,BW^4}{72\,\pi \int_0^a a\left(f\left(a/W\right)\right)^2 da} \tag{1}$$

where $f(a/W)$ is a factor dependent on the geometry of the specimen such that

$$K_I = \sigma\sqrt{\pi a}\; f(a/W) \tag{2}$$

The governing equation of flexural vibration according to Euler-Bernoulli theory is given by,

$$EI\frac{\partial^4 y}{\partial x^4} + \rho A\frac{\partial^2 y}{\partial t^2} = 0 \tag{3}$$

where $y(x,t)$ is the transverse deflection, E is the modulus of elasticity, A the area of cross-section, I the area moment of inertia and ρ the density of beam material. The model based on vibration of edge-cracked TPB is obtained by dividing the given beam into two halves and connecting them with a mass-less rotational spring at the middle of the span.

The governing equation for half segment $(0 < \beta < 1/2)$ is given by,

$$\frac{\partial^4 Y(\beta)}{\partial \beta^4} - \lambda^4 Y(\beta) = 0, \qquad 0 < \beta < 1/2 \tag{4}$$

where, $\lambda^4 = \frac{\omega^2 \rho A L^4}{EI}$, $\beta = x/L$ and ω is the natural frequency.

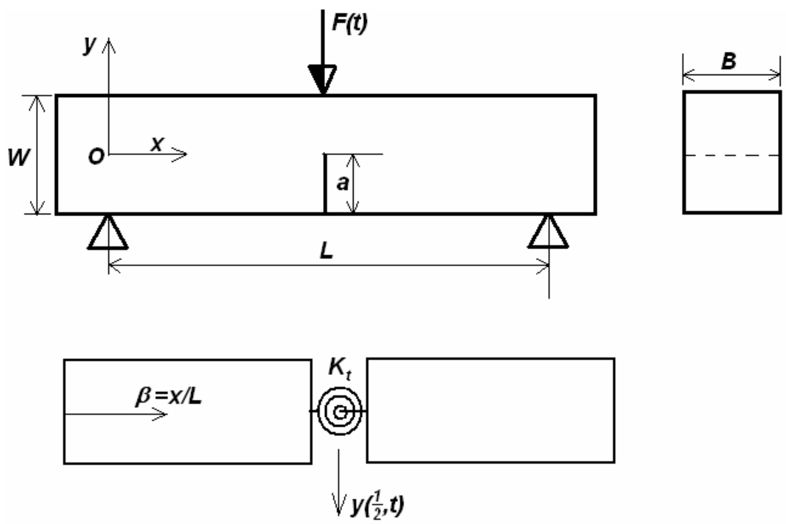


Figure 1. Representation of pre-cracked TPB through rotational spring.

The solution of Equation (4) in terms of arbitrary constants representing the mode shapes is given as follows,

$$Y(\beta) = C_1 \cosh \lambda\beta + C_2 \sinh \lambda\beta + C_3 \cos \lambda\beta + C_4 \sin \lambda\beta \tag{5}$$

Using the boundary conditions $Y\big|_{\beta=0} = 0$ and $\left.\frac{d^2Y}{d\beta^2}\right|_{\beta=0} = 0$, it is obtained that $C_1 = 0$ and $C_3 = 0$. The anti-symmetric modes do not contribute to the mid-span displacement. Hence for these modes, $C_2 = 0$. The frequencies of anti-symmetric modes are obtained using $\lambda/2 = n\,\pi$. In case of symmetric modes however, the presence of crack at mid-span is responsible for the jump in slope between the two halves. Due to symmetry of beam, the change in slope for any symmetric mode shape can be written as,

$$\theta = \frac{2}{L}\left(\frac{dY}{d\beta}\right)_{\beta=1/2} \tag{6}$$

The strain energy released due to the extra rotation is assumed to be stored in rotational spring. Hence, the rotation of spring is given by,

$$\theta_S = \frac{M}{K_t} = -\frac{EI}{K_t L^2}\left(\frac{d^2Y}{d\beta^2}\right)_{\beta=1/2} \tag{7}$$

Combining Equations (5), (6) and (7)

$$C_2\left(\frac{\lambda}{\mathrm{K}}\sinh\frac{\lambda}{2}+2\cosh\frac{\lambda}{2}\right)+C_4\left(-\frac{\lambda}{\mathrm{K}}\sin\frac{\lambda}{2}+2\cos\frac{\lambda}{2}\right)=0 \tag{8}$$

where, $\mathrm{K}=\frac{K_t L}{EI}$ is the non-dimensional rotational-spring stiffness. It may be noted that the rotational spring stiffness, $K_t(a/W)$ given by Equation (1) is assumed to be constant for all the modes. In the case of natural vibrations, additional condition for the symmetric modes is obtained through, $\left.\frac{d^3Y}{d\beta^3}\right|_{\beta=1/2}=0$, i.e. shear force is zero,

$$C_2\cosh\frac{\lambda}{2}-C_4\cos\frac{\lambda}{2}=0 \tag{9}$$

To ensure non-trivial solution for C_2 and C_4,

$$\begin{vmatrix} \cosh\frac{\lambda}{2} & -\cos\frac{\lambda}{2} \\ \frac{\lambda}{\mathrm{K}}\sinh\frac{\lambda}{2}+2\cosh\frac{\lambda}{2} & -\frac{\lambda}{\mathrm{K}}\sin\frac{\lambda}{2}+2\cos\frac{\lambda}{2} \end{vmatrix} = 0 \tag{10}$$

This gives the equation to solve for the natural frequencies of the symmetric modes. Further, assuming $C_{4i}=1$, the symmetric mode shapes can be written as follows,

$$Y_i(\beta)=\frac{\sinh\lambda_i\beta\ \cos\frac{\lambda_i}{2}+\sin\lambda_i\beta\ \cosh\frac{\lambda_i}{2}}{\cosh\frac{\lambda_i}{2}} \tag{11}$$

Finally the solution for transverse deflection at the mid-span under a concentrated load $F(t)$ can be written through convolution integral.

$$y\left(\frac{1}{2},t\right) = \sum_{i=1}^{\infty}\frac{Y_i^2(1/2)}{\omega_i W_i L}\int_0^t F(\tau)\sin\omega_i(t-\tau)\,d\tau \tag{12}$$

where $W_i=\rho A\int_0^1 Y_i^2(\beta)\,d\beta$, and the beam is assumed to be initially at rest.

Since the stress intensity factor is proportional to the magnitude of displacements in the vicinity of crack tip, it can be assumed that

$$K_I(t)=\kappa\ y\left(\frac{1}{2},t\right) \tag{13}$$

The proportional factor κ is determined by using the static solution. The static deflection at mid-span may be expressed as

$$y_s\left(\frac{1}{2}\right)=\sum_{i=1}^{\infty}\frac{Y_i^2(1/2)}{\omega_i^2 W_i L}F_{static} \tag{14}$$

Noting that $K_I^{static}=\kappa\ y_s(1/2)$, κ is obtained from

$$\kappa=\frac{K_I^{static}}{\sum_{i=1}^{\infty}\frac{Y_i^2(1/2)}{\omega_i^2 W_i L}F_{static}} \tag{15}$$

Finally, the dynamic stress intensity factor (DSIF) is given by

$$\frac{K_I(t)}{K_I^{static}} = \frac{\sum_{i=1}^{\infty} \frac{Y_i^2(1/2)}{\omega_i W_i L} \int_0^t F(\tau)\sin\omega_i(t-\tau)\,d\tau}{\sum_{i=1}^{\infty} \frac{Y_i^2(1/2)}{\omega_i^2 W_i L} F_{static}} \tag{16}$$

The relative contribution of any of the higher modes can be quantified through

$$\frac{Z_m}{Z_1} = \frac{\dfrac{Y_m^2(1/2)}{\omega_m W_m}}{\dfrac{Y_1^2(1/2)}{\omega_1 W_1}} \tag{17}$$

The numerical results for $\left(Z_m/Z_1\right)$, $m = 2,3$ and 4 are listed in Table 1 for several values of a/W ratio. It suggests that contribution due to all the modes except for the fundamental mode can be neglected. Thus Equation (16) can be approximated by considering only the fundamental mode

$$\frac{K_I(t)}{K_I^{static}} = \frac{1}{F_{static}}\omega_1 \int_0^t F(\tau)\sin\omega_1(t-\tau)\,d\tau \tag{18}$$

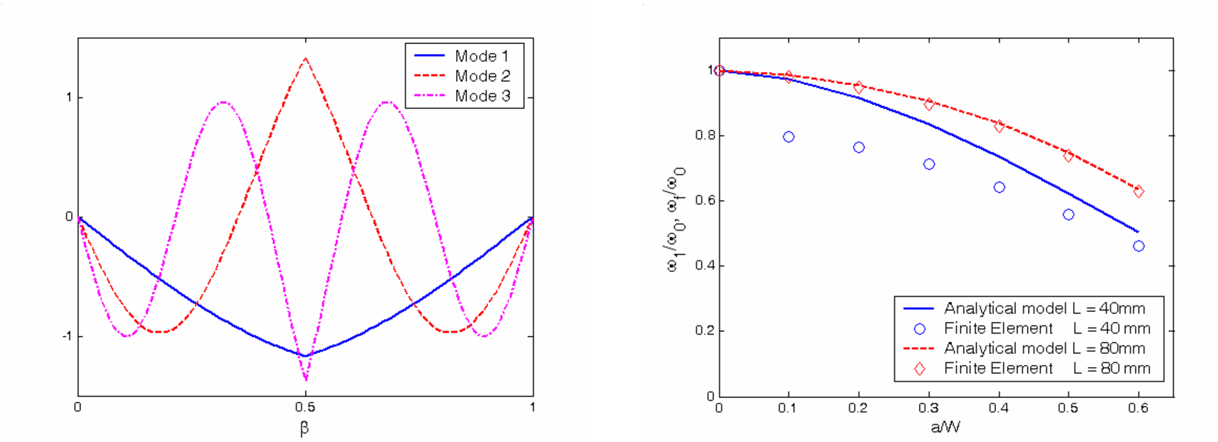
Table 1. Numerical values of $\left(Z_m/Z_1\right)$ in Equation (18).

a/W	Z_2/Z_1	Z_3/Z_1	Z_4/Z_1
0.1	3.8x10^{-2}	1.7x10^{-5}	1.4x10^{-8}
0.2	6.6x10^{-3}	3.9x10^{-6}	4.1x10^{-9}
0.3	3.5x10^{-3}	2.7x10^{-6}	3.2x10^{-9}
0.4	2.8x10^{-3}	2.5x10^{-6}	3.1x10^{-9}
0.5	2.5x10^{-3}	2.4x10^{-6}	3.0x10^{-9}
0.6	2.2x10^{-3}	2.2x10^{-6}	2.8x10^{-9}

3. NUMERICAL SOLUTIONS

The First three symmetric mode shapes of two TPB specimens, with dimensions $L = 40$mm and 80mm, $B = 10$mm and $W = 10$mm, with crack size $a = 5$mm were computed using Equation (11). These are shown in Figure 2(a). The fundamental frequencies obtained using the present model are compared with the results of Finite Element (FE) analysis in Figure 2(b), where the frequencies are normalized by the fundamental frequency of the corresponding simply supported beam without crack.

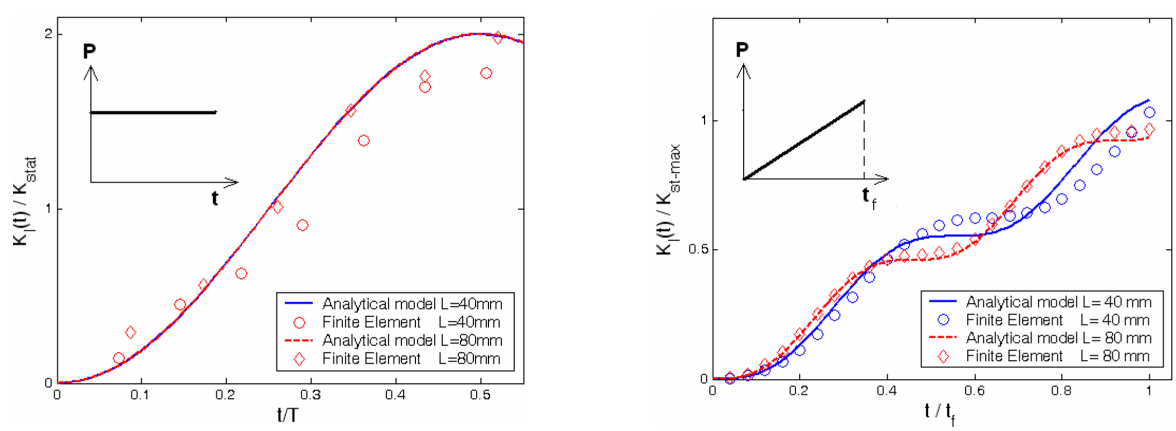
For the two TPB specimens the DSIFs normalized with respect to the quasi-static stress intensity factor are shown in Figure 3(a) for the case of a step load. Similar results for the ramp load are shown in Figure 3(b).



(a) (b)

Figure 2. (a) The symmetric mode shapes of TPB specimen. (b) Comparison of analytical (ω_1) and FE (ω_f) frequencies for fundamental mode of TPB specimen.

The DSIFs are also computed through two-dimensional transient finite element (FE) analysis. The analysis was done using four node plane-strain quadrilateral elements. Implicit time integration method is applied for solution of FE equations. The stress intensity factor is evaluated using modified crack-closure integral (CCI) method.

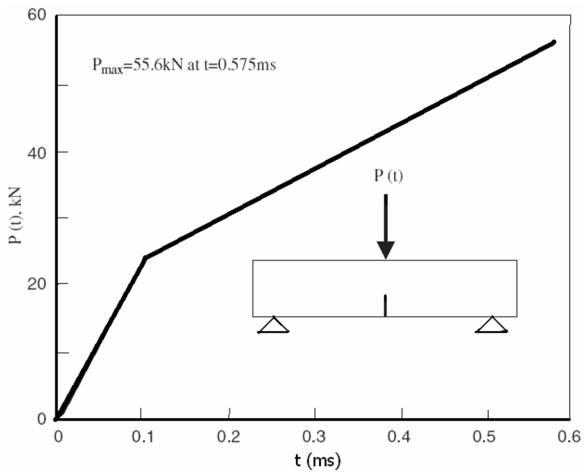


(a) (b)

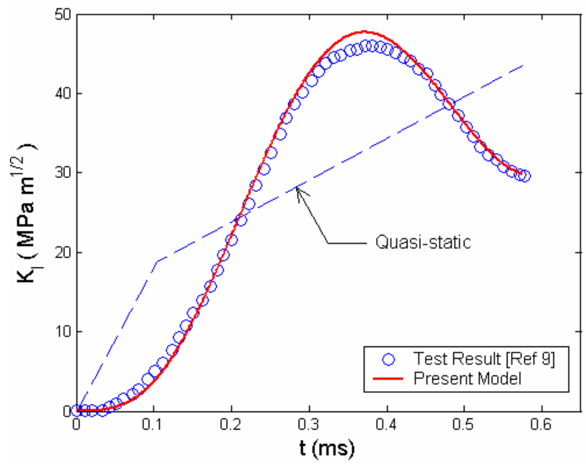
Figure 3. Comparison of analytical and FE results for beam span 40mm and 80mm (a) for step load and (b) for ramp load.

A reasonably good agreement is obtained between the analytical and computed DSIF for S = 80mm with the inclusion of only first mode in the approximation. The maximum difference is 8%. Since the Euler-Bernouli theory is not strictly valid for short beams, the analytical results in case of S = 40mm do not compare well with those obtained through the FE analysis. The maximum difference observed is 35%. The inclusion of shear deformation and rotational inertia effects through Timoshenko beam equations are expected to improve the accuracy.

Aberson et al. [9] attempted a numerical analysis for three point bend specimen of steel subjected to a falling load with the loading history shown in Figure 4(a). The TPB specimen dimensions are $S = 250\,mm$, $B = 25mm$, $W = 76\,mm$ and $a = 25\,mm$. Figure 4(b) shows a comparison of the dynamic stress intensity factor computed using present formulation (solid line), and the test results (circles) of Aberson et al.[9]. The stress intensity factor obtained through quasi-static formula is also shown (dashed line) in Figure 4(b). The fracture toughness K_{IC} was calculated as 43.7 $MPa\sqrt{m}$ for the peak load $P_{\max} = 55.6\,kN$.



(a)



(b)

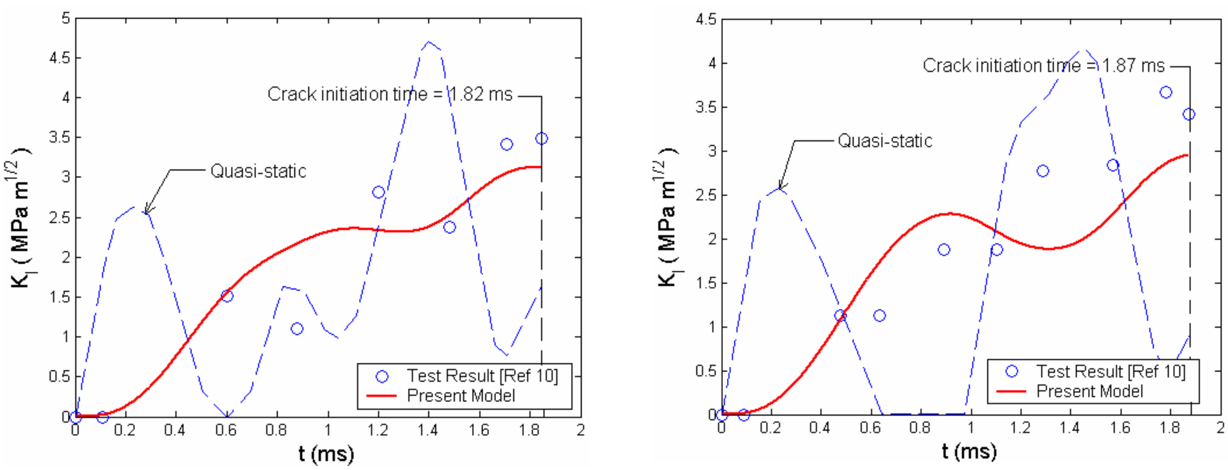
Figure 4. (a) Loading history, (b) Dynamic stress intensity factors.

The quasi-static SIF is evaluated neglecting inertia effects. There is a considerable difference between the dynamic and quasi-static responses. This is mainly due to the specimen inertia. When specimen inertia is included, the peak stress intensity factor rises by more than 8% above the quasi-static maximum. More importantly, the peak occurs at a different instant of time. Hence, the results of dynamic analysis are in contradiction with the assumption that the crack begins to propagate at the instant when load attains its peak value.

Bohme and Kalthoff [10] have presented the DSIFs for TPB specimens made from the epoxy resin Araldite B. In their experiments, $K_I(t)$ was measured by means of the method of caustics. Specimens were impacted by a drop weight of 4.9kg mass at velocity of 1 m/s. The specimens were 412 and 550mm long, 100mm wide and 10mm thick. A fixed support span of 400mm was used. The notch size used is 30mm. Their results are compared with the present analytical results in Figure 5. The quasi-static SIF is calculated directly using the instantaneous load neglecting the inertia effects. The critical DSIFs are presented in Table 2.

Table 2. Critical DSIF corresponding to crack initiation time.

Critical DSIFs $MPa\sqrt{m}$	*Test 1*	*Test 2*
Test Results [*Ref. 10*]	3.48	3.42
Present model	3.15	3.03



(a) (b)

Figure 5. Comparison of analytical and experimental DSIFs for impact tests on TPB specimens of Araldite B material [10] for (a) Test 1 and (b) Test 2.

4. CONCLUSION

A multi degree of freedom model to study the dynamic behavior of a TPB specimen under impact loading is presented. The modeling is based on representation of a crack in beam by a rotational spring. The beam modeling is done through the Euler-Bernouli beam theory. It is shown that contribution from only fundamental mode is sufficient for analysis of test results. The frequencies obtained by the present modeling compare closely with FE results. However for short specimens the accuracy of present model is not good. The dynamic SIF for TPB specimens have been determined under a step and a ramp loading. These results are in good agreement with those based on transient two-dimensional FE analysis. Possibility of an application of present method of analysis to instrumented impact tests is shown through a comparison of results with those published in the literature.

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