ICSV14
Cairns • Australia
9-12 July, 2007

14th International Congress on Sound & Vibration

VIBRATION ANALYSIS OF MINDLIN'S SANDWICH PLATE (FSDT) UNDER RANDOM EXCITATION

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Abstract

In this paper, vibration analysis of a sandwich plate modeled within the First Order Shear Deformable Theory (FSDT) under random excitation is presented. It was considered that the sandwich plate is thick, and the CPT (Classic Plate Theory) is no longer applicable, therefore, one of the shear deformable plate theories, the FSDT, is adapted for the structural formulations. The sandwich plate is assumed to be simply supported with movable edges. One of the faces of the sandwich plate is exposed to Gaussian stationary random loads with zero mean. Both uniformly distributed and point random loads are considered. Thermal effect is also taken into account. Root Mean Squares (RMS) of deflection responses are computed and compared by using both the CPT and the FSDT. Spectral density of the responses is also presented. A parametric study is conducted to show the effects of scattering in the geometry of the sandwich plate (both face and core), characteristics of loading and temperature change on the sandwich deflection response.

1. INTRODUCTION

The use of sandwich type structures continues to grow around the world. Due to their high strength and, the need for low weight structures, laminated composite plates and sandwich panels are being increasingly used in high –performance systems such as aeronautical and aerospace constructions [1]. In sandwich structures, there are bottom and top face sheets, which primarily resist bending loads. Transverse normal and shear stresses are transmitted through the core. Transverse shear deformations can be important especially when the core is thick and has relatively low stiffness [2]. There exist several theories/studies on the free and forced vibrations of laminated plates and sandwich panels. They are basically the Classical Plate Theory (CPT), which neglects the effect of transverse shear strain; and the shear deformable theories (such as the First-order Shear Deformation Theory (FSDT, Mindlin), and second and higher order theories e.g. the Third Order Shear Deformation Theory(TSDT)), which include the effect of transverse shear strain[3-6]. The free and forced vibration of sandwich panels is investigated by using the CPT, FSDT and/or higher-order theories in [7-12]. Random vibrations of composite structures by using different plate theories are given in

[13]. In this study, vibration analysis of a sandwich plate modeled within the First Order Shear Deformable Theory (FSDT) under random excitation is presented.

2. STRUCTURAL FORMULATION

For the sandwich panel, it is assumed that deformation through thickness is continuous. $w(x,y,t)$ is the normal deflection of mid-plane, $\psi_x(x,y,t)$ is the rotation about x axis, $\psi_y(x,y,t)$ is the rotation about y axis. The equations of motion for Mindlin's Sandwich Plate [3] by added thermal and damping terms [2,12,13] can be written as

$$\begin{aligned}&\left(D_c + D_f\right)\frac{\partial^2 \psi_x}{\partial x^2} + \left(\nu_c D_c + \nu_f D_f\right)\frac{\partial^2 \psi_y}{\partial x \partial y} \\ &+ \frac{1}{2}\left[\left(1-\nu_c\right)D_c + \left(1-\nu_f\right)D_f\right]\left(\frac{\partial^2 \psi_x}{\partial y^2} + \frac{\partial^2 \psi_y}{\partial x \partial y}\right) \\ &- K_s\left[G_c h_c + 2G_f h_f\right]\left(\psi_x + \frac{\partial w}{\partial x}\right) - \frac{\partial M_{xx}^T}{\partial x} = \left(\rho_c h_c + \rho_f h_f\right)\frac{\partial^2 \psi_x}{\partial t^2} + \lambda \frac{\partial \psi_x}{\partial t}\end{aligned} \tag{1}$$

$$\begin{aligned}&\left(D_c + D_f\right)\frac{\partial^2 \psi_y}{\partial y^2} + \left(\nu_c D_c + \nu_f D_f\right)\frac{\partial^2 \psi_x}{\partial x \partial y} \\ &+ \frac{1}{2}\left[\left(1-\nu_c\right)D_c + \left(1-\nu_f\right)D_f\right]\left(\frac{\partial^2 \psi_y}{\partial x^2} + \frac{\partial^2 \psi_x}{\partial x \partial y}\right) \\ &- K_s\left[G_c h_c + 2G_f h_f\right]\left(\psi_y + \frac{\partial w}{\partial y}\right) - \frac{\partial M_{yy}^T}{\partial y} = \left(\rho_c h_c + \rho_f h_f\right)\frac{\partial^2 \psi_y}{\partial t^2} + \lambda \frac{\partial \psi_y}{\partial t}\end{aligned} \tag{2}$$

$$K_s\left[G_c h_c + 2G_f h_f\right]\left(\frac{\partial \psi_x}{\partial x} + \frac{\partial \psi_y}{\partial y} + \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = \left(\rho_c I_c + 2\rho_c h_c\right)\frac{\partial^2 w}{\partial t^2} + \lambda \frac{\partial w}{\partial t} - p^r\left(x,y,t\right)) \tag{3}$$

where $D_c = \dfrac{E_c I_c}{\left(1-\nu_c^2\right)}$ and $I_c = \dfrac{h_c^3}{12}$ (4a,b)

$$D_f = \frac{E_f I_f}{\left(1-\nu_f^2\right)} \text{ and } I_f = \frac{2}{3}h_f\left(h_f^2 + \frac{3}{4}h_c^2 + \frac{3}{2}h_c h_f\right) \tag{5a,b}$$

where subscripts/superscripts c and f stand for the core and face, respectively. E is the modulus of elasticity, ν is the Poisson's Ratio, G is the shear modulus, ρ is the material density, h is the thickness, λ is the damping coefficient, K_s is the shear correction factor, $p^r\left(x,y,t\right)$ is the random pressure acting on the top surface of the sandwich plate.

Thermal moment is

$$M_{xx}^T = M_{yy}^T = M^T = \int_{-\frac{h_c}{2}-h_f}^{\frac{h_c}{2}+h_f} \frac{E\alpha}{1-\nu} T\left(x,y,z\right)z\,dz \tag{6}$$

where α is the coefficient of thermal expansion, $T(x,y,z)$ is the temperature changes from a stress free reference temperature. The proper material constants for the integrand are used within the integral limits. Boundary Conditions of a Mindlin's Plate for a simply supported with in-plane "movable ends" as follows:

$$w(0,y,t)=w(a,y,t)=w(x,0,t)=w(x,b,t)=0 \tag{7}$$

$$M_{xx}(o,y,t)=M_{xx}(a,y,t)=M_{yy}(x,0,t)=M_{yy}(x,b,t)=0 \tag{8}$$

$$\psi_y(0,y,t)=\psi_y(a,y,t)=\psi_x(x,0,t)=\psi_x(x,b,t)=0 \tag{9}$$

where the bending moments are [3,12]

$$M_{xx}(x,y,t)=\left(D_c+D_f\right)\frac{\partial\psi_x}{\partial x}+\left(\nu_c D_c+\nu_f D_f\right)\frac{\partial\psi_y}{\partial y}-M_{xx}^T \tag{10}$$

$$M_{yy}(x,y,t)=\left(D_c+D_f\right)\frac{\partial\psi_y}{\partial y}+\left(\nu_c D_c+\nu_f D_f\right)\frac{\partial\psi_x}{\partial x}-M_{yy}^T \tag{11}$$

2.1 Free Vibration Analysis

For the undamped free vibration analysis, by setting the all mechanical and thermal load to zero, it can be shown that following functions satisfy the boundary conditions, Eqs. (7-9),

$$\psi_x(x,y,t)=\hat{X}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{\hat{i}\omega t} \tag{12}$$

$$\psi_y(x,y,t)=\hat{Y}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{\hat{i}\omega t} \tag{13}$$

$$w(x,y,t)=\hat{W}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{\hat{i}\omega t} \tag{14}$$

By inserting Eqs.(12-14)into the equations of motion (1-3), and arranging the terms, one can obtain

$$\left([K]-\omega^2[M]\right)\{\eta\}=0 \tag{15}$$

where $\{\eta\}^T=\left\{\hat{X}_{mn},\hat{Y}_{mn},\hat{W}_{mn}\right\}$ (16)

and, $[K]$ and $[M]$ are the stiffness and mass matrices, and ω is the natural frequency of the vibration. For every (m,n), there are three natural frequencies ω_{mni}^2 and corresponding X_{mni},Y_{mni},W_{mni} natural modes $(i=1,2,3)$.

2.2 Random Vibration Analysis

For the forced vibrations, natural mode method is employed. The response is expanded in terms of natural modes [3, 14]

$$\psi_x(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{i=1}^{3}X_{mni}(x,y)\,q_{mni}(t) \tag{17}$$

$$\psi_y(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{i=1}^{3}Y_{mni}(x,y)\,q_{mni}(t) \tag{18}$$

$$w(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{i=1}^{3}W_{mni}(x,y)\,q_{mni}(t) \tag{19}$$

where $q_{mni}(t)$ is the generalized coordinate.
Temperature is also expanded in double Fourier sin series as

$$T(x,y,z)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}T_{mn}(z)\,\sin(\alpha_m x)\sin(\beta_n y) \tag{20}$$

$$T_{mn}(z)=\frac{4}{a\,b}\int_0^a\int_0^b T(x,y,z)\,\sin(\alpha_m x)\sin(\beta_n y)dx\,dy \tag{21}$$

In this study, the temperature is assumed to be distributed linearly through thickness, i.e.

$$T(x,y,z)=T_0\left(1+c\frac{z}{h}\right) \tag{22}$$

where T_0 is the reference temperature at $z=0$, and c is the slope of the temperature gradient, $h=2h_f+h_c$ is the total thickness of the sandwich plate.
From Eqs. (20-22) and Eq. (6), the thermal moment can be written as

$$M^T(x,y)=M_{xx}^T(x,y)=M_{yy}^T(x,y)=\overline{M}^T\sum_{\substack{m\\odd}}\sum_{\substack{n\\odd}}\frac{1}{m\,n}\sin(\alpha_m x)\sin(\beta_n y) \tag{23}$$

$$\text{where}\quad \overline{M}^T=\frac{16T_0\,c}{\pi^2 h_t}\left[D_c(1+\nu_c)\alpha_c+D_f(1+\nu_f)\alpha_f\right] \tag{24}$$

Substituting Eqs. (17-19) and (23) into Eqs. (1-3), and from free vibration analysis and the orthogonality of the natural modes, one can obtain

$$M_{mni}\frac{d^2q_{mni}}{d\,t^2}+C_{mni}\frac{d\,q_{mni}}{d\,t}+M_{mni}\,\omega_{mni}^2\,q_{mni}(t)=F_{mni}^T+F_{mni}^r(t) \tag{25}$$

where

$$M_{mni}=\int_0^a\int_0^b\left[I_1X_{mni}^2+I_1Y_{mni}^2+I_2W_{mni}^2\right]dx\,dy \tag{26}$$

$$C_{mni}=\int_0^a\int_0^b\lambda\left[X_{mni}^2+Y_{mni}^2+W_{mni}^2\right]dx\,dy \tag{27}$$

$$F_{mni}^r(t)=\int_0^a\int_0^b p^r(x,y,t)\,W_{mni}(t)\,dx\,dy \tag{28}$$

where $I_1=\rho_c I_c+\rho_f I_f$ and $I_2=\rho_c h_c+2\rho_f h_f$ (29a,b)

$$F_{mni}^T=-\overline{M}^T\left\{\int_0^a\int_0^b\left[X_{mni}\sum_{k\,odd}^{\infty}\sum_{l\,odd}^{\infty}\frac{\pi}{l\,a}\cos(\alpha_k x)\sin(\beta_l y)+Y_{mni}\sum_{k\,odd}^{\infty}\sum_{l\,odd}^{\infty}\frac{\pi}{k\,b}\sin(\alpha_k x)\cos(\beta_l y)\right]dx\,dy\right\} \tag{30}$$

Finally,

$$\frac{d^2q_{mni}}{d\,t^2}+2\,\xi_{mni}\,\omega_{mni}\frac{dq_{mni}}{d\,t}+\omega_{mni}^2\,q_{mni}(t)=Q_{mni}^T+Q_{mni}^r(t) \tag{31}$$

where $$2\,\xi_{mni}\;\omega_{mni} = \frac{C_{mni}}{M_{mni}} \tag{32}$$

$$Q^{T}_{mni} = \frac{F^{T}_{mni}}{M_{mni}} \tag{33}$$

$$Q^{r}_{mni}(t) = \frac{F^{r}_{mni}(t)}{M_{mni}} \tag{34}$$

In this study, it is assumed that random pressure is uniformly distributed, stationary and it has a zero mean, correlation function can be given as

$$R_w(x,y,\tau) = \sum_{m}^{\infty}\sum_{n}^{\infty}\sum_{i}^{3}\sum_{r}^{\infty}\sum_{s}^{\infty}\sum_{j}^{3} W_{mni}(x,y)\, W_{rsj}(x,y) Q^{T}_{mni}\, Q^{T}_{rsj}\, H_{mni}(0) H_{rsj}(0) \tag{35}$$

$$+ \int_{-\infty}^{\infty} S_W(x,y,\omega) e^{\hat{i}\,\omega t} d\omega$$

where the response spectral density can be computed from [14]

$$S_w(x,y,\omega) = \sum_{m}^{\infty}\sum_{n}^{\infty}\sum_{i}^{3}\sum_{r}^{\infty}\sum_{s}^{\infty}\sum_{j}^{3} W_{mni}(x,y) W_{rsj}(x,y) H^{*}_{mni}(\omega) H_{rsj}(\omega) S_{Q^{r}_{mni}Q^{r}_{rsj}}(\omega) \tag{36}$$

where $$S_{Q^{r}_{mni}Q^{r}_{rsj}}(\omega) = \frac{1}{M_{mni}}\frac{1}{M_{rsj}} \int_0^a\int_0^b\int_0^a\int_0^b S_p(x,y,\omega) W_{mni}(x,y) W_{rsj}(x,y)\, dx\, dy\, dx\, dy \tag{37}$$

where $$H_{mni}(\omega) = \frac{1}{\omega^2_{mni} - \omega^2 + 2\hat{i}\,\xi_{mni}\,\omega_{mni}\,\omega} \tag{38}$$

and $*$ indicates complex conjugate .

3. NUMERICAL RESULTS AND CONCLUSION

Following geometric and material properties are used (unless indicated otherwise): planar dimensions: $a = b = 0.6\,m$, $h = 0.04\,m$ (i.e. $a/h = 15$); the aluminum (2014-T6) face sheets: $E_f = 73.1\times10^9\,Pa$, $\nu_f = 0.35$, $\rho_f = 2790\,kg/m^3$; and the PVC foam core: $E_c = 103.63\times10^6\,Pa$, $\nu_c = 0.32$, $\rho_c = 130\,kg/m^3$ .The structural damping $\xi = 0.05$ and shear correction factor $\kappa = 5/6$ are taken. The core thickness is taken 90% of the total panel thickness. First nine natural frequencies of the sandwich panel as function of the side to thickness ratios (a/h) for the FSDT and CPT are compared in Table 1. All natural frequencies for the CPT are greater than those for the FSDT. However, discrepancies in natural frequencies between these two theories are larger for the thicker sandwich construction. For the forced vibrations, the input random pressure p^r acting on the sandwich panel is assumed to be a uniformly distributed band limited Gaussian white noise. The spectral amplitude S_o can be computed from

$$S_o = \frac{p_o^2}{\Delta\omega} 10^{SPL/10} \tag{39}$$

where p_0 is the reference pressure($p_0 = 2\times10^{-5}\,Pa$), SPL is the sound pressure level expressed in decibels and $\Delta\omega$ is the frequency bandwidth ($rad/\sec$). $SPL = 160\,dB$ is assumed for uniformly distributed random pressure, the responses are computed at the mid point ($x = a/2$, $y = b/2$). Figure 1 shows the deflection Root Mean Square (RMS) values as function of the side to thickness ratios (a/h). The RMS responses for the FSDT are greater than those for the CPT in the thicker sandwich construction (i.e. $a/h \leq 20$), they are indistinguishable for the thinner sandwiches panels.

Table 1. Natural frequencies $\omega_{mn1}\ (rad/s)$ for square sandwich panels $a = 0.6\,m$. $(h_c/h = 0.9)$.

a/h	Theory	1,1	1,2	1,3	2,1	2,2	2,3	3,1	3,2	3,3
5	FSDT	10882	21532	34006	21532	29502	39970	34006	39970	48488
	CPT	14368	35920	71840	35920	57472	93392	71840	93392	129312
10	FSDT	6573	14797	25922	14797	21764	31612	25922	31612	40003
	CPT	7184	17960	35920	17960	28736	46696	35920	46696	64656
15	FSDT	4592	10861	20079	10861	16547	25054	20079	25054	32647
	CPT	4789	11973	23946	11973	19157	31130	23946	31130	43104
20	FSDT	3506	8476	16118	8476	13147	20384	16118	20384	27058
	CPT	3592	8980	17960	8980	14368	23348	17960	23348	32328
100	FSDT	705	1786	3571	1786	2859	4637	3571	4637	6407
	CPT	718	1796	3592	1796	2873	4669	3592	4669	6465

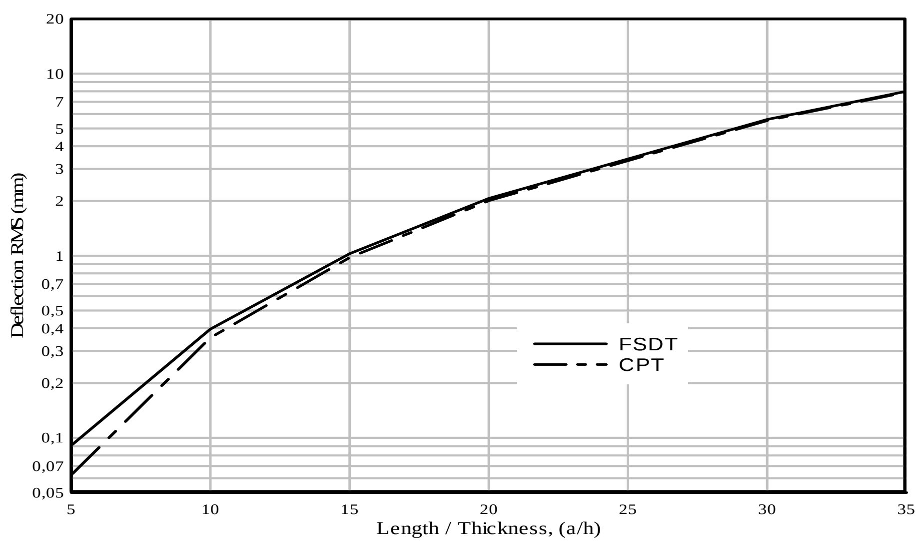

Figure 1. Deflection RMS values vs. the length to thickness ratios for two plate theories

The effect of the core shear modulus on the RMS responses for two theories is shown in Figure 2. Discrepancies are larger for the lower core shear modulus. Deflection response spectral density is given in Figure 3. As it can be seen from the figure, there exist response peaks corresponding to the natural frequencies of the sandwich panel. Peaks are shifted to the right for the CPT. Temperature effect is investigated for the sandwich panel under uniformly distributed random pressure of $SPL = 160dB$. The thermal expansion coefficients are taken $\alpha_f = 23\times10^{-6}\ /^0C$ for the aluminum face sheets and $\alpha_c = 23\times10^{-7}\ /^0C$ for the foam core. Effects of the temperature on the deflection RMS values (RMS/h_f) are compared in Table 2. With increased temperature, the RMS response increases, and at elevated temperatures, both theories give almost the same RMS values. The deflection RMS due to a random point load acting at ($x^* = 025m, y^* = 0.25m$) with specified spectral densities is given in Figure 4. Discrepancies between these two theories are distinguishable only for the larger input spectral densities. In Conclusion, it is necessary to use a refine theory for an accurate analysis of sandwich panels under random excitation.

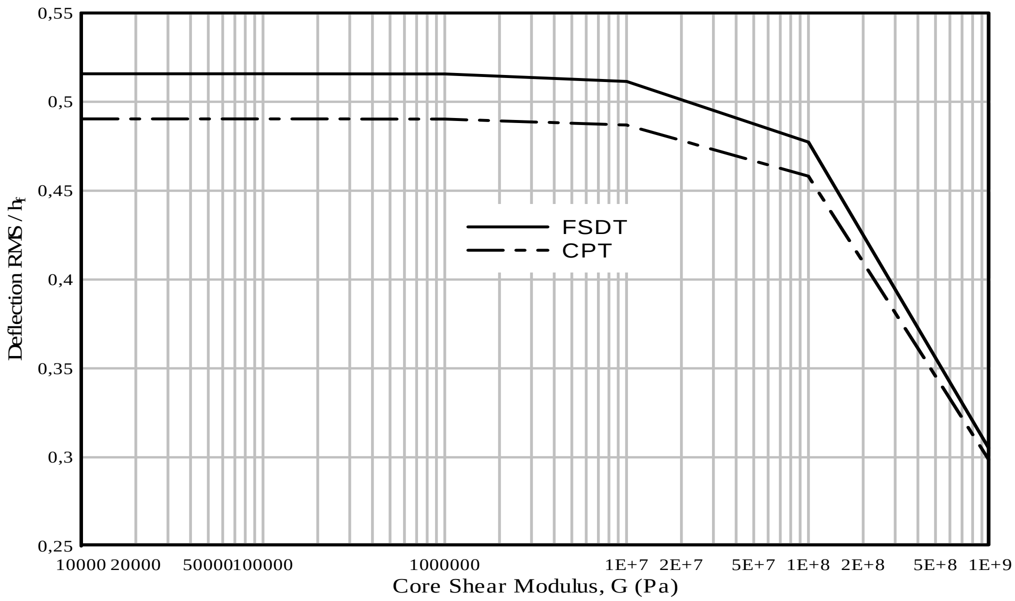

Figure 2. Deflection RMS values for different Core Shear Modulus.
($a = 0.6\,m$, $a/h = 15$, $\left(h_c/h = 0.9\right)$).

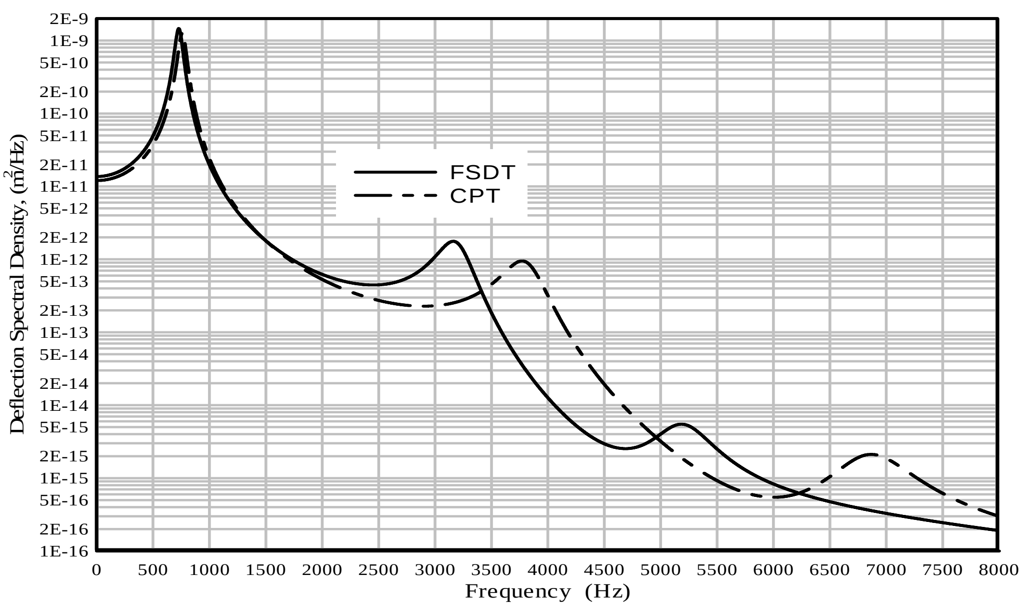

Figure 3. Deflection Response Spectral Density.

Table 2. Temperature effect on the RMS response ($c = 2$)

Theory	$T_0 = 0^{o}C$	20	40	60	80	100	120
FSDT	0.5140	0.6521	0.9531	1.3091	1.6856	2.0714	2.4622
CPT	0.4890	0.6331	0.9412	1.3017	1.6811	2.0692	2.4618

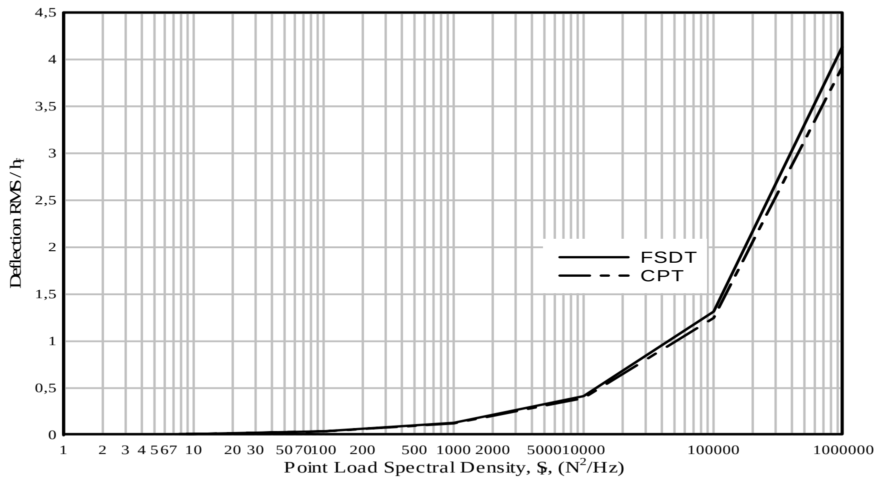

Figure 4. Response to random point load

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