A STUDY OF INPUT MOBILITY FUNCTIONS AT A VIOLIN'S BRIDGE

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Abstract

Experimental comparison of individual notes from different violins has revealed some differences between the tonal qualities of violins. These differences offer clues towards an objective determination of a violin’s acoustical quality. This paper reports some preliminary findings of the correlation between the tonal quality and input mobility functions at locations where the strings attach of the bridge of two different violins. It is demonstrated that the mobility functions are related to the violin’s bowing sensitivity and to the overall sound radiation to the performer and listeners.

1. INTRODUCTION

The understanding of violin acoustics has long been a passion and endeavour of violin players, violin makers and acousticians. Several important advances have been made in the understanding of violin’s acoustics, including the self-sustained string oscillations due to sticking and sliding frictional forces between the bow and string [1], rocking and vertical vibration modes of violin bridges [2, 3] and “bi-tri octave” tuning of the back and top free plates for the best instrument sound [4]. In parallel with those advances, effort have been made in investigating a large number of violins with high and moderate qualities by comparing their bridge admittance [5] and short time Fourier transforms of glissandi on each string [6]. These efforts are aimed towards the development of objective criteria for the acoustical quality of violins.

In this paper, we concentrate on a study of input mobility functions at violin bridge. Different from the overall bridge admittance described in Jansson’s measurement [5], we are interested in the measurement of the input mobility functions at the locations where each string attaches to the bridge. Those mobility functions play an important role in the determination of the violin’s bowing sensitivity on each string and sound power radiated from the violin. This paper attempts to address the following issues: (1) a violin’s acoustical quality accessed by player and listeners, (2) what can be learnt from steady state notes played? (3) measurement of mobility functions at a violin’s bridge, (4) the correlation of the tonal quality and mobility
functions, (5) the effect of the mobility on a violin’s bowing sensitivity, and (6) the effect of mobility on a violin’s sound power radiation.

2. VIOLIN’S ACOUSTICAL QUALITY

The objective description of a violin’s acoustical quality is based on the timbre of each note played, which is also called tonal quality and is determined by the relative amplitude of the sound at the fundamental frequency heard as pitch, and higher frequency overtones. The envelope of the sound and the variations in timbre with time may also play an important role in determining the tonal quality, but the discussion of this is beyond the scope of this paper. When a music scale or piece is played, the relative loudness of the notes played contributes to the degree of sound balance of the instrument. It is important to note that the “apparent” quality of a violin is often perceived differently by the player from that heard by listeners. As illustrated in Figure 1, the production of violin sound by a player involves a sound feedback mechanism, and sound received is also dependent on the quality of the room acoustics. By using this feedback mechanism, a skilful player may be able to maintain the balance of the note by adjusting the bowing pressure. The feedback mechanism also includes the player hand and chin response to the bow/string interaction, and the mechanical vibration of the instrument body. That is why a skilful player may sometimes bring a listener into tears even with a student violin. However, with a student violin, the player has to work harder to produce the same effect. Two violins (Figure 2) were selected for a subjective comparison test of playing the same piece of music and music note by the same player (from WAYO). Although, the listeners were not able to identify overall differences between the two violins (indeed the detailed tonal difference is usually identified by trained ears or by frequency analyser), the player was. Bacher violin was made in 1753 by Marimilian Bacher of Breßlau in Germany. It is very light, very sweet and quite responsive, although not as loud as desirable for a top-of-the-range instrument. The student violin was purchased for the purpose of comparison with the high quality instruments. No details exist on when it was made. It has a number carved into its neck- a definite sign of being factory made. To the player, its sound is quite satisfactory and responsive. However it is not at all even, as is easily apparent in playing tests.

Figure 1 Flow diagram of production of violin sound.
3. QUALITY OF STEADY STATE NOTES

The detailed tone-to-tone quality comparisons were conducted between the two violins. Figure 3 shows the power spectral density of the open G strings, which were produced by the same player and measured at the same location with respect to the violins in an anechoic room. The player kept the same bowing speed and pressure as much as possible when bowing different violins. It is assumed that the relative amplitudes of the higher frequencies overtones with respect to that at the fundamental frequency do not vary significantly with the bowing pressure of the player. This assumption implies that the feedback mechanism of violin playing may affect the overall loudness of a tone, but not the relative amplitudes, which is used to describe the tonal quality.

![Figure 3. PSD of G3 (196 Hz) of Bacher and student violins. Little circles in the figures are the identified peaks in the narrow frequency spectrum. The circle with largest value in a major peak is used to represent the peak value of the fundamentals and overtones.](image)

The comparison of the relative amplitudes at the higher frequency overtones of the two violins shows significant level difference at a three frequency regions as shown in Figure 4.
Those differences occur at the third harmonics (588 Hz) by 5 dB, a frequency range centered at 2kHz by more than 15 dB and around 3300Hz by more than 10dB. For all these three cases, the Bacher violin has higher amplitudes.

Similar comparison results were true for the A3 notes (220Hz). As shown in Figure 5, level differences of the overtones between the two violins occur at the same three frequency regions as identified in Figure 4. An extra region of amplitude difference is also observed at frequencies near 1000Hz.

A question was raised at this point that if such level differences at overtones are related to the
characteristics of the violins, which motivated the measurement of the mobility functions at the violin bridges.

4. MEASUREMENT OF MOBILITY FUNCTIONS AT VIOLIN’S BRIDGE

The experiment is designed to measure the mobility functions at the bridge locations where the strings are attached. The dominant mobility functions at each location are the tangential mobility:

\[ M_t(\omega) = \frac{v_t}{F_t}, \]  

and normal mobility

\[ M_n(\omega) = \frac{v_n}{F_n}, \]

where \( F_t \) and \( F_n \) respectively are the tangential and normal forces (as shown in Figure 6 for the G string location on the bridge), \( v_t \) and \( v_n \) are the corresponding tangential and normal velocities due to the forces.

![Figure 6. Tangential and normal forces at G string location of the bridge.](image)

The forces and velocities at the bridge are generated by a B&K mini-shaker and measured by a B&K impedance head. Figure 7 shows the experimental set-up and a connector which joins the impedance head and the string on the bridge.

![Figure 7. Mobility measurement experimental set-up and a connector between impedance head and the string on the bridge. (a) connector for normal mobility, (b) mini-shaker, impedance head and connector, (c) connector for tangential mobility.](image)
The mobility functions of at the G string location of the bridge of the two violins are shown in Figure 7. In the mobility measurement, the vibration of the strings is damped. The difference between the mobility functions of the violins is also identified at the same frequency regions where the difference of the overtone amplitudes is significant. The only exception is that the differences of the mobility functions near 2000Hz are small, while that of the overtone amplitudes is more than 10 dB.

Figure 8. Measured tangential and normal mobility functions of at the G string location of the two violins.

The properties of the overtone amplitude distribution seem to be explainable using the mobility functions. Figures 8 and 9 suggest:

1. The 5dB difference at 588 Hz is mainly due to that the third harmonics of the open G string of the Bacher violin overlaps with the peak frequency of the tangential mobility, while that of the student violin does not.

2. The 10dB difference near 3000Hz correlates well with the drop of the tangential mobility (15 dB) of the student violin at that frequency.

3. The difference near 1000Hz when A3 is played may due to the Bacher string being coupled well with the violin body through the normal mobility (which is nearly 10 dB higher than that of the tangential mobility) at this fingering position, while the student string drives the violin only tangential.

Figure 9. Measured tangential and normal mobility functions of at the G string location of the two violins.
The overtone level difference in the region of 2000Hz indicates that the string energy transmission into the Bacher violin is through the normal mobility, while that into the student violin still relies on the tangential mobility.

5. EFFECT OF THE MOBILITY ON VIOLIN’S BOWING SENSITIVITY

The relevance of the bridge mobility functions to the bowing sensitivity can be illustrated by a simple tensioned string model as shown in Figure 10, where the effect of the entire violin on the mobility at the bowing location is represented by the bridge mobility function (only tangential mobility is used here for illustration).

![Figure 10. Simple string and bridge mobility model for studying bowing sensitivity.](image)

The analysis of the string transverse vibration shown that at the nth overtone of the string, the mobility at the bowing position $x_d$ (bowing sensitivity) is related to the bridge mobility at $x_b$ by:

$$M_d(\omega_n) = \frac{V(x_d, \omega_n)}{F_d(\omega_n)} \equiv \frac{n\pi x_d}{L} M_b(\omega_n).$$

(3)

It then becomes clear that the bowing sensitivity of a violin string is directly proportional to the bridge mobility when $x_d$ is close to $x_b$. If the fundamental frequency or any of the overtone frequency overlaps with the one of the peak frequencies of the mobility function, then a large vibration at the bridge location can be excited. On the other hand, if the frequency of string oscillation is at the valley of the mobility, then a large amount of bowing pressure is required to generate the same bridge vibration. It seems desirable to have all the frequencies associate with important tones of a violin coinciding with those frequencies at which the mobility functions have similarly high values (at or near the their peaks). This will result in a relatively even or uniform bowing sensitivity.

6. EFFECT OF MOBILITY ON VIOLIN’S SOUND POWER RADIATION

The mobility functions at the bridge allow the evaluation of the mechanical power transmitted into the violin at each of the oscillation frequency of the bowed string:

$$P(\omega_n) = \frac{1}{2} \text{Re}\{M_t(\omega_n)\} \left|F_t(\omega_n)\right|^2 + \frac{1}{2} \text{Re}\{M_n(\omega_n)\} \left|F_n(\omega_n)\right|^2$$

(4)

where $F_t$ and $F_n$ are the tangential and normal forces at the string attachment position of the bridge. This transmitted mechanical power is the supply of the kinetic energy in violin structure, which is dissipated by mechanical and radiation damping mechanisms. Thus the
energy-balance equation linking this input power to the spatial averaged velocity $\overline{V}(\omega_n)$ of the violin structure can be expressed as:

$$P(\omega_n) = \eta \frac{1}{2} m |\overline{V}(\omega_n)|^2$$

(5)

where $m$ is the violin mass, and

$$\eta = \eta_m + \eta_r$$

(6)

is the loss factor consisting of mechanical ($\eta_m$) and radiation ($\eta_r$) loss factors. $\eta_m$ is a simple function of frequency. $\eta_r$ is determined by the radiation efficiencies of all violin body modes and coupling of these modes with the sound field within the violin cavity at $\omega_n$. Although the determination of the loss factors requires further experimental and computational efforts, the spatial averaged body velocity of the student violin measured by a laser scanning vibrometer already shows a promising correlation between the input mobility and the body velocity (see Figure 11). It becomes clear that the characteristics of the mobility functions are exhibited in the averaged velocity and therefore the radiated sound power. However, the relative amplitudes of the power and sound pressure at each of the oscillating frequencies of a bowing string will be adjusted by $\eta_m$, $\eta_r$, radiation directivity and characteristics of the room, which makes the violin research more challenging.

Figure 11. Spatial averaged velocity, mobility and sound pressure (inside violin body) of the student violin.

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REFERENCES