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STUDY ON THE CHARACTERISTICS OF VIBRATION AND ACOUSTIC RADIATION OF DAMAGED STIFFENED PANELS

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Abstract

The characteristics of dynamic and acoustic radiation of a damaged stiffened panel are investigated by using FEM/BEM. The modes of damages are introduced into the finite element analysis. Isotropic damage modes are modeled by globally and isotropically softening the stiffness. Based on Mindlin theory, a shell element model is built to model the healthy and damaged structures and to calculate the dynamic characteristics and response of a structure surface. With the linear boundary element, the sound pressure of structure radiating outwards can be calculated and the radiated power and directivity can then be obtained. The influences of various locations and extents of damages on vibration and acoustic characteristics are studied. An analysis method has been established to analyze acoustic behavior of a damaged structure. Conclusions have been drawn from the analyses of some typical examples. The proposed method is useful for assessing the influences of the damages present in a stiffened panel on its acoustic radiation properties.

1. INTRODUCTION

Under complicated loading conditions, the offshore structures cannot avoid damages. Therefore, it is of significance to investigate the properties of dynamic and acoustic radiation of the stiffened panels which have been widely used in the field of ship and offshore structures due to their high load-carrying capability and light weight.

Ever since the introduction of modes of damages in the 1950s, damaged structures have been a topic of primary interest in the field of offshore research. For example, Banks and Emeric have studied the property of frame structure with asymmetric damages and the dynamic equations were obtained. The lower vibration modes of the stiffened structure can be easily solved by Finite Element Method, but it fails when dealing with the coupled interaction of structure and acoustic medium. If the acoustic medium is light fluid, the weak coupled fluid-structure problem can be solved via decomposition. In case of heavy fluid, the fluid is strongly coupled with the structure and it becomes a daunting task. Although last decade has witnessed intensive studies of the strong coupled problems by using FEM/BEM, more work is required for the better understanding of the problem.

In this work, the modes of damages are introduced into the finite element analysis. Isotropic damage modes are modeled by globally and isotropically softening the stiffness. Anisotropic damage modes are modeled by using reduction coefficients for elastic damages in x and y directions. A shell element model is built to model the healthy and damaged structures and to calculate the dynamic characteristics and response of a structure surface. With the linear boundary element, the sound pressure of structure radiating outwards can be calculated and the radiated power and directivity can then be obtained. The influences of various locations and extents of damages on natural frequencies, vibration and acoustic radiation modes, radiation power, and directivity pattern are studied. An analysis method has been established to analyze acoustic behavior of a damaged structure. The proposed method is useful for assessing the influences of the damages present in a stiffened panel on its acoustic radiation properties.

2. BASIC THEORIES

The FEM/BEM have been employed to evaluate the characteristics of dynamic and acoustic radiation of a damaged stiffened panel. The damaged structure is solved by FEM while the action of the acoustic medium on the damaged panel is analyzed by BEM. With the response of the structure surface, the sound pressure of structure radiating can be calculated through the Rayleigh's integral formula.
In the presence of the acoustic medium coupled to the structure, the dynamics equations of the stiffened panel can be expressed in a discrete manner,

$$M\ddot{w} + Kw = f \qquad (1)$$

Where M and K are global mass and stiffness matrices, respectively, and w and f the nodal displacement vector and acoustic medium coupled vector, respectively.
The plate in the infinite baffleplate vibrates at the monochromatic frequency of ω. The radiation acoustic pressure at point P in the the semi-domain can be obtained by the following The Rayleigh's integral formula using normal vibration velocity $u_n(S,t)$ of source point S,

$$P(P,t) = \frac{j\omega\rho_0}{2\pi}\iint_s u_n(S,t)G(S,P)ds \qquad (2)$$

Where ρ_0 is the density of acoustic medium, k wave number, r the distance between source point S and point P, $G(S,P) = e^{-jkr}/r$, the Green Function.

2.1 Damaged plate and shell model

Based on the assumption of *Kachanov*, the damage of the structure can be modeled by the reduction of elastic modulus. By setting ψ as the ratio of the total damaged areas and the cross-sectional area, the reduction coefficient of the elastic modulus in the x-direction can be defined as $K_{dx} = 1 - \psi_x$. The elastic modulus in the x- and y-directions can be given as $E_{xx}^{'} = K_{dx}E_{xx}$ and $E_{yy}^{'} = K_{dy}E_{yy}$, respectively. It is worthy noting that the elastic modulus in the z-direction is independent of the damages in the other two directions.
The stress-strain relationship in the anisotropic plate is complicated. With the assumption of isotropic damage modes, the stress-strain relationship in the damaged material can be generated. Assuming the thickness of the plate is infinitesimal, $G_{yz} = G_{zx} = G$, $D = \sqrt{D_x D_y}$ and $\mu = \sqrt{\mu_x \mu_y}$, the stress-strain relationship of the damaged plate can be given on the basis of *Mindlin* assumpation,

$$\varepsilon_x = \frac{\sigma_x}{K_{dx}E} - \frac{\mu\sigma_y}{E} \qquad \varepsilon_y = \frac{\sigma_y}{K_{dy}E} - \frac{\mu\sigma_x}{E}$$

$$\gamma_{xy} = \tau_{xy} \frac{2(1+K_{dx}\mu)}{K_{dx}E} \qquad \gamma_{yz} = \frac{\tau_{yz}}{G} \qquad \gamma_{zx} = \frac{\tau_{zx}}{G} \tag{3}$$

2.2 Fluid added effect

The investigations of the dynamics structure and acoustic medium coupling problem involve two aspects: 1) the characteristics of the acoustic radiation such as the dependence of the sound pressure on the vibration of structure, the distribution of the sound pressure and directivity, etc. 2) the counteraction of the sound filed on the vibration of the structure.
If the acoustic medium is light fluid, the weak coupled fluid-structure problem can be solved via decomposition. In case of heavy fluid, the fluid is strongly coupled with the structure and the problem should be solved simultaneously for the behaviors of the dynamic structure and the sound field in water. Taking the coupled fluid-plate with a coupling parameter λ as an example. The parameter can be expressed as:

$$\lambda = \frac{\rho_0 c}{\rho h \omega} \tag{4}$$

Where c is the sound velocity in the medium, ρ and h are density and thickness of the plate, respectively, ω the vibration frequency of the coupled system.
The magnitude of λ determines how strongly the system is coupled. If $\lambda < 1$, the system is coupled weakly while it is strongly coupled if $\lambda > 1$.

2.3 Acoustic radiation mode

Acoustic radiation mode is the distribution of the acoustic intensity excited by the structural vibration. The sound pressure is determined by the vibration modes. Supposed the structure vabrates at the r-th mode, the normal vibrational velocity $u_{nr}(S,t)$ of a point s at the coupled surface can be given by,

$$u_{nr}(S,t) = j\omega_r \phi_{nr} e^{j\omega_r t} \tag{5}$$

Substituting Eq. (5) into Eq.(2), the distribution of the sound pressure and the corresponding r-th acoustic radiation mode shapes can be obtained as,

$$p_r(P,t) = -\frac{\rho\omega_r^2}{2\pi} \iint_s \phi_{nr} G_r(S,P) ds e^{j\omega_r t} \tag{6}$$

$$\phi_{pr}(P) = -\frac{\rho\omega_r^2}{2\pi} \iint_s \phi_{nr} G_r(S,P) ds \tag{7}$$

It is noted that the acoustic radiation mode is the function of the normal velocity of the coupled surface. The reduction of the dynamic properties of the damaged structure will definitely lead to the change of the acoustic radiation.

2.4 Acoustic damping

The acoustic damping will lead to a reduction of the system's energy. Assuming a rectangular plate vibrates at the frequency of ω and the distribution of the responding velocity at point (x,y) of the coupled surface between the plate and fluid medium is in the form of $u(x,y,t) = U(x,y)e^{i\omega t}$, the radiation energy in the fluid medium per period can be expressed as

$$E_R = \left(\frac{2\pi}{\omega}\right) A\rho_0 c |u|^2 S(x,y) \tag{8}$$

Where A and $S(x,y)$ are the area of the plate and the eigenvalue of the vibration, respectively. The average velocity of the plate $|u|$ can be calculated by,

$$|u|^2 = \frac{1}{A}\int_0^b\int_0^a \frac{1}{2}[U(x,y)]^2 dxdy \qquad (9)$$

The total energy of the plate is given,

$$E_s = \frac{1}{2}\int_0^b\int_0^a \rho h[U(x,y)]^2 dxdy = \rho hA|u|^2 \qquad (10)$$

Based on the theory of vibration dissipation, acoustic damping coefficient can be expressed as,

$$\psi = \frac{E_R}{E_s} = \left(\frac{2\pi}{k}\right)\left(\frac{\rho_0}{\rho}\right)S(x,y) \qquad (11)$$

The $S(x,y)$ can either be the i-th vibration mode shape $\varphi_i(x,y)$ or the superposition of all the vibration modes. Therefore, the acoustic damping coefficients for the resonance or non resonance vibration can be furnished,

$$\psi_i = \frac{E_R}{E_s} = \left(\frac{2\pi}{k}\right)\left(\frac{\rho_0}{\rho}\right)\varphi_i(x,y) \text{ 或 } \psi = \frac{E_R}{E_s} = \left(\frac{2\pi}{k}\right)\left(\frac{\rho_0}{\rho}\right)\sum_i A_i\varphi_i(x,y) \qquad (12)$$

Where A_i is the weighting coefficient of each mode.
In view of the relationship between the acoustic radiation energy and density, the radiation and the vibration energy of the plate can be described as,

$$E_R = \sum \rho_0\left(\frac{1}{\pi h^2}\right)^2 \left|\pi h^2 u^2\right|^2 \left[(kh)^2 + \frac{1}{2}\right]\cdot A \qquad (13)$$

$$E_S = \sum \frac{1}{2} Ah\rho|u|^2 \qquad (14)$$

The acoustic damping coefficient can then be expressed as the ratio of the radiation energy,

$$\psi = \frac{E_R}{E_S} = \sum \frac{\rho_0}{2\rho h}|u|^2\left[(kh)^2 + \frac{1}{2}\right] \qquad (15)$$

3. NUMERICAL EXAMPLES

The computer program is developed to investigate the properties of dynamic and acoustic radiation of a damaged stiffened panel, in which the structure is investigated using FEM model of 4-node shell elements while fluid is modeled by BEM model of 4-node linear elements. The influence of damage on stiffed structure on the aspects of acoustic radiation modes, sound power, directivity pattern and the damping of acoustic radiation have been explored.

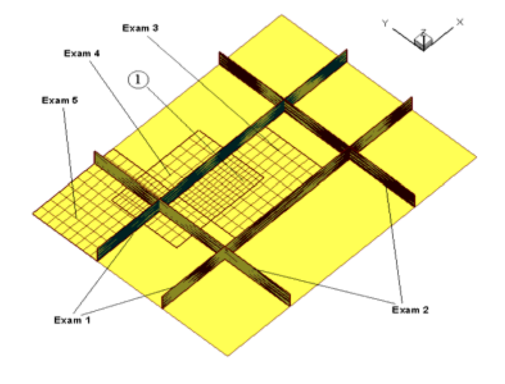
3.1 Geometrical and physical description of the damaged stiffened panel

The damaged stiffened panel is simply supported along four sides. It is of 1.2m long in x direction, 0.9m wide in y direction and 5mm thick. The panel is stiffened evenly in x and y directions with the height of 0.1m and thickness of 5mm. Young's modulus and Poisson's ratio of the material are taken as $2.01\times10^{11} N/m^2$ and 0.3, respectively.
Table 1 shows the details of the modulus, the location and the area of the damage. If the damage is located within the region of $a1 \le x \le a2$ and $b1 \le y \le b2$, the center of the damage will be located at the node of (c,d). To facilitate the study, the location, area and modulus of the damage are expressed in nondimentional forms.

- Non-dimensional damage area: $A = (a2 - a1)(b2 - b1)/ab$

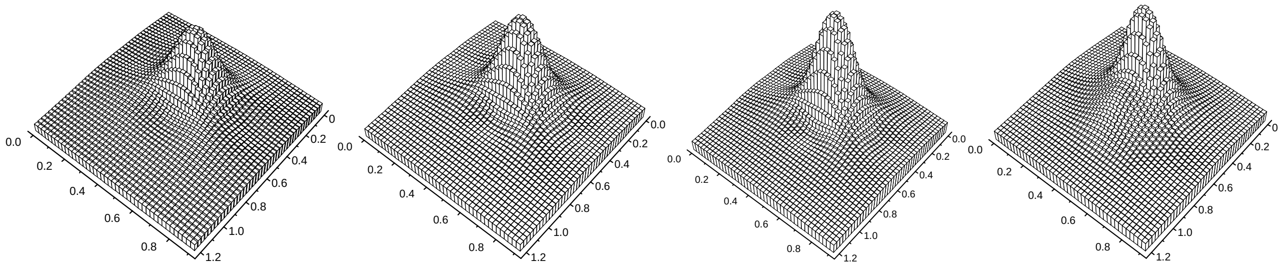
- Non-dimensional damage location: $dx = c/a$, $dy = d/b$
- Non-dimensional damage modulus: $k_x = E_x / E$, $k_y = E_y / E$

Table 1 Distribution of the damage of the simply supported, damaged stiffened panel

	damage modulus k	damage area a	damage location （dx，dy）	
1	0.3 0.6 0.9	0.083	（0.5,0.5）	
2	0.4	0.009 0.083 0.148	（0.5,0.5）	
3	0.4	0.083	（0.125,0.5） （0.625,0.5）	

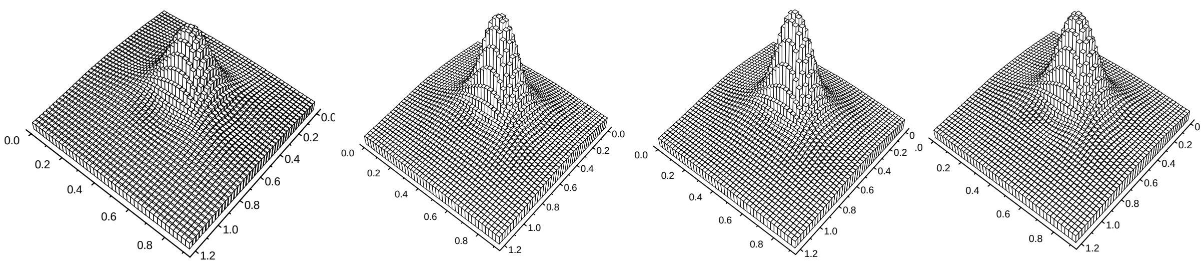
3.2 Study of the aberrance of acoustic radiation modes

Figure 1 shows the influence of the damage modulus on acoustic radiation modes. It can be observed that the peak value of the acoustic radiation mode increases obviously with the increase of the damage modulus. As the weakening of the damaged part will cause the strengthening of the swing, the acoustic radiation mode only changes a little within the domain of damage.



(a) healthy (b)k = 0.3 (c) k = 0.6 (d) k = 0.9

Fig.1 influence of the damage modulus on acoustic radiation modes
（damage area a =0.083and damage location（0.5,0.5））

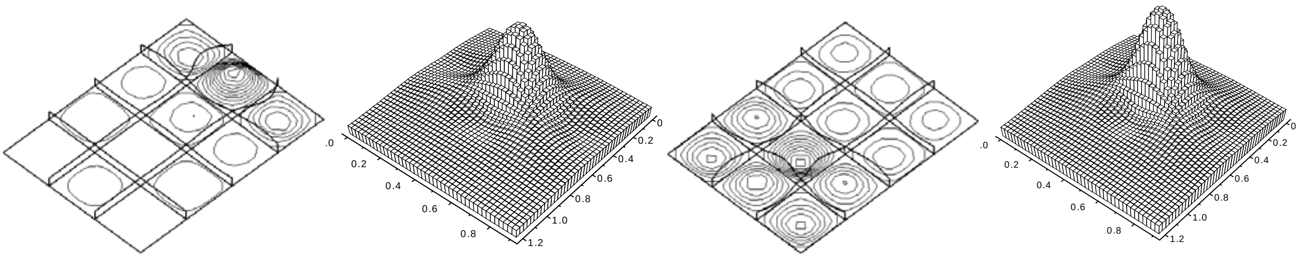


(a) healthy (b) a = 0.009 (c) a = 0.083 (d) a = 0.148

Fig.2 influence of damage area on acoustic radiation mode
（damage modulus k = 0.43 and damage location（0.5,0.5））

Figure 2 depicts the influence of damage area on acoustic radiation mode. It can be found out that the increase of the damage area will cause the weakening of the stiffness, and lead to the increase of the influence on acoustic radiation mode.

Figure 3 shows the influence of damage location on acoustic radiation mode. It is observed that the acoustic radiation modes appear to be local vibration within the area of damage, and the location of the peak value of the vibration will move with the moving of the damage area.



(a) （0.125，0.5） (b)（0.625，0.5）

Figure 3 influence of damage location on vibration mode and acoustic radiation mode
（damage modulus k = 0.43and damage area a = 0.083）

3.3 Study of the aberrance of sound power and directivity pattern

Figure 4 shows the influence on sound power caused by different modulus, area and location of the damage, respectively. Figure 5 indicates directivity patterns on 300Hz with altering damage modulus. With the increase of damage modulus, it can be found that natural frequency reduces, and vibration modes tend to be local vibration near the damage location. The increase of damage modulus may cause the increase of model three of nature frequency. Directivity direction is found to be maximum at 120° when $k = 0.6$, while the maximum value appear around 0° in other situations. With the increase of damage modulus, shape of the directivity pattern is found to be more obtuse and the branch petal becomes smaller.

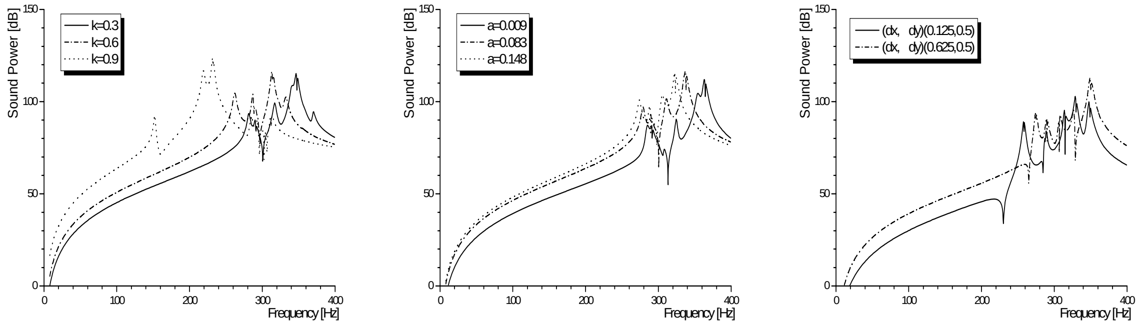
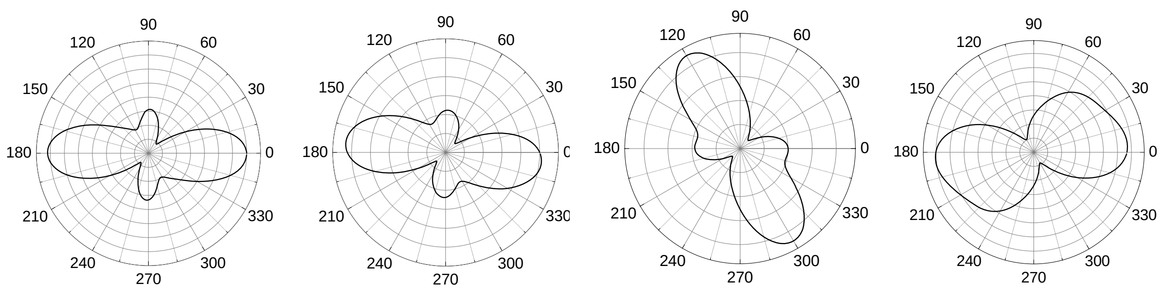


Fig. 4 Sound intensitys with altering modulus, area and location of damage

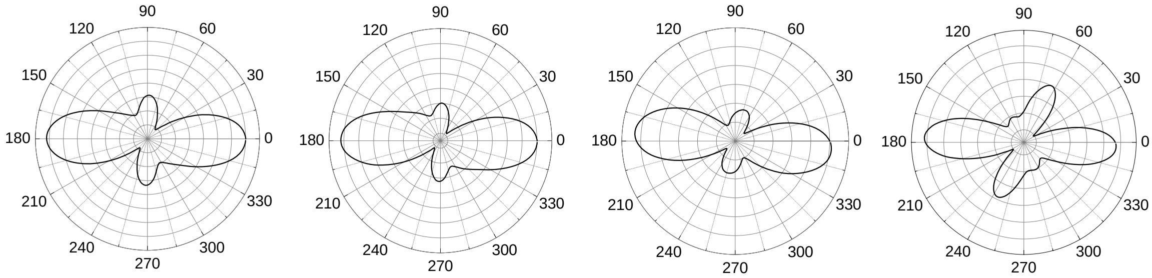


(a) k = 0 (b) k = 0.3 (c) k = 0.6 (d) k = 0.9

Fig.5 Directivity patterns on 300Hz with altering damage degree

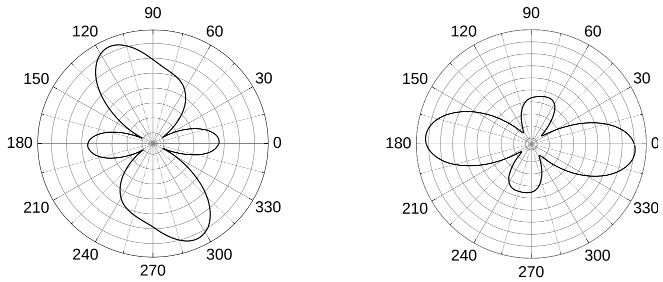
Figure 6 displays the influence on sound power and directivity pattern caused by the damage area. It can be concluded that small damage area cause little influence on natural frequency and vibration mode. Order of the sound intensity will increase with the increase of damage area. The picture indicates that the direction of the primary maximum of directivity pattern rarely changes; the secondary maximum increases and more branch petals appear with the increase of damage area.

Figure 7 shows the influence on sound power and directivity patterns caused by altering damage locations. It can be found that the damage location affects much on vibration mode. The vibration is found to be stronger within the area of damage, and sound intensity changes a little for different damage locations. Results of directivity pattern show that when the damage locates closer to the primary maximum, the direction will approach to 0° and the shape of the pattern may be more acute.



(a) *a* = 0 (b) *a* = 0.009 (c) *a* = 0.083 (d) *a* = 0.148

Fig.6 Directivity patterns on 300Hz with altering damage areas

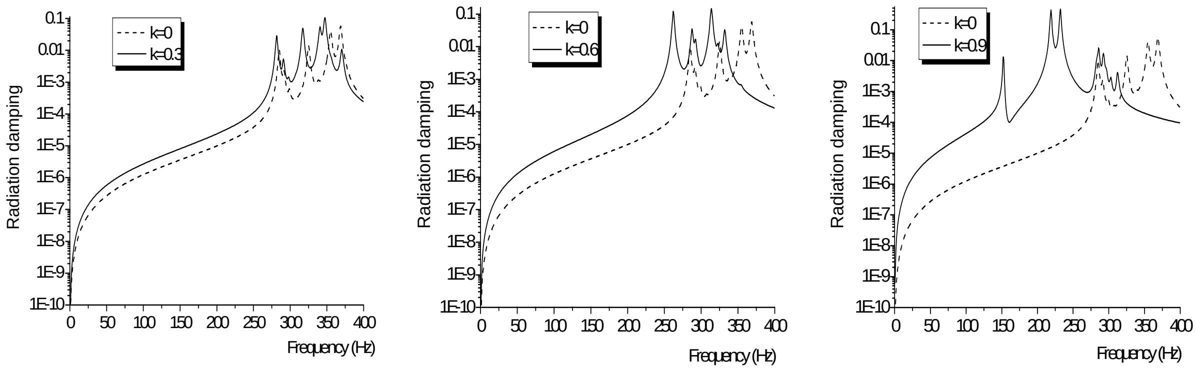


(a) Directivity patterns with damage location in (0.125,0.5) (b) Directivity patterns with damage location in (0.625,0.5)

Fig.7 Directivity patterns on 300Hz with altering damage locations

3.4 Study of the aberrance of acoustic radiation damping

Figure 8 shows the influence of damage modulus on acoustic radiation damping, in which the dashed line denotes the radiation damping of healthy structure and the solid line denotes the radiation damping of damaged structure with k = 0.3、k = 0.6 and k = 0.9, respectively. The figure indicates that the damage may weaken the structure and reduce the natural frequency of the structure. The resulted weak structure will increase the swing of the vibration and enhance the acoustic radiation damping.

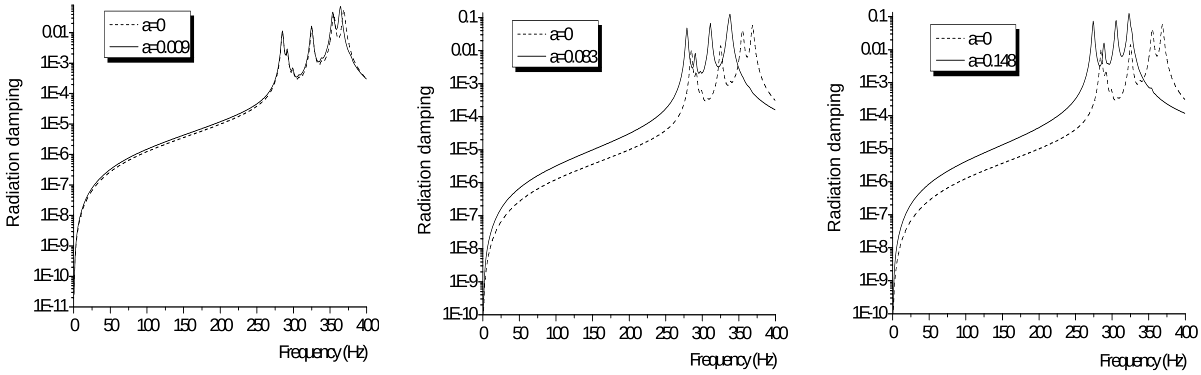


(a) Damage modulus k = 0.3 (b) Damage modulus k = 0.6 (c) Damage modulus k = 0.9

Fig.8 Acoustic radiation damping with altering damage degree

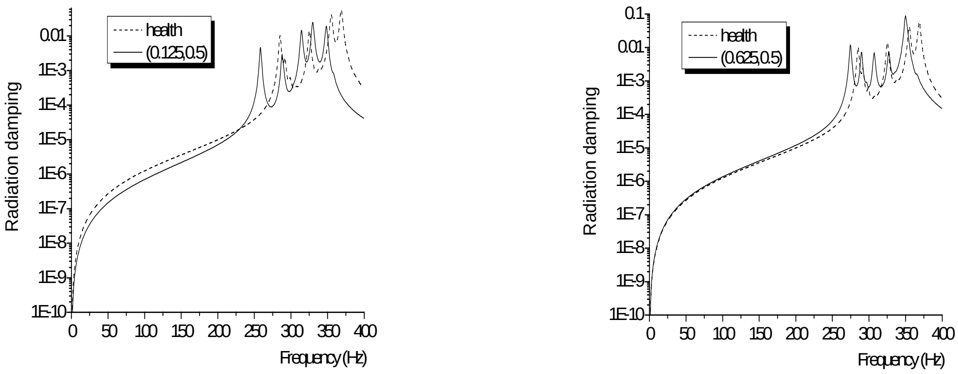
Figure 9 shows the acoustic radiation damping with different damage areas. It indicates that the acoustic radiation damping changes a little when the damage area is not very big. With the increase of the damage area, the damage may weaken the structure and cause the increase of the radiation damping.

Figure 10 shows the influence of damage location on radiation damping. It can be found that even the damage locates near the structure boundaries, radiation damping of the damaged structure just becomes a little lower than that of the healthy one. This situation is caused by the local vibration near the structure boundaries. It indicates that the damping location hardly affect the radiation damping. When the damage locates near the center of the structure, the radiation damping of the damaged structure matches well with that of the healthy one, except the location of the peak value changes a little and becomes closer to the ordinate.



(a) Damage area a = 0.009 (b) Damage area a = 0.083 (c) Damage area a = 0.148

Fig.9 Acoustic radiation damping with altering damage areas



(a) damage location in (0.125,0.5) (b) damage location in (0.625,0.5)

Fig.10 Acoustic radiation damping with altering damage locations

4. CONCLUSION

Presented herein are the characteristics of dynamics and acoustic radiation of a damaged stiffened panel investigated by the FEM/BEM. The following conclusions are drawn based on the numerical analysis:

- in case of the structure coupled with fluid, the noncompressive fluid model can be used with ease for the lower frequency. However, as the frequency increases, compressive fluid model should be adopted for more reasonable results, but the computing cost increases accordingly
- the introduction of the added mass of fluid results in an asymmetric mass. In the present work, the eigenvalue problems are solved by Lanczos and QR algorithms
- with the increase of the damage modulus, the peak value of radiation mode increases. The peak points of the radiation power and damping move into the direction of lower frequency. Directivity direction is found to be maximum at 120° when $k = 0.6$
- with the increase of the damage areas, the peak of radiation mode is very difference. The peakvalue of radiation power and damping moves towards low frequency. The main petal keeps fixedness but the accidental petal is strong gradually
- for different damage locations, the peak of radiation moves greatly. The radiation power and damping is changes a little. But the main petal shows big difference.