



RADIATION EFFICIENCY OF ACOUSTIC-EXCITED PLATES WITH STRINGER ATTACHMENTS

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Abstract

The average radiation efficiency of acoustic-excited plate with stringer attachments is investigated by using a modal expansion method. This work is an extension to author's previous publication (JSV 2007), where the predicted and measured sound transmission loss for curved aircraft panels was reported. For the plate and frequency range studied here, numerical results reveal that the radiation efficiency of acoustic-excited plate is very sensitive to the damping of the structure. Increasing the structure loss factor significantly increases radiation efficiency far below the plate critical frequency.

1. INTRODUCTION

To reduce radiated noise from large structure like ships and aircrafts, it is important to know the acoustic radiation efficiency on built-up plate. Radiation efficiency is defined as a ratio of the power radiated from a structure to that of a piston of the same size moving with the same average velocity. Unlike the sound reduction index (or sound transmission loss) which reflects a ratio of transmitted acoustic power to incident acoustic power, the radiation efficiency gives information of the radiated sound power and as well the plate response.

Much work has been devoted on the theory of sound radiation from baffled plates [1-3]. Among them, Maidanik[1] presented a theory based on energy transfer analysis, and his theory was further developed by Leppington et al [2] twenty year later. The energy method gives good agreements with measurement data in the multi-mode range. However, there was a need for a way to predict radiation efficiency in the low-frequency range. Such a method was presented in 1972 by Wallace[3], who derives a formula for the modal radiation efficiency from the full wave equation.

For understanding radiation from plates with mechanic excitation, it is also important to understand the influence of the exciting force and its near field for simple load cases (i.e. point- and line forces). This can be described by simple models developed by Fahy [4] and Cremer et al.[5]. To describe the radiation efficiency of a plate with random force excitation are recently discussed by Xie et al.[6].

Using modal expansion method, the average radiation efficiency of acousticexcited panel with stringer attachments is investigated in this paper. This work is relevant to author's previous publication [7], where the predicted and measured sound transmission loss for curved aircraft panels was reported. The effects of damping and stringers on radiation efficiency are numerically investigated and corresponding discussions are given as well.

2. MATHEMATICAL MODEL

Consider a simply supported, rectangular panel with stiffener attachments in the lateral direction (corresponding to stringer attachments for aircraft panels), see Fig.1. Sound is incident from one side and transmitted though the panel to other side. The differential equation governing the vibration of panel is given in reference [7]



Fig.1 Schematics of stiffened rectangular panel

$$D\nabla^4 w - m_p \omega^2 w = 2p^i - 2p^r - \sum_{s=1}^{S} q_s \delta(y - L_s) - \sum_{s=1}^{S} \kappa_s \delta'(y - L_s), \qquad (1)$$

where w is the normal displacement of the panel, an the incident wave acting on the panel surface may be written as $p^i = p_0 \exp[j(\omega t - k_x x - k_y y - k_z z)]$, p^r denotes the acoustic pressure radiated by the panel, $D = Eh^3/12(1-v^2)$ is the bending stiffness, E the Young's modulus, v the Poisson ratio, h the thickness of the panel, m_p the surface density of the panel, and j the unit imaginary $\sqrt{-1}$. The air density and sound speed are ρ_0 and c_0 , respectively.

The governing equations of the stiffener flexural and torsional displacement is given in reference [8]

$$\left(D_s d^4/dx^4 - m_s \omega^2\right) w_s = q_s \quad , \tag{2}$$

$$\left(T_{s} d^{2}/dx^{2} - EI_{w} d^{4}/dx^{4} + \rho_{s} I_{p} \omega^{2}\right)\theta_{s} = \kappa_{s} , \qquad (3)$$

where D_s and T_s are respectively the bending and torsional stiffness of the beam stiffener, I_w the warping constant of the stiffener, m_s the mass per unit length of the stringer, η_s the loss factor of the stiffeners, I_p the polar moment of inertia for the beam.

An eigenfunction describing the panel deflection is assumed to be

$$\phi_{mn}(x, y) = \phi_m(x)\phi_n(y) = \frac{2}{\sqrt{ab}}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b},$$
(4)

and following the same procedure in reference [7], the modal velocities of the panel are then

$$V_{mn} = Y_{mn} \left(P_{mn}^{t} - \sum_{n'} P_{mn}^{t} Y_{mn'} \boldsymbol{\Phi}_{n'} \right),$$
(5)

where P_{mn}^{t} is the modal force, and the skin/stringer coupling function $\Phi_{n'}$ is given by reference [7]. Now the modal admittance Y_{mn} can be written as

$$Y_{mn} = \frac{j\omega}{m_p} \left\{ \omega_{mn}^2 \left[1 + j\eta_{mn}^e \right] - \omega^2 \right\}^{-1},$$
(6)

and ω_{mn} is $(m,n)^{th}$ eigenfrequency and η_{mn}^{e} is the effective modal loss factor, defined by

$$\omega_{mn}^{2} = \frac{D}{m_{p}} \left[\left(\frac{m\pi}{a}\right)^{2} + \left(\frac{n\pi}{b}\right)^{2} \right]^{2} , \qquad (7)$$

$$\eta_{mn}^{e} = \eta + \frac{(\rho_1 c_1 + \rho_2 c_2)}{m_p} \frac{\omega \sigma_{mn}}{\omega_{mn}^2} , \qquad (8)$$

where η is the material damping, and σ_{mn} is the modal radiation efficiency. Regarding the modal radiation efficiency, one could refer to the recent contributions made by Li and Gibeling[9], who developed an analytical expression for the modal radiation efficiency, and substantially reduced the calculation time in comparison with the double integral formula due to Wallace [3].

The radiated sound power is then obtained, while neglecting cross terms of modal radiation efficiency, as

$$\Pi^{t} = \frac{1}{2} \operatorname{Re} \left\{ \int_{A} p^{r} \cdot v^{*} dA \right\} = \frac{1}{2} \rho_{0} c_{0} \sum_{mn} \sigma_{mn} |Y_{mn}|^{2} \left(P_{mn}^{t} - \sum_{n'} P_{mn}^{t} Y_{mn'} \Phi_{n'} \right)^{2}, \qquad (9)$$

where *A* is the surface area of the panel. The transmitted power and panel velocity in a diffuse field are obtained by averaging over all incident angles

$$\overline{\Pi^{t}} = \frac{\int_{0}^{2\pi\pi/2} \int_{0}^{1} \sin\theta \cos\theta d\theta d\phi}{\int_{0}^{2\pi\pi/2} \int_{0}^{2\pi\pi/2} \sin\theta \cos\theta d\theta d\phi}$$
(10)

$$\overline{v^2} = \frac{\int_{0}^{2\pi\pi/2} \int_{0}^{2\pi\pi/2} v^2 \sin\theta \cos\theta d\theta d\phi}{\int_{0}^{2\pi\pi/2} \int_{0}^{2\pi\pi/2} \sin\theta \cos\theta d\theta d\phi}$$
(11)

Where $v^2 = \frac{1}{2S} \sum |V_{mn}|^2$, Note that the average of $|P_{mn}^i|^2$ over all incident angles is related to the modal radiation efficiency

$$\int_{0}^{2\pi\pi/2} \int_{0}^{2\pi/2} \left| P_{mn}^{i} \right|^{2} \sin\theta d\theta d\varphi = \frac{16\pi^{2}}{Ak^{2}} \sigma_{mn} .$$

$$\tag{12}$$

The radiation efficiency is defined by a ratio of the power radiated from a structure to that of a piston of the same size moving with the same average velocity, Viz.,

$$\sigma_{av} = \frac{\overline{\Pi^{t}}}{\rho c S \overline{v^{2}}}$$
(13)

The combination of Eqs. (5) and (9)-(13) gives rise to an expression for modal average radiation efficiency due to acoustic excitation.

$$\sigma_{av} = \frac{\sum \sigma_{mn} (\sigma_{mn} - 2\sigma_{mn} \operatorname{Re}\{Y_{mn}\Phi_{m}\} + \sum_{n'} \sigma_{mn'}Y_{mn'}\Phi_{n'})|Y_{mn}|^{2}}{\sum (\sigma_{mn} - 2\sigma_{mn} \operatorname{Re}\{Y_{mn}\Phi_{m}\} + \sum_{n'} \sigma_{mn'}Y_{mn'}\Phi_{n'})|Y_{mn}|^{2}}$$
(14)

For an isotropic plate without stringer attachments, the expression for the modal average radiation efficiency is simplified as

$$\sigma_{av} = \frac{\sum \sigma_{mn}^2 |Y_{mn}|^2}{\sum \sigma_{mn} |Y_{mn}|^2}$$
(15)

It is worthy to note that the modal average radiation efficiency [5] or the average radiation efficiency of point-excited isotropic plate [6] has the form of

$$\sigma_{av} = \frac{\sum \sigma_{mn} |Y_{mn}|^2}{\sum |Y_{mn}|^2}$$
(16)

For a slightly curved panel with stringer attachments, one could substitute the curved panel modal admittance $Y_{c,mn}$ [7] for Y_{mn} in Eq. (14) to calculate the radiation efficiency. Eqs. (14) and (15) are then used in the prediction of the radiation efficiency of acoustics excited plates with and without stringer attachments.

3. NUMERICAL RESULTS AND DISCUSSION

The numerical study starts from the calculation of a reference plate for parameters appropriate to a typical aircraft panel. The plate is assumed with an axial length of 0.55m, lateral width 1m, skin area density $5.4 kg/m^2$ and bending stiffness 51 Nm. The critical frequency for this plate is about 6kHz.

Fig. 2 shows the radiation efficiency of the reference plate with different loss factor under the condition of acoustic excitation. It is of interest to notice that radiation efficiency is very sensitive to the damping of the structure. Increasing the structure loss factor significantly increases radiation efficiency. The reason is that the damping reduces the plate response dramatically while has less effects on the radiated/transmitted acoustic power. It is well known that the radiated/transmitted sound power is not sensitive to the structure damping for the forced transmission. The damping is only effective when the radiated/transmitted sound power is resonance dominant. For the reference case and frequency range studied here, the forced transmission is dominant. Therefore the radiated/transmitted sound power is roughly same for the plate with different structural damping. But the plate response is significantly reduced with structural damping treatment.

Fig.3 shows an example of measured radiation efficiency for a steel plate with and without damping treatment, reproduced from ref.[10]. The steel plate has a size of $1m \times 2m$ and thickness of 1mm, the damping treatment is a constrained damping layer, viz., 0.6 mm rubber-like material plus a constraining sheet of 0.28 mm steel. It is evident that damping layer improves radiation efficiency dramatically in all frequency range of the measurement. However, the measured sound transmission loss for the same plate with and without damping treatments is roughly the same [10].



Fig.2 Predicted damping effect on the radiation efficiency of acoustic-excited plate



Fig.3 Measured damping effect on the radiation efficiency of acoustic excited steel plate with the size of 1m x 2m and 1mm thickness, the damping treatment is a constrained damping layer (0.6 mm rubber-like material plus a constraining sheet of 0.28 mm steel). Reproduction from ref.[10].

A comparison of measured and predicted radiation efficiency of acoustic-excited plate with stringer arrangement is also shown in Fig.4. The measured data is from a metallic aircraft panel with a size of $2.2m \times 1.67m$. A detailed describe of this panel can be found in ref. [7]. The plate used in calculation is assumed to have similar structural arrangements to the measured panel (with similar stringer attachments and the same damping, structural curvature and thickness), while has a size of $1.1m \times 0.55m$ (corresponds to a sub-area between two ring frames). The parameters for the stringers used in calculation are given in Table 1. The stringers used in calculation are assumed having rectangular cross-section and equally spaced with a distance of 0.2m.

Fig.4 shows that the predicted radiation efficiency has good agreements with the measured data above 300Hz, indicating the validity of this prediction model. The discrepancies below 200Hz could result from the smaller size of the plate used in calculation for which the boundary conditions start to play an important role in the plate response and sound radiation.

Table 1

Panel	Material	Thickness	Height	Density	Young's	Stiffener	Loss
		mm	mm	kg/m^3	modulus	Number	factor
A	Aluminum	1.5	30	2700	6.85E+10	4	0.015



Fig.4 A comparison of predicted and measured radiation efficiency of acoustic excited plates with structural curvature and stringer attachments.

4. CONCLUSIONS

A numerical model based on modal expansion method is developed in this paper to predict the average radiation efficiency of acoustic-excited plate with stringer attachments. This model gives good prediction in comparison with a measurement for a large aircraft panel.

For the plate and the frequency range studied here, numerical results conclude that the radiation efficiency of acoustic excited plate is very sensitive to the damping of the structure. Increasing the structure loss factor increases radiation efficiency dramatically. The reason is that the damping significantly reduces the plate response while has less effects on the radiated/transmitted acoustic power. Such characteristics are not similar to the sound transmission loss for which is not sensitive to the structural damping when the transmitted power is dominated by forced transmission.

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