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# FREE VIBRATION OF VERTICAL PUMP 

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#### Abstract

Vertical pumps are widely used owing to the fact that they occupy small floor space. In this type of pumps, however, the vibration problems are very important, since in many cases, they have less stiffness in comparison with later pumps. This study presents a simple solution method for calculating the natural frequencies and modes of vertical pumps. In this study, a model of a vertical pump was developed and the no dimensional parameters for the vibration characteristics of it were determined. Added mass was calculated for the effects of water and the transfer matrix method was used. Also the result of the calculation was demonstrated by the practical test of power plant vertical pump.


## 1. INTRODUCTION

A structure in or in contact with a fluid has significantly varying dynamic characteristics such as natural frequency and damping, with the resistance of the fluid. The effects of fluid on a structure element vibrating in a fluid include inertia effect and damping effect: the former is usually studied in the manner of introducing the concept of added mass. In the past, as it was considered that except for structures at fine intervals, damping was not important in general, the relevant matters were limited primarily to studies on lubrication. Studies on fluid-related structures were intended to protect all kinds of vessels and pipes against damages caused by flow or earthquake, which was one of concerns in the petrochemical industry and power plant. After that, with the technological advance in revolutionary machines such as turbine blade and vertical pump, pressure vessel, heat exchanger and nuclear reactor components, many studies on the coupled fluid-structure systems were actively performed, including study on the effects of liquids on the dynamic motions of immersed solids [1], theoretical study on the added mass and damped vibration coefficient of vibrating rod in viscous fluids [2], and study on the analysis of added mass with finite-element method [3]. Based upon the results of these studies, Shimogo [4] conducted testing and interpretation on the vibration of vertical pump by replacing the effect of water pressure with added mass, and reported that resonance frequency decreased by approx. $10 \%$ in a pipe filled up with water, however the rotational shaft and water flow in the pipe had little or no influence upon resonance frequency. In addition, Yang [5] calculated the added mass of water with referring to the formula suggested by Fritz [1] and analyzed the time response to periodic external force and the eigenvalue of vertical pump with lumped mass method. Recently, in connection with the vibration of liquid vessel, the relevant
studies are in progress [6-8]. Although numerous studies on the vibration of beam have been conducted till now, a majority of them were limited to beams with relatively simple shapes and were difficult to apply their results to the practice of beam design because of very significant differences from the vibration of actual structural elements or too long time required to calculate the actual values. Also if the analysis model was modified a little, the existing analysis methods became inefficient. For actual structures, their beams are often supported even halfway. In the case of double-span beam like this, both the beams of the central support point affect vibration characteristics each other. In this study, a vertical pump was modeled with double-span beam, transfer function matrix was induced in consideration for added mass and the free vibration of vertical pump model was analyzed.

## 2. THEORETICAL BACKGROUND\&ANALYSIS

### 2.1 Analysis Model

A vertical pump consists of motor section in the upper part, impeller section in the lower part and column section in the middle part. As a model for analyzing the vibration of vertical pump, a double-span beam immersed partially in fluids is assumed as shown in Figure 1. Here, $M_{1}$ and $M_{2}$ means the mass of motor and impeller, respectively. In the model shown in Fig. 1, since the length of column section is much larger than its diameter, assuming that it is a beam, its shear effects and rotary inertia effects are neglected


Figure 1. Vertical pump model.

### 2.2 Kinematic Equations \& Transfer Function Matrix

Station $i$ is combined with translational spring, rotatory spring, lumped mass and rotational mass moment of inertia, and a multi-span beam whose length of field $i$ (interval between station $i$ and station $i+1$ ), bending stiffness and mass per unit length are equivalent to $l_{i}, E I_{i}$ and $m_{i}$, respectively is considered. The equation of bending vibration in the field $i$ is:

$$
\begin{equation*}
E I_{i} \frac{\partial^{4} w_{i}}{\partial x_{i}^{4}}+m_{i} \frac{\partial^{2} w_{i}}{\partial t^{2}}=0,0 \leq x_{i} \leq l_{i} \tag{1}
\end{equation*}
$$

Assuming simple harmonic motion, the following equation is formulated

$$
\begin{equation*}
w_{i}\left(x_{i}, t\right)=w_{i}\left(x_{i}\right) \sin w t \tag{2}
\end{equation*}
$$

Displacement $w_{i}\left(x_{i}\right)$ is calculated as follows

$$
\begin{gather*}
W_{i}\left(x_{i}\right)=c_{1} \cos p_{i} x_{i}+c_{2} \sin p_{i} x_{i}+c_{3} \cosh p_{i} x_{i}+c_{4} \sinh p_{i} x_{i}  \tag{3}\\
p_{i}^{4}=\frac{m_{i}}{E I_{i}} w^{2} \tag{4}
\end{gather*}
$$

Both the ends of field $i$ are regarded as station $i$ and station ( $i+1$ ), and in field $i$, the displacement and its derivative at the rightmost and leftmost points, respectively are defined as follows:

$$
\begin{gather*}
W_{(i+1) L}^{(n)}=\lim _{x_{i} \rightarrow l_{i}} \frac{d^{(n)} W_{i}\left({ }_{i}\right)}{(n)}  \tag{5-a}\\
W_{i R}^{(n)}=\lim _{x_{i} \rightarrow 0} \frac{d^{(n)} W_{i}}{d x_{i}}, n=0,1,2,3 \tag{5-b}
\end{gather*}
$$

where $W_{(i+1) L}^{(n)}$ means the $\mathrm{n}^{\text {th }}$-degree derivative of displacement just at the left side of station ( $i+1$ ). And, to induce nondimensional equations, the coefficient of nondimensional natural frequency, nondimensional displacement and its derivative are defined in sequence as follows:

$$
\begin{gather*}
\Omega_{i}=p_{i} l_{i}  \tag{6}\\
y_{(i+1) L}^{(n)}=\frac{1}{p_{i}^{n}} W_{(i+1) L}^{(n)}  \tag{7-a}\\
y_{i R}^{(n)}=\frac{1}{p_{i}^{n}} W_{i R}^{(n)}, n=0,1,2,3 \tag{7-b}
\end{gather*}
$$

Next, after substituting equation (3) for equation (5), $c_{1}, c_{2}, c_{3}$, and $c_{4}$ are eliminated. Then, $W_{(i+1) L}^{(n)}(n=0,1,2,3)$ can be expressed in the function of $W_{i R}^{(n)}$, and equations (6) and (7) are applied again, inducing four non-dimensional equations. Assuming that $y_{i R}, y_{i R}^{\prime}, y_{i R}^{\prime \prime}$ and $y_{i R}^{\prime \prime \prime}$ indicate the non-dimensional displacement, 1st-degree derivative, 2nd-degree derivative and 3rd-degree derivative, respectively right at the right side of station $i$, the station vector is defined as:

$$
\begin{equation*}
Y_{i R}^{T}=\left[y_{i R} y_{i R}^{\prime} y_{i R}^{\prime \prime} y_{i R}^{\prime \prime \prime}\right]^{T} \tag{8}
\end{equation*}
$$

The above-mentioned four non-dimensional equations can be expressed in the following determinant.

$$
\begin{equation*}
Y_{(i+1) L}=F_{i} Y_{i R} \tag{9}
\end{equation*}
$$

where matrix $F_{i}$ meets the followings.

$$
2 F_{i}=\left[\begin{array}{cccc}
\cos \Omega_{i}+\cosh \Omega_{i} & \sin \Omega_{i+} \sinh \Omega_{i} & -\cos \Omega_{i}+\cosh \Omega_{i} & -\sin \Omega_{i}+\sinh \Omega_{i}  \tag{10}\\
-\sin \Omega_{i}+\sinh \Omega_{i} & \cos \Omega_{i}+\cosh \Omega_{i} & \sin \Omega_{i}+\sinh \Omega_{i} & -\sin \Omega_{i}+\sinh \Omega_{i} \\
-\cos \Omega_{i}+\cosh \Omega_{i} & -\sin \Omega_{i}+\sinh \Omega_{i} & \cos \Omega_{i}+\cosh \Omega_{i} & \sin \Omega_{i}+\sinh \Omega_{i} \\
-\sin \Omega_{i}+\sinh \Omega_{i} & \cos \Omega_{i}+\cosh \Omega_{i} & \sin \Omega_{i}+\sinh \Omega_{i} & -\cos \Omega_{i}+\cosh \Omega_{i}
\end{array}\right](
$$

### 2.3 Conditions for Inter-span Combination

Since the displacements and tilt angles should be identical before and after station $i$, the formulas below should be established.

$$
\begin{align*}
& W_{i L}=W_{i R^{\prime}},  \tag{11-a}\\
& \Theta_{i L}=\Theta_{i R}, \tag{11-b}
\end{align*}
$$

Moreover, assuming that bending moment and shear force in the section of beam are defined as $M_{b}$ and V, respectively, the equilibrium conditions of shear force and moment for station $i$ are formulated as follows:

$$
\begin{gather*}
-k_{i} W_{i R}+V_{i R}-V_{i L}=-M_{i} w^{2} W_{i R}  \tag{12-a}\\
-t_{i} \Theta_{i R}+\left(M_{b}\right)_{i L}=-j_{i} w^{2} \Theta_{i R} \tag{12-b}
\end{gather*}
$$

The relation among the displacement, tilt angle and shear force of beam is applied and the above equations are integrated with referring to equations ( $7-a$ ) and (7-b). Then, the following non-dimensional determinant is deduced ( $S_{i}$ is a matrix).

$$
\begin{equation*}
Y_{i R}=S_{i} Y_{i L}, i=2,3, \ldots . ., n \tag{13}
\end{equation*}
$$

### 2.4 Added mass

To determine the reaction of water acting on a vibrating beam in fluids, it is required first to calculate the pressure distribution around the beam. This pressure distribution can be calculated with Navier-Stokes equations and continuity condition. If the amplitude of beam is small, nonlinear terms in Navier-Stokes equations can be neglected, and for nonviscous fluids, pressure distribution is expressed in Laplace's equations. In this study, as the length of beam is much larger than its sectional dimension, it can be regarded as two-dimensional problems, and governing equations and boundary conditions for the pressure distribution are as follows:

$$
\begin{align*}
& \frac{\partial^{2} p}{\partial x^{2}}+\frac{\partial^{2} p}{\partial \partial^{2}}=0,  \tag{15}\\
& \alpha \frac{\partial p}{\partial x}+\beta \frac{\partial p}{\partial y}=-\rho a_{n} \tag{16}
\end{align*}
$$

Where $n$ means the direction in which it turns towards fluids perpendicularly to structure surface; $\alpha$ and $\beta$ mean the contacted form by the axes x and y against direction $n$, respectively. It is known that $p$ meeting the two equations above is consistent with the variational problem minimizing the following function [9].

$$
\begin{equation*}
\phi=\frac{1}{2} \iint_{A}\left[\left(\frac{\partial P}{\partial X}\right)^{2}+\left(\frac{\partial P}{\partial y}\right)^{2}\right] d x d y+\int_{c} \rho a_{n} p d s \tag{17}
\end{equation*}
$$

where $A$ represents the inside of fluid and c the boundary of fluid. Now, to minimize $\phi$ in equation (17), the node pressure of finite element $e$ is expressed in vector $p_{e}$. Assuming that pressure is the function for the coordinates $x$ and $y$, matrix D meeting the following equations can be induced with using the relation among node pressure $p_{e}$ and node coordinates $x$ and $y$.

$$
\begin{equation*}
\binom{\frac{\partial p}{\partial x}}{\frac{\partial p}{\partial y}}=D P_{e} \tag{18}
\end{equation*}
$$

Thus, the first term of function $\phi$ for element $e$ is:

$$
\begin{gather*}
\phi_{e}=\frac{1}{2} p_{e}^{T} g_{e} p_{e},  \tag{19}\\
g_{e}=D^{T} D A_{e} \tag{20}
\end{gather*}
$$

where $A_{e}$ means the area of finite element $e$. Then, assuming that the $b$ th boundary is a segment connecting two nodes $i$ and $j$, its length $L_{b}$, and its vertical acceleration $a_{b}$, the following equation is obtained.

$$
\begin{gather*}
\int_{c_{b}} \rho a_{b} p d s=\frac{1}{2} \rho a_{b} L_{b}\left(p_{i}+p_{j}\right)  \tag{21}\\
h_{b}^{T}=\frac{1}{2} \rho a_{b} L_{b}[1,1] \tag{22}
\end{gather*}
$$

The second term of the function $\phi$ in equation (17) is formulated as follows:

$$
\begin{equation*}
\phi_{b}=h_{b}^{T} P_{b} \tag{23}
\end{equation*}
$$

where $P_{b}$ indicates the vector composed of pressure at two nodes $i$ and $j$ forming the $b$ th boundary. As a result of summing up $\phi_{e}$ and $\phi_{b}$ in equations (19) and (23) for all the elements and boundaries in fluids, function $\phi$ is formulated as follows:

$$
\begin{equation*}
\phi=\frac{1}{2} P^{T} G P+H^{T} P \tag{24}
\end{equation*}
$$

where $G$ and $H$ represent the combinations of the nodes corresponding to $g_{e}$ and $h_{b}$, respectively. The condition to minimize $\phi$ for node pressure vector $\boldsymbol{P}$ is induced as follows:

$$
\begin{equation*}
G P+H=0 \tag{25}
\end{equation*}
$$

The force acting on solid elements moving at accelerated rates in contact with fluids is:

$$
\begin{equation*}
F=\frac{1}{2} H^{T} P \tag{26}
\end{equation*}
$$

As dividing this force by the acceleration, the added mass is obtained.

## 3. BOUNDARY CONDITIONS AND EIGENVALUES

In station 1, the equilibrium condition of shear force and moment can be expressed in the following equations.

$$
\begin{align*}
& -k_{1} W_{1 R}+V_{1 R}=-M_{i} w^{2} W_{1 R}  \tag{27}\\
& \quad-t \Theta_{1 R}+\left(M_{b}\right)_{1 R}=-j_{1} w \Theta_{1 R} \tag{28}
\end{align*}
$$

The relation among the displacement, tilt angle and bending moment and shear force of the beam is applied and integrated, and the equations of equilibrium condition in station $n$ can be formulated as the non-dimensional equations below.

$$
\begin{align*}
& y_{(n+1) L}^{\prime \prime}=\left(-\frac{t_{n+1}}{E I_{n}} \frac{1}{p_{n}}+\frac{j_{n+1}}{m_{n}} p_{n}^{3}\right) y_{(n+1) L}^{\prime}  \tag{29-a}\\
& y_{(n+1) L}^{\prime \prime \prime}=\left(-\frac{k_{n+1}}{E I_{n}} \frac{1}{p_{n}^{3}}-\frac{M_{n+1}}{m_{n}} p_{n}\right) y_{(n+1) L} \tag{29-b}
\end{align*}
$$

Inducing the equations of equilibrium condition in station $n$ in the same manner, the following equations are obtained.

$$
\begin{gather*}
y_{(n+1) L}^{\prime \prime}=\left(-\frac{t_{n+1}}{E I_{n}} \frac{1}{p_{n}}+\frac{j_{n+1}}{m_{n}} p_{n}^{3}\right) y_{(n+1) L}^{\prime}  \tag{30-a}\\
y_{(n+1) L}^{\prime \prime \prime}=\left(-\frac{k_{n+1}}{E I_{n}} \frac{1}{p_{n}^{3}}-\frac{M_{n+1}}{m_{n}} p_{n}\right) y_{(n+1) L} \tag{30-b}
\end{gather*}
$$

Using the equations induced till now, the relation among the displacement in stations 1 and $(n+1)$, and its derivatives, is formulated as follows:

$$
\begin{equation*}
Y_{(n+1) L}=F_{n} S_{n} \ldots \ldots . . F_{2} S_{2} F_{1} Y_{1 R} \tag{31}
\end{equation*}
$$

Applying the boundary condition of equation (15) to this equation, $\Omega_{1}$ can be calculated. And, based on $E I_{1}, m_{1}$, and the full length of beam $l$, the coefficient of non-dimensional natural frequency $\Omega$ is defined as follows:

$$
\begin{gather*}
\Omega=\frac{l}{l_{1}} \Omega_{1}  \tag{32}\\
w=\left(\frac{\Omega}{l}\right)^{2}\left(\frac{E I_{1}}{m_{1}}\right)^{1 / 2} \tag{33}
\end{gather*}
$$

## 4. ANALYSIS RESULTS \& REVIEW

In accordance with the equations induced previously, the added mass matrix was obtained, and subsequently, field matrix, state matrix and boundary condition equation were combined to prepare a computer program capable of solving the problem of Eigenvalues. The results of calculation are as follows.

### 4.1 Kovats’Model

Figure 2 shows the model suggested by Kovats [10] which was used to analyze the vibration of the upper structure of vertical axial pump. This model has a translational spring in the middle of cantilever beam and lumped mass at the free ends, where $l_{2} / l_{1}=1, E I_{2} / E I_{1}=1$, $m_{2} / m_{1}=1, M / m_{1} l_{1}=1$, and $k l_{1}^{3} / E I_{1}=80$. Pak[11] divided this model into twos beams, applied eight boundary conditions to two equations of motion and calculated the natural frequency.


Figure 2 Kovats' model for vertical pump.
Table 1 summarizes the natural frequency coefficients $\Omega$ and the results of the analysis performed by Pak, suggesting that they are consistent with each other. Table 1 presents the non-dimensional natural frequency coefficient $\Omega$ of Kovats’ Model .

Table 1. Natural frequency.

| Mode 1Mode 2Mode 3 Mode 4Mode 5 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Pak | 1.587 | 4.54 | 7.15 | 10.28 |  |
| Analysis | 1.587 | 4.5 | 7.15 | 10.2887 |  |

### 4.2 Double Span Beam Model

The resources of the double-span model shown in Figure 1 are as follows: $\mathrm{M}_{1}=120 \mathrm{~kg}$; the mass of motor , $\mathrm{M}_{2}=25 \mathrm{~kg}$; the mass of impeller, $\mathrm{d}_{\mathrm{s}}=50 \mathrm{~mm}$; the diameter of rotating shaft, $\mathrm{d}_{\mathrm{pi}}=200 \mathrm{~mm}$; the inner diameter of column pipe $\mathrm{d}_{\mathrm{po}}=215 \mathrm{~mm}$; the outer diameter of column pipe $d_{b}=450 \mathrm{~mm}$; the inner diameter of barbell $d_{i}=300 \mathrm{~mm}$; the outer diameter of impeller $t=40 \mathrm{~mm}$; the thickness of support flange $\rho=7790 \mathrm{~kg} / \mathrm{m}^{3}$; density $\mathrm{E}=2 \times 10^{11} \mathrm{~N} / \mathrm{m}^{2}$; Young's modulus $l_{1}=1000 \mathrm{~mm}, l_{2}=3600 \mathrm{~mm} l_{3}=100 \mathrm{~mm}, l_{\mathrm{w}}=1600 \mathrm{~mm}$. The motor and impeller were regarded as lumped mass. As barbell has far larger bending stiffness than that of column pipe or rotating shaft, it was regarded as a rigid body. It is known that if the outer diameter of disk with diameter $\mathrm{d}_{\mathrm{o}}$ is completely fixed and the column of a rigid body with diameter $\mathrm{d}_{\mathrm{i}}$ is fixed at the center of the disk, the corresponding rotational stiffness is as follows [12].

$$
\begin{equation*}
k_{\theta}=\frac{E t^{3}}{\alpha} \tag{34}
\end{equation*}
$$

where $t$ is the thickness of disk. $\alpha$ is as given in Table 2.

Table 2 Values of $\alpha$.

| $\mathrm{d}_{\mathrm{i}} / \mathrm{d}_{\mathrm{o}}$ | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 |
| :---: | :--- | :--- | :--- | :--- | :--- |
| $\alpha$ | 0.595 | 0.320 | 0.167 | 0.081 | 0.035 |

Accordingly, this study assumed that the support flange corresponds to torsional stiffness with constant $1.279 \times 10^{8} \mathrm{Nm}$. As a result of analyzing the model, the value of added mass matrix is as follows:

$$
m_{a d}=\left|\begin{array}{cccc}
57.39 & 0.00 & -93.65 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 \\
-93.65 & 0.00 & 252.49 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00
\end{array}\right|
$$

where the unit is $\mathrm{kg} / \mathrm{m}$. Depending on the presence of guarder, the values of natural frequency are given in Table 3, and Figure 3 shows the natural modes. Figure 4 shows changes in the primary natural frequency depending upon the inner diameter of barbell without guarder. As the inner diameter of barbell gets close to the outer diameter $(0.30 \mathrm{~m})$ of impeller, the magnitude of added mass increases rapidly and consequently the natural frequency decreases rapidly.

Table 3. Natural frequencies of the model of Figure 3.

| Mode | Without <br> guarder | With <br> guarder |
| :--- | :--- | :--- |
| 1st | 7.137 Hz | 37.92 Hz |
| 2nd | 48.33 Hz | 52.33 Hz |
| 3rd | 52.86 Hz | 133.61 Hz |
| 4th | 149.58 Hz | 272.91 Hz |
| 5th | 290.26 Hz | 469.38 Hz |



Figure 3. Natural frequency of the model.


Figure 4. Natural frequency of first mode as diameter of barbell varies.

## 5. CONCLUSION

This study aimed to analyze the bending vibration of vertical pump by modeling a vertical pump with double-span beam immersed in confined fluids. The reaction of fluids to vibrating beam was replaced with added mass, and a program capable of analyzing the natural frequency and natural mode of pump model was produced. Since assuming that the structure of pump is a continuous system using beam, transfer matrix was applied, it was possible to calculate accurate results for the pump model with less efforts, as well as to draw the non-dimensional design parameters related to the natural vibration of pump structure.

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