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# IMPROVED STRUCTURE-ACOUSTIC INTERACTION MODELS, PART I: MODEL DEVELOPMENT

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**Abstract**

An improved pressure-field governing equation has been developed for the modelling of external acoustic field interacting with flexible structures. The proposed interaction model is aimed to accurately capture both low and medium frequency interactions and to offers a frequency tailoring capability for a wider range of applications. The theoretical basis of the present model is the use of a combination of the retarded Kirchhoff potential and advanced potential to arrive at a stable second-order parametrerized approximate model. The present model is compared to the classical models of the first-order and second-order Doubly Asymptotic Approximations (DAA1 and DAA2) is shown to possess certain accuracy advantages. Numerical evaluations of the proposed interaction model are carried out in a companion paper, Improved Structure-Acoustic Interaction Models, Part II: Model Evaluaion.

## 1. INTRODUCTION

Computationally tractable models of external acoustic fields interacting with flexible structures have received intense interest over the past three decades. A dominant approach adopted in the development of acoustic-structure interaction models has been to improve the classical plane wave approximation models[16, 12, 8, 5, 9]. In addition, the finite element based approach has also been pursued, which is well documented in a survey article by Astley[2]. The fidelity of various approximate interaction models are then evaluated by comparing the results obtained by the approximate models either with the solution from the retarded acoustic potential equation or the continuum wave equation[3, 12].

One of the computationally tractable approximate acoustic-structure interaction equations is the Doubly Asymptotic Approximation(DAA) proposed by Geers and co-researchers[8]. In deriving their DAA models, the two limiting cases have been modeled: early-time approximation(ETA) and late-time approximation(LTA) by employing the initial-value and final-value theorems of the Laplace tranform to the series expansion terms of the Kirchhoff's spherical acoustic integral wave equation [5]. The DAA models are then constructed by comparing the parameterized first-oder or second-order forms with the ETA and LTA limits. The DAA models have proved to be adequate for characterizing the fluid acoustic radiation damping affecting the structural responses that are dominated by low-frequency components. For medium and high-frequency transients, however, most existing approximate structure-external acoustic interaction models suffer from both frequency distortions and inaccurate radiation damping.

This has motivated the present authors to develop acoustic models that can capture predominant acoustic modes, as distinct from structural modes, and yet that are computationally attractive. From the theoretical point of view, the well-known Kirchhoff's retarded potential equation may be considered as the foundation of all the existing approximate models. Under this premise, different approximate acoustic models originate from the corresponding different approximations of the retarded (or delayed) operator.

A key departure of the present acoustic-structure interaction model derivation is the use of a combination of the retarded potential and the advanced potential, and a precursor to the present improved model was presented in 2006[18, 13]. In employing the advanced potential, we are keenly mindful of the disagreement between Ritz and Einstein[19] on the validity of the advanced potential and the subsequent discussions that appear to suggest that the use of the advanced potential may be untenable in relativistic electromagnetic theory[7]. Even to this date, the 1908 Ritz-Einstein disagreement continues to rouse intense arguments and counter-arguments[1, 15]. However, our use of the advanced potential in deriving approximate acoustic models is justified primarily by the observation that the classical laws of physics (to which the acoustics field belongs) discovered by Galileo, Newton and Einstein are time-symmetric and secondly by recent applications of the time-reverse concept in acoustics[4, 6]. In other words, as acoustic signals are invariant under time-reversal, each packet of sound that comes from a source can be reflected, refracted or scattered. Consequently, a set of reflected waves can retrace all of the scattering paths, converging at the original source just as if time was going backwards. The rest of the paper is organized as follows.

Section 2 introduces the Kirchhoff's retarded potential and its conjugate advanced potential in the Laplace-transformed domain. A new potential is constructed by a linear weighting of the two potentials and the two-term expansions of the Laplace-transformed delay and advancing exponentials provide the basic second-order parameterized acoustic-structure interaction model. It is shown that the present basic model is stable provided the the weighting parameter is chosen properly and obviate the asymptotic matching procedures employed in the derivation of the various DAA models.

The present basic parameterized model is then modified, first, to accommodate the early time plane-wave phenomenon in the continuum case. Second, the weighting parameter is generalized as a mode-by-mode parameter by comparing the accuracy of the present model with the analytical solution for a uniform spherical shell. Third, the mode-by-mode weighting parameters are then converted into a discrete matrix for general structural surface geometries. The resulting parameterized model thus derived show that: (1) the maximum convergent temporal order of the coupled acoustic pressure equation is at most two; (2) several existing approximate models fail to satisfy the initial impulse response condition, thus they may yield erroneous impulse responses that are important for inverse identification applications; (3) the present parameterized approximate model may be tailored to specific applications for problems where the computation of pressure fields radiating from the flexible surface constitutes a key interest.

Thus, in comparison with our previous derivations[18, 13], A major improvement in the present interaction model is the parametrization of the weighting parameter and a construction of the discrete parameterized matrix so that the improved model can be applicable to general interaction surfaces. A critical evaluation of the present improved model is reported in the companion paper, Part II[14].

## 2. THEORETICAL BASIS

Kirchhoff's retarded potential formula for describing the expanding or radiating wave can be expressed as[3]:

$$4\pi\epsilon p(P,t) = \int_S \{\frac{\rho}{R}\dot{u}(Q,t_r) - \frac{1}{R^2}\frac{\partial R}{\partial n}p(Q,t_r) - \frac{1}{cR}\frac{\partial R}{\partial n}\dot{p}(Q,t_r)\}dS_Q \tag{1}$$

where $p$ and $u$ are the pressure and particle velocity, $R$ is the distance from $P$ to a typical point $Q$ on the surface $S$; $\partial/\partial n$ denotes differentiation along the outward normal to $S$; $\epsilon$ is the solid angle that takes on $(1, 0.5, 0)$ depending on whether the point $Q$ is within the acoustic domain, on the surface $S$, or inside the enclosed surface $S$, respectively. The $t_r = (t - \frac{R}{c})$ denotes the retarded time and the retarded potential terms are Laplace-Transformed as

$$\int_0^\infty e^{-st}g(Q,t_r)dt = e^{-sR/c}\overline{g}(Q,s), \quad \overline{g}(Q,s) = \int_0^\infty e^{-st}g(Q,t)dt \tag{2}$$

Hence, the retarded potential formula(1) can be expressed in the Laplace Transform domain as

$$\int_S \overline{p}(Q,s)\frac{\partial R}{\partial n}\frac{1}{R^2}(1+Rs/c)\,e^{-Rs/c}dS_Q + 4\pi\epsilon\overline{p}(P,s) = \rho\int_S s\overline{u}(Q,s)\frac{1}{R}\,e^{-Rs/c}dS_Q. \tag{3}$$

The Laplace-transformed counterpart(3) states that the contributions of Kirchhoff's retarded potential formula(1) from the previous states are expressed in terms of the delay operator $e^{-sR/c}$. It should be noted that various approximations, both in the time and the Laplace domain methods, amount to how this delay operator is approximated.

It was shown in[18] that (3) does not yield a computationally stable consistent approximation, primarily due to the inherent destabilizing delay exponential $e^{-sR/c}$, a well-known fact in delayed feedback theory. Hence, from the present perspective, the DAAs are stabilized forms by utilizing a parameterized model only to match the ETA and LTA asymptotes of (3). To obviate the inherent destabilizing property associated with the expansion of the retarded potential, we introduce the advanced acoustic potential defined as

$$\phi_a = \phi(Q,t_a) = \phi(Q,t+\frac{R}{c}), \quad \int_0^\infty e^{-st}\phi(Q,t_a)dt = e^{sR/c}\overline{\phi}(Q,s). \tag{4}$$

Combining the two-term Taylor series approximate Laplace-transformed expressions of the retarded and advanced equation in accordance of the weighting rule stipulated in [19],

$$\phi = \frac{1}{2}(1-\alpha)\phi_r + \frac{1}{2}(1+\alpha)\phi_a, \quad \phi_r = \phi(Q, t - R/c), \quad \phi_a = \phi(Q, t + R/c) \tag{5}$$

we obtain

$$\begin{aligned} &\int_S \overline{p}(Q,s)\frac{\partial R}{\partial n}\frac{1}{R^2}(1+Rs/c)\left\{1+\alpha Rs/c\right\}dS_Q \\ +4\pi\epsilon\overline{p}(P,s) \approx \rho &\int_S s\overline{u}(Q,s)\frac{1}{R}\left\{1+\alpha Rs/c\right\}dS_Q \end{aligned} \tag{6}$$

Rearranging the above approximate equation in the order of $s$-variable, we obtain

$$\alpha s^2 B\overline{p}(P,s) + (1+\alpha)scB_1\overline{p}(P,s) + c^2 B_2\overline{p}(P,s) = \alpha s^2\rho c A\overline{u}(P,s) + s\rho c^2 A_1\overline{u}(P,s) \tag{7}$$

where

$$\begin{aligned} B\overline{p}(P,s) &= \int_S \frac{\partial R}{\partial n}\overline{p}(Q,s)dS_Q, \quad B_1\overline{p}(P,s) \;=\; \int_S \frac{1}{R}\frac{\partial R}{\partial n}\overline{p}(Q,s)dS_Q \\ B_2\overline{p}(P,s) &= \int_S \frac{1}{R^2}\frac{\partial R}{\partial n}\overline{p}(Q,s)dS_Q + 4\pi\varepsilon\delta(P-Q) \\ A\overline{u}(P,s) &= \int_S \overline{u}(Q,s)dS_Q, \quad A_1\overline{u}(P,s) \;=\; \int_S \frac{1}{R}\overline{u}(Q,s)dS_Q \end{aligned} \tag{8}$$

It should be noted that the third-order term is parasitic in that as $R \to \infty$, the term becomes infinite. This implies that, from the context of the present theoretical basis, the maximum order of stable approximate interaction models is two.

## 3. PROPOSED EXTERNAL ACOUSTIC-STRUCTURE INTERACTION MODEL

The basic second-order external acoustic-structure interaction model derived in (7), while it does not require ETA and LTA matching as was necessary in the derivation of the DAAs, needs two modifications: plane wave approximation and a parameterized representation of the weighting factor $\alpha$ employed for a combination of the retarded and advanced potential. We present these two modifications below.

### 3.1. Plane Wave Approximation

The approximate model for external acoustic field interacting with flexible structures derived in (7) has been obtained by expanding the delay and advance exponential to their first order. This implies, by virtue of the initial($s \to \infty$) and final($s \to 0$) value theorems of the Laplace transform, that the approximate model thus derived would offer higher model fidelity for the late-time response than for early-time response. This means that among the five coefficient operators $(B, B_1, B_2, A, A_1)$ in the approximate acoustic model(7), the two zeroth-order terms $(B_2, A_1)$ should need no further modifications. This leaves the two remaining operators, viz., $(B, B_1)$, as potential modification candidates in order to improve the model fidelity for the early-time responses.

The plane wave approximation was investigated for the early-time responses by Mindlin and Bleich[17] and Fellipa[5], among others, which we apply to the present approximate model for the early time responses. For the present formulation the plane wave approximation is effected by invoking:

$$\text{Plane wave approximation:} \quad \frac{\partial R}{\partial n} \rightarrow 1 \tag{9}$$

such that $B$ and $B_1$ are modified as

$$B_1\overline{p}(P,s)|_{\frac{\partial R}{\partial n}\rightarrow 1} = A_1\overline{p}(P,s), \;\; B\overline{p}(P,s)|_{\frac{\partial R}{\partial n}\rightarrow 1} = A\overline{p}(P,s) \tag{10}$$

This is because, in physical terms, the direction of the wave path and the normal to the interaction surface remain parallel for plane waves. With the above modifications, we arrive at the present parameterized second-order external acoustic model given by

$$\boxed{\alpha s^2 A\overline{p}(P,s) + (1+\alpha)scA_1\overline{p}(P,s) + c^2B_2\overline{p}(P,s) = \alpha s^2\rho cA\overline{u}(P,s) + s\rho c^2A_1\overline{u}(P,s)} \tag{11}$$

It is noted that the present parameterized model(11) has not resorted to asymptotic matching as was the case for the DAAs.

### 3.2. Modal Form of the Present Parameterized Acoustic Interaction Equation

In our earlier work reported in [18, 13] it was discovered that it is preferable to tailor the weighting parameter ($\alpha$) employed for the combination of the retarded and advanced potential to specific dominant interaction modes. This can be viewed as a compensation for the gross approximation committed in the plane wave approximation introduced in equation(11). This is because the weighting parameter shows up only for the three terms affected by the plane wave approximation. In the parametrization search we have been guided by the exact mode-by-mode interaction characteristic roots for a sphere[12] and the works of Geers and Felippa[9] and Geers and Zhang[10, 11]. The present parametrization is achieved as follows.

First, the approximate model(11) is specialized to a spherical case. In so doing, for the simplicity of subsequent algebra, dimensionless variables are used: $t = Tc/a$, $u = w/a$, $r = R/a$ and $p$ is normalized by $\rho c^2$, where $T$ is the time($sec$). With these non-dimensionalizations, (11) takes the form:

$$\alpha_n s^2\overline{p}_n + (1+\alpha_n)s\overline{p}_n + (1+n)\overline{p}_n = \alpha_n s^2\overline{w}_n + s\overline{w}_n \tag{12}$$

where the weighting parameter, $\alpha$, is now parameterized for each mode to be $\alpha_n$.

The coupled analytical mode-by-mode interaction equations are given by[8]:

$$\begin{bmatrix} \lambda_n s^2 + A_n^{vv} & A_n^{vw} & 0 \\ A_n^{vw} & s^2 + A_n^{ww} & \mu \\ 0 & s^2\kappa_n(s) & s\kappa_n'(s) \end{bmatrix} \begin{Bmatrix} v_n \\ w_n \\ p_n^s \end{Bmatrix} = \begin{Bmatrix} 0 \\ -\mu p_n^0 \\ s\kappa_n(s)u_n^0 \end{Bmatrix} \tag{13}$$

$$A_n^{vv} = \lambda_n(1+\beta)\xi_n\gamma_0, \quad A_n^{vw} = \lambda_n(1+\nu+\beta\xi_n)\gamma_0, \quad A_n^{vw} = [2(1+\nu)+\lambda_n\beta\xi_n]\gamma_0 \tag{14}$$

where $\mu = (\rho/\rho_s)(a/h)$, $\gamma_0 = c_s^2/c^2$, $\beta = (h/a)^2/12$, $\lambda_n = n(n+1)$, $\xi_n = (\lambda_n - 1 + \nu)$, and the exact pressure impedances for the first three modes are given by

$$\frac{\overline{p}_n(s)}{\overline{w}_n(s)} = \frac{\kappa_n'}{\kappa_n} = \begin{cases} s/(s+1), & \text{for n=0} \\ (s^2+s)/(s^2+2s+2), & \text{for n=1} \\ \frac{s^3+3s^2+3s}{s^3+4s^2+9s+9}, & \text{for n=2} \end{cases} \tag{15}$$

The DAA$_1$ and DAA$_2$ can be assessed in the same way. Geers et al[8, 11] give the following Laplace-transformed equations:

$$\text{DAA}_1 \quad : \quad s\overline{p}_n + (1+n)\overline{p}_n = s\overline{w}_n \tag{16}$$

$$\text{DAA}_2(1978) \quad : \quad s^2\overline{p}_n + (1+n)s\overline{p}_n + (1+n)^2\overline{p}_n = s^2\overline{w}_n + (1+n)s\overline{w}_n \tag{17}$$

$$\text{DAA}_2(1994) \quad : \quad s^2\overline{p}_n + (1+n)s\overline{p}_n + n(1+n)\overline{p}_n = s^2\overline{w}_n + ns\overline{w}_n. \tag{18}$$

*Remark 1:* It is noted that the discrete forms of the DAA$_1$ and the DAA$_2$(1978) exist, but to date no corresponding discrete form of the DAA$_2$(1994) has been reported.

*Remark 2:* The modal form of the present model(12) is seen to specialize to the DAA$_1$ with $\alpha_n = 0$ and to the DAA$_2$(1994) with $\alpha_n = 1/n$, but not the DAA$_2$(1978).

The early time consistency is important for inverse acoustic problems. Applying the initial value theorem to the impedances of the exact and the present parameterized models, one obtains

$$\lim_{t\to 0}\left[\frac{p_n(t)}{u_n(t)}\right] = \delta(0) - 1, \quad \text{for the exact and present model cases with } \alpha_n \neq 0 \tag{19}$$

$$\lim_{t\to 0}\left[\frac{p_n(t)}{u_n(t)}\right]_{DAA} = \begin{cases} \delta(0) - (n+1) & \text{for } DAA_1 \\ \delta(0) & \text{for } DAA_2(1978) \end{cases} \tag{20}$$

Hence, neither the DAA$_1$ nor the DAA$_2$(1978) satisfies the early-time consistency requirement. The only exception is for the case of the DAA$_1$ with the breathing mode of $n = 0$.

*Remark 3:* As the mode number increases, the number of roots for the analytical homogeneous pressure equation increases as seen from (15). However, the present modal model given by (12) possesses only two regardless of its order. This inability is an inherent deficiency of the present pressure equation(11), and also for the DAAs.

In order to alleviate this deficiency, we have carried out a mode-by-mode examination of the dominant interaction characteristic roots, which suggests the following modal relation(see Part II[14] for details):

$$\chi_n = \alpha_n^{-1} = \begin{cases} 1, & \text{when } n = 0 \\ b_0 n + b_1, \quad b_0 \approx 1 \text{ and } |b_1| << 1, & \text{when } n \geq 1 \end{cases} \tag{21}$$

### 3.3. Coupled Discrete Acoustic-Structure Interaction Equations

Now that we have identified the mode-by-mode parameterized weighting $\alpha_n$, its synthesis into its corresponding discrete form is all that remains for its general applicability. To this end, we divide (11) by $\alpha$ to obtain the following discrete form

$$\mathbf{A}\ddot{\mathbf{p}} + c(\mathbf{I}+\mathbf{X})\mathbf{A}_1\dot{\mathbf{p}} + c^2\mathbf{X}\mathbf{B}_2\mathbf{p} = \rho c\mathbf{A}\ddot{\mathbf{u}} + \rho c^2\mathbf{X}\mathbf{A}_1\dot{\mathbf{u}} \tag{22}$$

where $\mathbf{X}$ is a discrete parameterized matrix that leads to $\chi_n$ given in (21) for the case of a spherical shell.

After various trial matrices, we have selected the following discrete parameterization matrix:

$$\mathbf{X} = b_0\mathbf{B}_2\mathbf{N}^{-1} - (b_0 - b_1)\mathbf{I} + 2\mathbf{B}_1\mathbf{A}_1, \quad \mathbf{N} = \mathbf{A}_1\mathbf{A}^{-1}\mathbf{A}_1 \tag{23}$$

which specializes to $\chi_n$(21) for the case of a spherical shell.

*Remark 4:* The choice of $(b_0 = 1, b_1 = 0)$ in the present model specializes to the modal form of the DAA$_2$(1994) presented in Geers and Zhang[10, 11]. To the best of our knowledge, the discrete parameterized external acoustic equation given in equation(22) is the first of its kind, which is distinctly different from that of the discrete form of the DAA$_2$(1978) proposed in [8].

The numerical evaluation of the present descrete parameterized acoustic interaction model(22) is presented in Part II[14].

## 4. CONCLUSIONS

An improved pressure-field governing equation has been developed for the modelling of external acoustic field interacting with flexible structures. The present model can be implemented using the available boundary integral matrices. Numerical evaluations of the proposed interaction model are carried out in a companion paper, Improved Structure-Acoustic Interaction Models, Part II: Model Evaluaion[14].

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