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IMPROVED STRUCTURE-ACOUSTIC INTERACTION MODELS, PART II: MODEL EVALUATION

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Abstract

A parameterized external acoustic model interacting with flexible structures derived in the companion paper, Part I[4], is evaluated for its model fidelity employing a spherical shell. The mode-by-mode interaction equations of the present discrete model reveal that the model parameters introduced in the present acoustic interaction model can be tailored to match with either the dominant acoustic-structural interaction frequency or its damping ratio, but not simultaneously both. A least-squares fit of the mode-by-mode parameters leads to a parameterized matrix representation so that the present model can be implemented for general interaction surfaces. Comparisons of the present parameterized model with the classical Doubly Asymptotic Approximate(DAA) models[3] show that the present model offers improved accuracy, especially for medium-frequency ranges.

1. INTRODUCTION

In the companion paper, Part I[4], a parameterized acoustic-flexible structure model has been derived, which is considered a significant improvement over the earlier model presented in[8, 7]. A novel feature of the earlier models is the use of a linearly weighted combination of the retarded and advanced potentials. It was shown therein that the approximate models thus derived bypass the need for asymptotic matching, are stable and consistent with respect to general admissible initial conditions such as impulse incident wave and other types of incident pressure conditions. This early-time consistency condition is considered an important property for inverse identification applications, in addition to transient acoustic-structure interaction simulations, because accurate determination of impulse response functions in time or frequency domain identification methods is critical.

A major improvement presented in Part I[4] over our precursor model[8, 7] is the parametrization of the weighting parameter and a construction of the discrete parameterized matrix so that the improved model can be applicable to general interaction surfaces.

For evaluation of the present improved model, first, we have focused on the comparison of the mode-by-mode dominant interaction characteristic roots of the present parameterized model with those of the exact case and the classical DAA models[3]. Second, we have employed distributed impulsive incident waves and examined both the frequency responses and temporal solutions. Simulation of a plane step incident pressure wave problem was also carried out but not reported herein due to page limitation. The results indicates that the present parameterized acoustic-model offers improved accuracy both in terms of frequency responses, especially for low and medium frequency ranges, and also for early-time responses due primarily for its consistency property.

2. COUPLING PROCEDURE

2.1. Structure Equation

The governing equation for a structure, the equation of motion, is expressed as

$$\mathbf{M}_s\ddot{\mathbf{x}} + \mathbf{K}_s\mathbf{x} = \mathbf{f} \tag{1}$$

where **x** is the displacement, $\mathbf{M}_s$ and $\mathbf{k}_s$ are the structural mass and stiffness matrices, respectively; **f** is the external forces including interaction forces emanating from acoustic field surrounding the structure, and a superscipt dot designates temporal differentiation.

2.2. Approximate Equation for Acoustic Medium

The approximate model proposed in companion paper[4] is

$$\alpha\overline{A}\ddot{p} + (1+\alpha)cA_1\dot{p} + c^2B_2p \quad = \quad \rho c\alpha\overline{A}\ddot{u} + \rho c^2A_1\dot{u} \tag{2}$$

where p is the pressure on surface in normal direction, c is the speed of sound in acoustic fluid, ρ is the density of acoustic fluid and α is the free parameter that represents the weighting of the retarded potential$\frac{1}{2}(1-\alpha)$ vs the advanced potential$\frac{1}{2}(1+\alpha)$. Equation (2) consists of $\overline{A}$, A_1 and B_2 boundary integral equations. They are defined as

$$B_2p(t) \quad = \quad \int_S \frac{1}{R_{PQ}^2}\frac{\partial R}{\partial n}p(Q,t)dS \tag{3}$$

$$A_1p \quad = \quad \int_S \frac{1}{R_{PQ}}p(Q,t)dS \tag{4}$$

$$\overline{A}p(t) \quad = \quad \int_S \frac{1}{R_{PQ}}dS\left(2\int_S \frac{1}{R_{PQ}}\frac{\partial R_{PQ}}{\partial n}dS_p\right)^{-1}\int_S p(P,t)dS_p \tag{5}$$

where R_{PQ} is the distance from the pressure source P to a typical point on the structural surface Q, n is the normal vector going into fluid at Q point as shown in Figure 1. In physical terms, B_2 and A_1 represent the late-time operators, $\overline{A}$ represents the early-time plane wave responses, and $(1+\alpha)A_1$ is considered to account for intermediate frequency characteristics. The replacement of A by $\overline{A}$ in the above equation is a computational expediency which we have found feasible.

As stated in Part I[4], α^{-1} is given by the following parameterized discrete matrix

$$\alpha^{-1} = B_2 N^{-1} - I + 2B_1 A_1^{-1}, \quad N = A_1 \overline{A}^{-1} A_1 \tag{6}$$

2.3. Coupled Acoustic-Flexible Structure Interaction Equations

When the structure is placed in an acoustic medium, the structural equation (1) is subjected to the incident and scattered pressures whose governing equation is given by (2). The coupled external acoustic-structural interaction equations are now give as

$$\begin{gathered} \mathbf{M}_s \ddot{\mathbf{x}} + \mathbf{K}_s \mathbf{x} = -\mathbf{G}\mathbf{A}(\mathbf{p}_I + \mathbf{p}^s) \\ \alpha \overline{\mathbf{A}} \ddot{\mathbf{p}}^s + (1+\alpha) c \mathbf{A}_1 \dot{\mathbf{p}}^s + c^2 \mathbf{B}_2 \mathbf{p}^s = \alpha \rho c \overline{\mathbf{A}} (\mathbf{G}^T \ddot{\mathbf{x}} - \ddot{\mathbf{u}}_I) + \rho c^2 \mathbf{A}_1 (\mathbf{G}^T \ddot{\mathbf{x}} - \dot{\mathbf{u}}_I) \end{gathered} \tag{7}$$

where $\mathbf{G}$ is the transformation matrix from normal force on the fluid mesh to nodal force on the structural mesh, $\mathbf{A}$ is the elemental area matrix, $\mathbf{p}^s$ is the scattered pressure on the surface and $\mathbf{p}_I$ and $\mathbf{u}_I$ are incident pressure and velocity, respectively. We assume that the $\mathbf{p}_I$ and $\mathbf{u}_I$ are known.

3. APPLICATION TO A SPHERICAL SHELL

In a spherical polar coordinate, the pressure and displacement on the surface of a sphere can be expanded as Legendre polynomial series. Using this series expansion, Huang[1], Zhang and Geers[2] have obtained the exact solutions for the interaction problems of a submerged spherical shell excited by an incident plane step wave excitation. We will adopt the spherical shell subjected to specific excitation forces to evaluate the performance of the proposed interaction model (2) as compared with the exact solution of Huang[1] and the results obtained by the DAA$_2$(1978) model[3].

3.1. An Elastic Spherical Shell Surrounded by Acoustic Medium

Figure 1 shows a flexible elastic spherical shell of radius a, thickness h, an isotropic material with Young's Modulus E, density ρ_s, and Poisson's ratio ν. The shell thickness-to-radius ratio h/a is small enough to apply thin shell theory and the longitudinal wave speed of the shell is denoted by $c_s = \sqrt{E/\rho_s(1-\nu^2)}$.

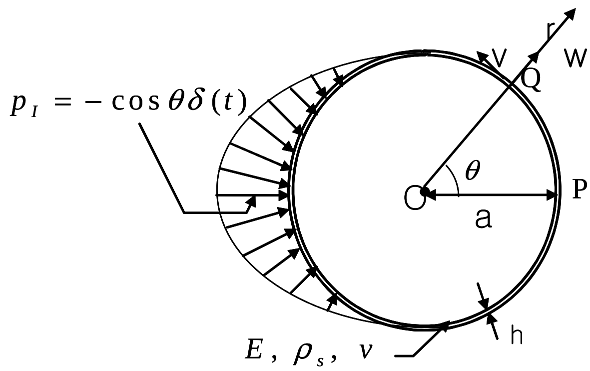


Figure 1. A submerged spherical shell excited by a cosine-type impulse force

The shell geometry is described using a spherical coordinate (R, θ) with its origin at O

and an in-vacuo condition of its interior. The radial and meridional displacements of the shell are denoted by $W(\theta,t)$ and $V(\theta,t)$, respectively. For subsequent analysis, dimensionless variables are introduced: time(t), pressure(p) and length are normalized to a/c, ρc^2 and a, respectively. The external wave pressure, $p(r,\theta,t)$, is the sum of the $p_I(r,\theta,t)$ and $p^s(r,\theta,t)$. For numerical simulation, the submerged spherical shell as shown in Figure 1 has the parameters: $h/a = 0.01$, $\rho_s/\rho = 7.7$ and $c_s/c = (13.8)^{1/2}$.

3.2. Modal equations for a sphere

For a sphere, the acoustic wave equation in spherical coordinates yields the modal solution in terms of Legendre polynomials as

$$\phi^s\ (r,\theta,s) = \sum_{n=1}^{\infty} \phi_n^s(r,t)\ P_n(cos\theta) \tag{8}$$

where $P_n(x)$ is the n^{th} Legendre polynomials. and ϕ_n^s is the component of ϕ^s for n^{th} Legendre polynomial. Applying (8) to the Laplace transformed wave equation, the ordinary differential equation is obtained as

$$r^2 \frac{d^2\overline{\phi}_n^s}{dr^2} + 2r\frac{d\overline{\phi}_n^s}{dr} - [n(n+1) + r^2 s^2]\overline{\phi}_n^s = 0 \tag{9}$$

whose regular solution [6] is given by

$$\overline{\phi}_n^s(r,s) = B_n(s)\kappa_n(rs) \tag{10}$$

where s is the Laplace Transform variable, $B_n(s)$ is the constant to be determined from the geometrical compatibility conditions and $\kappa_n(rs)$ is the nth order modified spherical Bessel function of the third kind. The pressure and particle velocity of acoustic fluid can be related by

$$p(r,t) = \dot{\phi}(r,t) \quad u = -\frac{d\phi}{dr}(r,t). \tag{11}$$

Using (10) and (11), $B_n(s)$ can be obtained and the desired Laplace transformed analytical modal solution for the scattering pressure is obtained as

$$\overline{p}_n^s(s) = -\kappa_n'(s)/\kappa_n(s)\overline{u}_n(s). \tag{12}$$

It should be noted that the above analytical modal pressure equation needs to be coupled with the equations of motion for the elastic sphere to bring about the coupling of the flexible structure with the surrounding external acoustic medium.

Similarly, the DAA2[3] and proposed model[4] can be expressed in terms of Legendre polynomials, which are summarized below:

$$\text{DAA}_2(1978) \ : \quad s^2\overline{p}_n + (1+n)s\overline{p}_n + (1+n)^2\overline{p}_n = s^2\overline{u}_n + (1+n)s\overline{u}_n \tag{13}$$

$$\text{Present model} \ : \quad \alpha_n s^2\overline{p}_n + (1+\alpha_n)s\overline{p}_n + (1+n)\overline{p}_n = \alpha_n s^2\overline{u}_n + s\overline{u}_n \tag{14}$$

where α_n is the n^{th}-mode weighting parameter obtained from the parameterized matrix in (6) given by

$$\alpha_n = 1/n \quad \text{for} \quad n > 0 \quad \text{and} \quad \alpha_0 = 1 \quad \text{if } n = 0 \tag{15}$$

Figure 2 shows the poles of acoustic impedances for the exact solution (12), the DAA_2(1978) (13), and the present model (14) corresponding to the increasing order of the Legendre polynomial. In Figure 2, the number of exact poles increases by one as the Legendre polynomial order increases. But the DAA2 and the present model have only two poles regardless of the modal order. For 0^{th} and 1^{st} modes, the present model captures the exact solution's poles whereas the DAA_2(1978) does not. For the other modes, the magnitudes of real and imaginary values of the poles calculated by the DAA_2(1978) and the present model linearly increase. According to Figure 2, the present model appears to approximates each of the dominant roots more closely than the DAA_2(1978).

As an additional consideration, a structure interacting with acoustic medium has two kinds of poles : lightly damped structural poles and highly damped pressure poles. Among them, the most dominant poles are the lightly damped structural poles corresponding to the radial displacement. Figure 3 shows the free-vibration root loci of a spherical shell equations[9] interacting with acoustic medium modeled by (12), the DAA_2(1978) (13) and the present model (14) for $n = 2$. Material and geometric parameters of these equations are chosen according to Figure 1. In particular, the roots-locus of the present model as the the parameter α_2 is varied is shown, which indicates that the present model may be tailored to accurately capture the frequency. Table 1 presents the mode by mode dominant structural poles of the exact solution, DAA_2(1978) and the present model. Note that the free parameter(α) of the present model has been chosen so that the dominant poles of the present model are as close as possible to those of the exact solution. Notice $1/\alpha$ increases almost one by one as the mode increases, hence the formula adopted in (15). Therefore, the discrete matrix parameterization of α defined in Part I[4] constitutes a good choice.

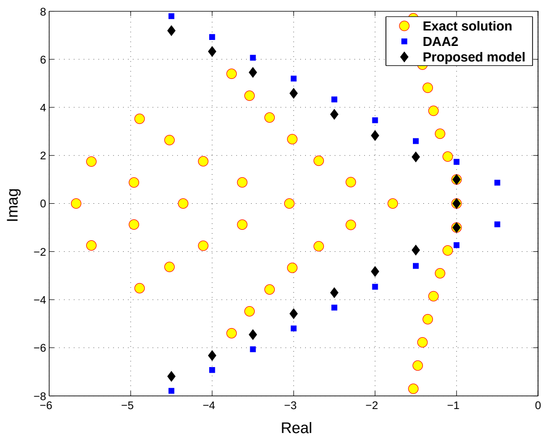


Figure 2. Acoustic impedance poles of exact solution, discrete DAA2 and proposed model order by order

3.3. A Submerged Spherical Shell Excited by Impulsive forces

The submerged spherical shell excited by a plane step wave has been analyzed in many papers. For this example problem, many of the existing approximate models perform rather well. For

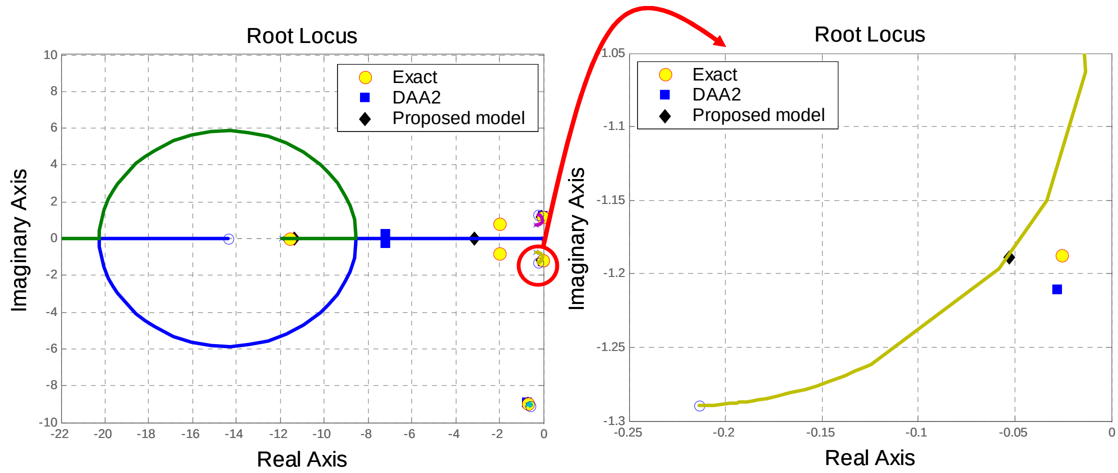


Figure 3. Free-vibration root loci for a spherical shell surrounded with water, n= 2 and the roots-locus of proposed model by α

Order	Optimal 1/α	Dominant structural roots			
		Exact	DAA2	Proposed (Optimal 1/ α)	Proposed (1/ α=n)
2	1.83	−0.026+1.192i	−0.0286 + 1.215i	−0.0466 + 1.181i	−0.0529 + 1.189i
3	2.92	−0.0069 + 1.505i	−0.0283 + 1.510i	−0.0421 + 1.489i	−0.0445 + 1.488i
4	3.83	−0.0010 + 1.723i	−0.0234 + 1.715i	−0.0282 + 1.696i	−0.0336 + 1.696i
5	4.91	−0.0001 + 1.889i	−0.0187 + 1.876i	−0.0230 + 1.864i	−0.0253 + 1.860i
6	6.00	−0.0000 + 2.027i	−0.0150 + 2.012i	−0.0197 + 2.005i	−0.0195 + 1.997i
7	6.93	−0.0000 + 2.148i	−0.0140 + 2.126i	−0.0122 + 2.133i	−0.0153 + 2.119i

Table 1. Mode-by-mode dominant structural roots for a spherical shell surrounded by water

the present evaluation, we have chosen the submerged spherical shell excited by a cosine-type impulse force as shown in Figure 1. As we will see, this example problem is characterized by the velocity and pressure transient response components that are dynamically changing from the early time to the late time period. This means that a large number of modes will participate with different weights at different time window, thus directly exposing the roles of different characteristic interaction poles as shown in Figure 2.

Figure 4 shows the radial velocity responses on a submerged spherical shell at $\phi = 180$ and shows, at early time, that the radial velocity is rapidly changing, and followed by the periodic responses. Observe that for the late-time period, the DAA_2(1978) and the present model follow with a reasonable phase and amplitude fidelity the exact radial velocity responses. However, the DAA_2(1978) over-estimates the early time peak while the present model predicts the early time responses with high accuracy. This difference is caused by the inaccuracy of the low-mode poles of the the DAA_2(1978) shown in Figure 2. Figure 5 shows the frequency response at $\phi = 180$. Observe the DAA_2(1978) captures more accurately the low-mode peak amplitude than the present model; however, the present model accurately estimates high-frequency peaks.

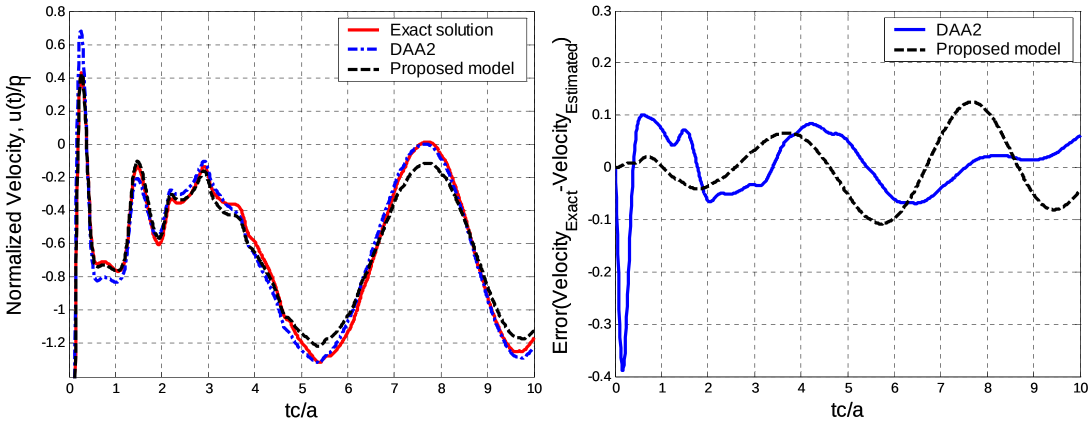
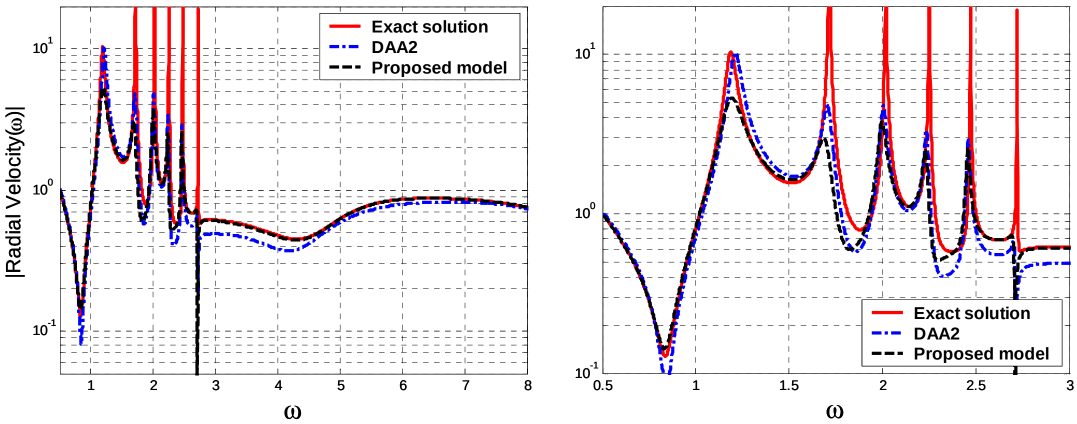


Figure 4. Radial velocity response and the error of radial velocity responses with respect to exact solution on a submerged spherical shell at $\phi = 180^o$ in detail



(a) FRF (b) FRF in detail

Figure 5. Frequency Response Function of radial velocity responses on a submerged spherical shell at $\phi = 180^o$

4. CONCLUSIONS

In the present Part II, the approximate acoustic model derived in Part I [4] has been evaluated for its performance and validation. For evaluation purposes, a submerged elastic spherical shell excited by a cosine-type impulse force has been used, and compared with the exact solutions and the DAA_2(1978) results. This example shows the characteristic of early time responses estimated by both approximations. Although not reported due to page limitation, we have also carried out evaluation for the case of step plane wave excitation, which show a similar performance. Clearly, the present model captures the early-time response accurately while introducing somewhat higher damping. Remedy for this and other aspects of the present model are being carried out and will be reported in a future communication.

It should be noted that the present model needs to be evaluated for its performance for more discrete general structural surface geometries, in particular, cylindrical shapes. Work requiring discrete structural models is actively pursued at present.

ACKNOWLEDGMENTS

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