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MODELLING COMPLICATED SYSTEMS WITH PARTLY UNKNOWN DYNAMIC PROPERTIES AS "STRUCTURAL FUZZY"

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Abstract

From experiments it is well-known that the vibration response of complicated, built-up systems often show more damping than structural losses in the components can account for. The "Theory of structural fuzzy" is intended for modelling of such high damping induced by resonant structures. When modelling a complicated system, it is divided into a deterministic master structure and one or more fuzzy substructures, which are known only in some statistical sense. In the present paper the theory of fuzzy structures is outlined and a special method of including spatial memory in the modelling of fuzzy substructures with continuous boundaries is examined. The high damping effect of the structural fuzzy is demonstrated by a numerical model of a simple master structure with fuzzy attachments. It is shown that the introduction of spatial memory reduces the damping effect of the fuzzy and may cause the resonance frequencies of the complex system to increase.

1. INTRODUCTION

Many complicated systems of practical interest consist basically of an outer shell- or a box-like *master* structure and a complicated *internal* structure. Examples of such structures varying from small to large sizes comprise hearing aids, machines, aircrafts and ship hulls. The outer master structure is often well defined and its vibration can be predicted using conventional methods. In contrast, the dynamic properties of the internals are only partly known and therefore they have to be modelled by using an alternative method such as the "Fuzzy structure theory" [1]-[4]. It is commonly known from experiments that vibrations of the master in a complicated system often is more damped than its damping lossfactor accounts for. Fuzzy structure theory is intended for an overall and simple prediction of the damped vibration of the master, and the theory considers the internals as one or more independent "fuzzy substructures", which are known in some statistical sense only.

In some complex systems the fuzzy substructures are attached to the master through a continuous boundary. This implies that spatial coupling within the fuzzy can only be neglected up to a certain frequency. Above this frequency the displacement of the continuous junctions is varying significantly with position due to vibration in the master structure, and therefore spatial coupling has to be accounted for. The present paper addresses the problem of including spatial coupling in the modelling of structural fuzzy. The starting point is taken in Soize's fuzzy law II [2] concerning fuzzy substructures with continuous boundaries, and this theory is simplified using an approach like that of Pierce, Sparrow and Russell [3], and Strasberg and Feit [4].

Figure 1 shows a complicated system comprising of three different fuzzy substructures attached to a master. Time harmonic force $Fe^{i\omega t}$ of angular frequency $\omega = 2\pi f$ excites the master and generates vibration in the whole system. At lower frequencies the master structure vibrates as a rigid body (see Fig. 1a), and the boundary displacement of the substructures is almost constant. This entails that the spatial coupling within each substructure has no significant effect on the response of the system. Now, increasing the excitation frequency introduces wave motion in the master structure. When the wavelength of the master becomes comparable with the dimensions of the fuzzy connection area then the spatial coupling begins to take effect. This is the case in Fig. 1b where the boundary displacement of substructure 3 is varying whereas those of substructures 1 and 2 are nearly constant. In Fig. 1c the frequency has been increased further and the boundary displacements of both substructures 1 and 3 are varying, while the boundary displacement of substructure 2 remains close to constant.

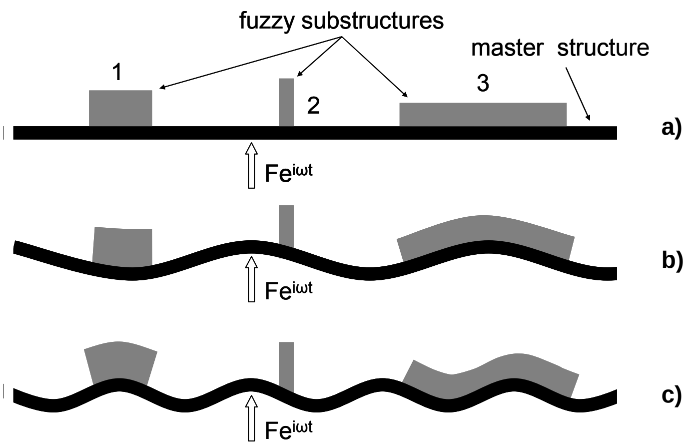

Figure 1. Three different fuzzy substructures attached to a master structure undergoing: a) rigid body motion, b) low freq. wave motion and c) high freq. wave motion.

Literature published on fuzzy structure modelling is extensive, but not many authors have addressed the subject of structural fuzzy with a continuous boundary. In 1993 Soize [2] introduced a method for including so-called spatial memory in the modelling of structural fuzzy and this is based on Ref. [5] where numerical examples involving fuzzy structures with continuous boundaries were presented.

In the following Section structural fuzzy theory will be explained briefly and the incorporation of spatial memory will be investigated. A numerical modelling example, which illustrates the effect of spatial memory within the fuzzy follows in Section 3.

2. OUTLINE OF STRUCTURAL FUZZY THEORY

The purpose of the fuzzy structure theory is to model the overall vibration response of a master, which is attached to one or more *resonant* substructures. When such a fuzzy substructure of multiple resonaters is attached to the master within a small area of virtually constant displacement, then the coupling forces eliminate one another and coupling can be neglected. This case of absent spatial coupling is exemplified in the following subsection.

2.1 Structural fuzzy without spatial memory

Fuzzy structure theory considers a single fuzzy substructure as comprising of many simple oscillators resonating at different frequencies and being attached to the master at their base. Consider a fuzzy substructure modelled by N simple oscillators that is attached to an area A of the master structure. An expression for the *boundary* impedance z_{fuzzy} of the substructure can be derived by assuming that the n'th simple oscillator of the fuzzy has the mass M_n and the complex spring stiffness $\underline{s}_n = s_n(1+i\eta)$, where η is the lossfactor. Introducing the oscillator's resonance frequency $f_{r,n} = \sqrt{s_n / M_n}/(2\pi)$, its impedance $Z_n = F_n / v_n$ at the base yields [4]

$$Z_n = i2\pi f M_n \left(\frac{f_{r,n}^2(1+i\eta)}{f^2 - f_{r,n}^2(1+i\eta)} \right). \tag{1}$$

Fig. 2 shows this impedance of the oscillator in a normalized form for three different values of η. Below and above its resonance frequency the oscillator is mass and spring controlled, respectively. Further, at resonance of the oscillator, where the impedance is very large and almost purely real, it will strongly oppose any movements of its base. It is this particular feature of the oscillator, which can imitate the damping effect of the fuzzy substructure.

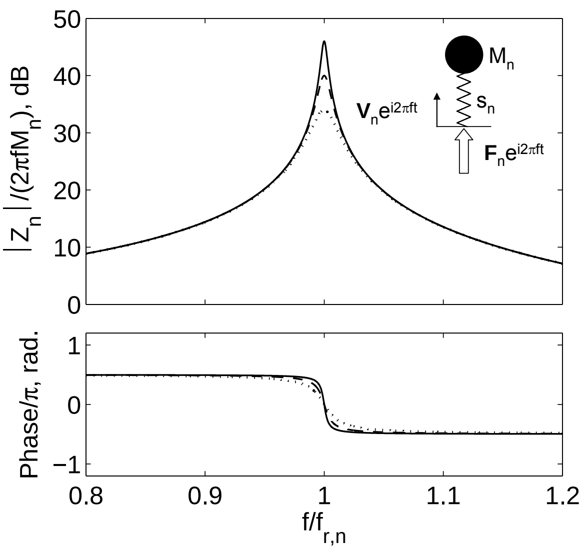

Figure 2. Impedance Z_n normalized: η=0.005 (——), η=0.01 (- - -) and η=0.02 (·······).

Generally, the oscillators of a fuzzy substructure have different masses and natural frequencies and they are attached randomly to the master structure within the considered fuzzy connection area. Also, the mass of all the oscillators equals the mass of the fuzzy substructure $M_{fuzzy} = \sum_{n=1}^{N} M_n$. Below a certain frequency, say $f_{r,lower}$, the oscillators will all be mass controlled. By increasing the frequency gradually from $f_{r,lower}$ to an upper limit, say $f_{r,upper}$, the oscillators will resonate one by one. Now, at each frequency within this "resonant" frequency band $f_{r,lower} \leq f_r \leq f_{r,upper}$ at least one oscillator will be close to its base anti-resonance and will oppose the motion of the master. If the oscillators are attached close to one another within the area A, which has a nearly constant boundary displacement, then the effective boundary impedance of all the oscillators, z_{fuzzy}, can be approximated as the sum of each oscillator's impedance Z_n divided by the attachment area A:

$$z_{fuzzy} \approx \frac{1}{A}\sum_{n=1}^{N} Z_n = \frac{1}{A}\sum_{n=1}^{N} i2\pi f M_n \left(\frac{f_{r,n}^2(1+i\eta)}{f^2 - f_{r,n}^2(1+i\eta)} \right) \tag{2}$$

This boundary impedance, however, requires specific knowledge about the properties of each oscillator and it is therefore conveniently replaced by an asymptotic and smoothed version. This is obtained by considering infinitely many oscillators resonating within $f_{r,lower} \leq f_r \leq f_{r,upper}$ and with a total mass M_{fuzzy} [3]. The smoothed impedance yields

$$z_{fuzzy} = \frac{i2\pi f}{A}\int_{f_{r,lower}}^{f_{r,upper}} m_{fuzzy}(f_r)\left(\frac{f_r^2(1+i\eta)}{f^2 - f_r^2(1+i\eta)}\right)df_r \tag{3}$$

where $m_{fuzzy}(f_r)df_r$ is the total mass resonating between the frequency f_r and $f_r + df_r$ and the total mass of the substructure is

$$M_{fuzzy} = \int_{f_{lower}}^{f_{upper}} m_{fuzzy}(f_r)df_r\,. \tag{4}$$

The damping effect of the fuzzy substructure is mainly governed by the frequency dependent resonating mass $m_{fuzzy}(f_r)$ [3]. Methods to find this parameter were suggested by Soize [6] and Pierce [7], and different prototype distributions were proposed by Pierce [3] and Strasberg [4].

2.2 Structural fuzzy with spatial memory

When the fuzzy substructure is attached to the master through an area for which the boundary displacement cannot be considered constant, then coupling within the fuzzy has to be taken into account. Soize [2] introduced a method to include such "spatial memory" into the modelling of the fuzzy boundary impedance; for ease of reference, the basics of this method will be given in the following.

Consider a fuzzy substructure connected to the master through a continuous boundary. A fuzzy substructure is generally attached to the master within an area, but for the sake of simplicity we shall consider a fuzzy attached to the master through a one-dimensional boundary of length L_{fuzzy}. Soize incorporates spatial memory in the fuzzy by introducing the "spatial oscillator" shown in Fig. 3a. The n'th spatial oscillator of a fuzzy substructure consists of a point mass of weight M_n / L_{fuzzy} placed at position x_1. The point mass is supported by springs of stiffness density $s_{\varepsilon,n}$ that are attached to the master structure at different positions $x \in [x_1 - \varepsilon, x_1 + \varepsilon]$. The width of the distributed springs (or spatial memory) is 2ε and the stiffness density is given as

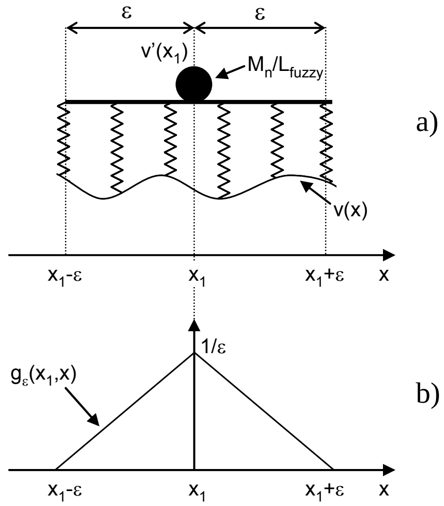

Figure 3. Fuzzy oscillator with spatial coupling: a) Oscillator attached to boundary and b) stiffness density distribution of the oscillator springs.

$$s_{\varepsilon,n} = (\underline{s}_n / L_{fuzzy})g_\varepsilon(x_1,x) = (M_n\omega_{r,n}^2 / L_{fuzzy})(1+i\eta)g_\varepsilon(x_1,x)\,. \tag{5}$$

Here $g_\varepsilon(x_1,x)$ is an even function of x_1 and $g_\varepsilon(x_1,x)$ is positive-valued and has an area of 1. For a one-dimensional spatial memory Soize suggests the function $g_\varepsilon(x_1,x)$ as the triangular distribution shown in Fig. 3b. This is determined as

$$g_\varepsilon(x_1,x) = \frac{\varepsilon - |x_1 - x|}{\varepsilon^2}1_{[x_1-\varepsilon, x_1+\varepsilon]} \tag{6}$$

where $1_{[x_1-\varepsilon,x_1+\varepsilon]}$ is a function which is equal to 1 for $x \in [x_1 - \varepsilon, x_1 + \varepsilon]$ and 0 elsewhere. Since the area of $g_\varepsilon(x_1,x)$ is 1 the oscillator in Fig. 3a has the same natural frequency $f_{r,n}$ as the simple oscillator with mass M_n and stiffness s_n.

Next, consider an infinite number of the n'th oscillator distributed over the fuzzy boundary, so that each location is associated with a point mass M_n / L_{fuzzy} as illustrated in Fig. 4. The point masses can vibrate independently whereas the springs overlap spatially at the connection boundary. These infinitely many *identical* oscillators constitute the *n*'th contribution to the total boundary impedance of the homogeneous fuzzy substructure. Fig. 4a illustrates the case with a large spatial memory since stiffness distributions overlap significantly. In Fig. 4b the springs overlap less than in Fig 4a because ε is smaller. Finally, in Fig. 4c the spatial memory is zero since $\varepsilon \to 0$, and the spatial stiffnesses become simple springs. The contribution to the force at x_2 due to the *n*'th oscillator $f_{\varepsilon,n}(x_2)$ is found as [2]

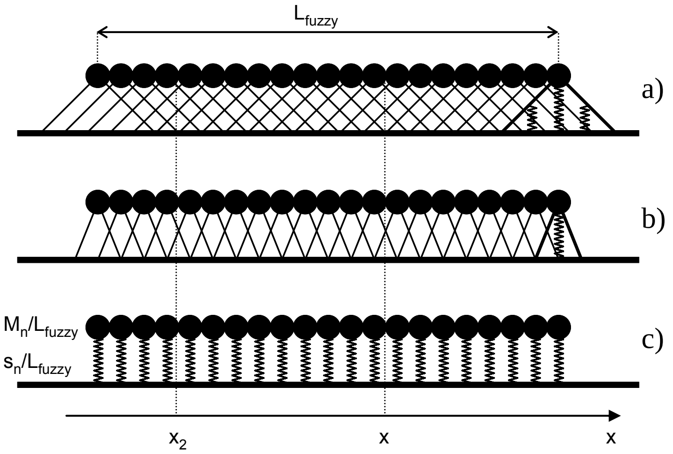

Figure 4. Structural fuzzy attached to the master with: a) high spatial memory, b) little spatial memory and c) no spatial memory.

$$f_{\varepsilon,n}(x_2) = \int_{L_{fuzzy}} z_{fuzzy,\varepsilon,n}(x_2 - x) v(x) dx \tag{7}$$

where $z_{fuzzy,\varepsilon,n}$ is the boundary impedance associated with the *n*'th oscillator. $z_{fuzzy,\varepsilon,n}$ can be derived as

$$z_{fuzzy,\varepsilon,n}(x_2 - x)dx = \frac{1}{i\omega} \frac{\underline{s}_n}{L_{fuzzy}} \left(\delta_0(x_2 - x) - \frac{\underline{s}_n}{\underline{s}_n - \omega^2 M_n} g(x_2 - x) * g(x_2 - x) dx \right) \tag{8}$$

where $*$ means convolution. By analogy to the smoothed impedance in eq. (3) the total force density applied to the connection boundary at x_2 due to *all* oscillators yields

$$f_\varepsilon(x_2) = \int_{f_{r,lower}}^{f_{r,upper}} \int_{L_{fuzzy}} z_{fuzzy,\varepsilon,f_r}(x_2 - x, f_r) v(x) dx \, df_r \tag{9}$$

A structural fuzzy constructed in this manner is homogenous because the boundary impedance due to all oscillators, $z_{fuzzy,\varepsilon}$, only depends on the distance between x and x_2. If $\varepsilon \to 0$, then eq. (8) reduces to the boundary impedance for structural fuzzy without spatial memory as given in eq. (3).

A numerical implementation of the fuzzy boundary impedance $z_{fuzzy,\varepsilon}$ is unfortunately rather complicated due to its *non-local* nature. This requires for instance the use a finite element model with special fuzzy elements. As mentioned earlier, the main purpose of the fuzzy structure theory is to serve as a simple modelling tool. Therefore Soize introduced an equivalent *local* oscillator, which can model the fuzzy with spatial memory in a simpler way. For the *n*'th equivalent oscillator this impedance $Z_{equ,n} = F_n / v_n$ becomes

$$Z_{equ,n} = \underline{s}_n \left(1 - \frac{\underline{s}_n}{\underline{s}_n - \omega^2 m_n} \alpha \right) = i2\pi f M_n \left(\frac{f_{r,n}^2 (1 + i\eta)\left(1 - (f_r / f)^2 (1 + i\eta)(1 - \alpha)\right)}{f^2 - f_{r,n}^2 (1 + i\eta)} \right) \tag{10}$$

Here $\alpha \in]0,1]$ is the so-called equivalent coupling factor. $Z_{equ,n}$ is shown in Fig. 5 for different values of α. As indicated in the figure, the equivalent oscillator represents a simple oscillator with spring stiffness $\underline{s}_1$ where the mass has been grounded by a second spring with stiffness $\underline{s}_2$. The relationship between the impedance in eq. (10) and the impedance of the grounded oscillator $Z_{ground,n}$ in Fig. 5 is $Z_{equ,n} = Z_{ground,n} / \alpha$ where $\alpha = \underline{s}_1 / (\underline{s}_1 + \underline{s}_2)$. Note that $\alpha \to 1$ when $\underline{s}_2 \to 0$ and $Z_{equ,n}$ approaches the impedance of a simple oscillator. By analogy to eq. (3) a smoothed version of the equivalent boundary impedance $z_{fuzzy,equ}$ of a structural fuzzy with spatial memory can be determined as

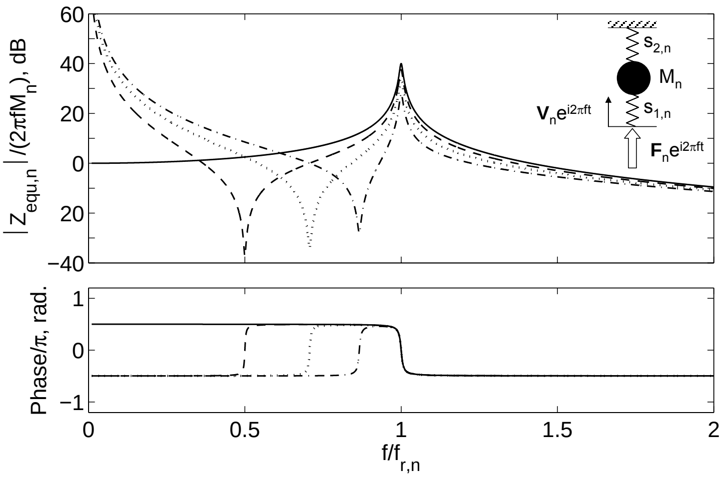

Figure 5. $Z_{equ,n}$ for different values of α: 1 (——), 0.75 (- - -), 0.5 (·······) and 0.25 (-·-·-).

$$z_{fuzzy,equ} \approx \frac{i2\pi f}{A} \int_{f_{r,lower}}^{f_{r,upper}} m_{fuzzy}(f_r) \left(\frac{f_r^2 (1+i\eta)\left(1-(f_r/f)^2(1+i\eta)(1-\alpha)\right)}{f^2 - f_r^2(1+i\eta)} \right) df_r \tag{11}$$

Soize states that it is not self-evident that the equivalent oscillator can model correctly a structural fuzzy with spatial memory. Perhaps one can imagine that the equivalent oscillator is capable of representing the direct impedance of eq. (8). However, coupling forces to other contact positions are ignored, due to its local nature. Therefore α has to be chosen carefully. Finding α requires matching of numerical simulations using the expressions in eq. (8) and eq. (10) frequency by frequency. This has been done by Soize [2] and by transforming his data, α can be determined as function of $2\varepsilon/\lambda$ as shown in Fig. 6. Here λ is the wavelength of vibration in the master structure, which has one-dimensional wave motion only. The data has been fitted with a 4'th-order polynomial and it reveals that a unique relationship exist between α and $2\varepsilon/\lambda$.

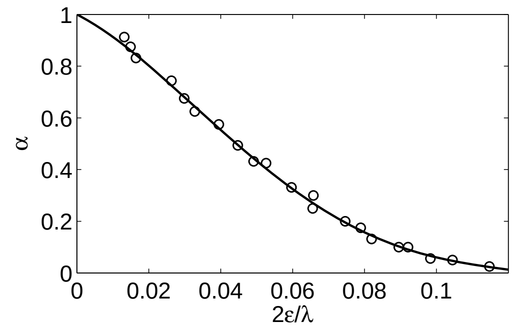

Figure 6. Variation of α. Data computed from results in Ref. [2].

3. BEAM MASTER STRUCTURE WITH STRUCTURAL FUZZY

In the following, a numerical example is presented. This illustrates the effect of structural fuzzy with and without spatial memory. The Finite Element Method [8] has been used to solve the vibration response of a simple Bernoulli-Euler beam, being free in space and considered as the master. A fuzzy substructure is attached on the whole length L of the beam, so $L_{fuzzy} = L$. The lossfactor of the beam is set to 0.005, whereas the lossfactor of the fuzzy oscillator springs has been chosen to 0.01. In this example the total mass of the fuzzy substructure M_{fuzzy} is taken to be one-tenth of the beam mass. The resonating mass per frequency $m_{fuzzy}(f_r)$ is chosen to be decreasing with frequency such that $m_{fuzzy}(f_r) = M_0 / f_r$ where

$M_0 = M_{fuzzy} \ln(f_{r,lower} / f_{r,upper})$. With this choice of mass distribution the boundary impedance of the fuzzy *without* spatial memory, eq. (3), becomes

$$z_{fuzzy} = \frac{i2\pi f M_0}{L_{fuzzy}} \ln\left(\frac{f^2 - (1+i\eta) f_{r,upper}^2}{f^2 - (1+i\eta) f_{r,lower}^2} \right). \tag{12}$$

Further, α is assumed to be constant with frequency implying that $2\varepsilon/\lambda$ is constant and that the spatial memory 2ε of the fuzzy decreases with frequency. For a constant value of α the boundary impedance of the fuzzy *with* spatial memory, eq. (10), becomes

$$z_{fuzzy,equ} = \frac{i\pi f M_0}{L_{fuzzy}} \left[\frac{(f_{r,upper}^2 - f_{r,lower}^2)(1+i\eta)(\alpha-1)}{f^2} + \alpha \ln\left(\frac{f^2 - f_{r,upper}^2 (1+i\eta)}{f^2 - f_{r,lower}^2 (1+i\eta)} \right) \right]. \tag{13}$$

Computed results are plotted in Fig. 7 as a function of the beam's non-dimensional frequency Ω defined as

$$\Omega = \omega \sqrt{\frac{12\rho}{E}} \frac{L^2}{h} \tag{14}$$

where h is the beam thickness, and ρ and E is the density and Young's modulus of the beam material, respectively. The oscillators of the fuzzy substructure are chosen to resonate in the inverval $50 \le \Omega \le 500$.

Fig. 7a shows the end driving point mobility $Y = v_0 / F_0$ of the master both with fuzzy and without fuzzy attachments as a reference. From these responses it is clearly seen that the structural fuzzy introduces significant damping in the master for frequencies $50 \le \Omega \le 500$. Yet, the damping is most significant in the case of no spatial coupling ($\alpha = 1$), where the system's resonance peaks are reduced by about 22 dB, as opposed to 17 dB for $\alpha = 0.05$. The structural fuzzy without spatial memory adds small extra mass to the master and therefore the resonances are shifted downwards compared to the reference. In

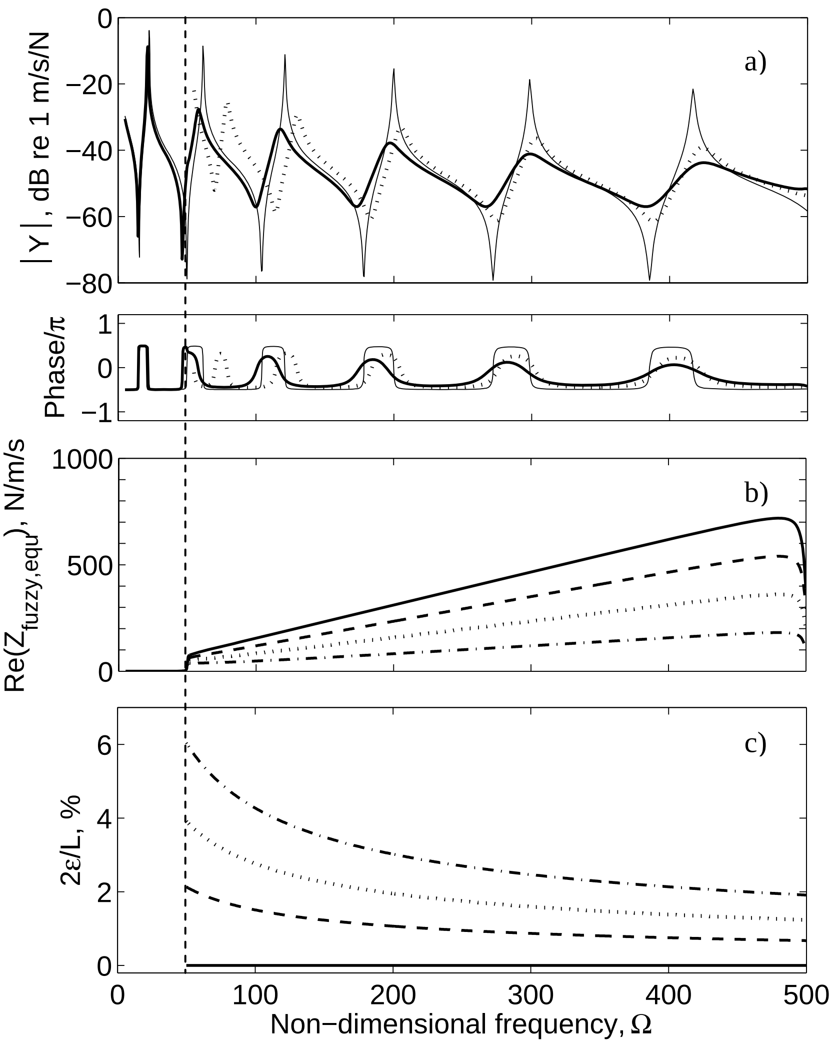

Figure 7. Flexural vibration of beam without structural fuzzy (—) and with structural fuzzy for different values of α: 1 (—), 0.75 (---), 0.5 (·······) and 0.25 (-·-·-).
a) End point mobility, b) apparent damping of the fuzzy impedance and c) variation of relative spatial memory 2ε/L.

contrast, the structural fuzzy with spatial memory adds both mass and stiffness to the master structure. For the chosen fuzzy this causes the resonance frequencies of the system to increase. Fig. 7b shows the apparent damping $\mathrm{Re}(z_{fuzzy,equ})$ of the fuzzy boundary impedance for four different values of α. It is observed that the damping effect of the fuzzy reduces with increasing spatial memory. Further, it is evident that the structural fuzzy introduces only little damping in the master for low values of α, where the fuzzy mainly add stiffness to the master. Soize [2] claims that the fuzzy only has a significant damping effect when $2\varepsilon / L < 5\%$. Fig. 7c shows $2\varepsilon / L$ as a function of Ω, and it is seen that $2\varepsilon / L \approx 4\%$ around $\Omega = 50$ for the case of $\alpha = 0.5$. If we consider the mobility in Fig. 7a again it is seen that Soize's assumption is in good agreement with the moderate damping effect occurring around $\Omega = 50$.

CONCLUSIONS

Complex systems with partly unknown properties can be modeled by Soize's fuzzy structure theory. Following a brief outline the emphasis is laid on examining a method for including spatial memory in the modeling of the structural fuzzy. Numerical simulations with a free-free beam as master structure and with an attached structural fuzzy weighing one-tenth of the beam show that the system's resonant response is reduced significantly by the fuzzy attachment. It is found that the simple fuzzy increases the system's apparent damping by a factor of 13. However, part of this effect is lost if the fuzzy has spatial memory, as typically will be the case towards higher frequencies. Also, it was shown that the spatial memory of the fuzzy increased the stiffness of the master and caused its resonance frequencies to shift upwards in the example presented.

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