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DYNAMIC RESPONSE OF A STRUCTURE WITH UNCERTAINTY IN ITS PARAMETERS AND APPLIED FORCES USING THE INTERVAL FACTOR METHOD

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Abstract

An interval factor method is presented to describe the dynamic response of a structure with lower and upper bounds applied to its parameters (material properties, geometric dimensions) and structural excitations (applied forces). In the interval analysis method, a bounded uncertain structural parameter can be described as an interval variable in terms of its lower and upper bounds. An interval variable can further be expressed as its mean value multiplied by its interval factor. The structural stiffness and mass matrices can then be divided into the product of two parts corresponding to the interval factors and the deterministic matrix. Computational expressions for the mean value, and lower and upper bounds of the structural dynamic responses are then derived by means of mode superposition and interval operations. An example of this method is applied to a complex truss structure. The bounded uncertain structural physical parameters, geometric dimensions and applied forces of the truss structure are considered as interval variables. The effects of these uncertainties on the dynamic displacement and stress responses are examined.

1. INTRODUCTION

Perturbation methods for the dynamic characteristics and responses of structures such as buildings, ships, vehicles, aerospace and offshore structures, that possess uncertainty due to variability in their geometric or material parameters and are generally under stochastic excitation, is a very significant research field [1-5]. A common approach to problems of uncertainty is to model the structural geometric and material parameters as random variables. However, probabilistic approaches cannot give reliable results unless sufficient experimental data or statistical information is available to validate the assumptions about the joint probability densities of the random variables or functions involved. In some cases, only the range of the structural parameters can be obtained. In this case, the interval analysis method will be more useful as an uncertain structural parameter can be described in terms of an interval variable, knowing only its lower and upper bounds.

The interval analysis method first appeared in the mid 1960s [6,7]. Using this method, linear interval equations, nonlinear interval equations and interval eigenvalue problems have been resolved. However, because of the complexity of the algorithms involved, it is difficult to apply these methods to practical engineering problems. Recently, the statistical response [8], eigenvalues [9] and dynamic responses [10] of structures with uncertain parameters have been investigated using interval analysis and matrix perturbation techniques. These perturbation methods use a combination of matrix perturbation theory, finite element method and Taylor series expansion to obtain the dynamic characteristics of structures with uncertainty. However, this method does not necessarily yield a conservative approximation, as the effect of neglecting the higher order terms can be significant [11].

In this paper, the dynamic responses of structures with uncertainty are investigated based on the interval factor method. A truss structure is used to illustrate applications of the method, in which structural physical parameters (Young's modulus and mass density), geometry (length and cross-sectional area of bar) and applied forces are considered as interval variables. The effects of these uncertainties on the dynamic displacement and stress responses are examined.

2. STRUCTURAL DYNAMIC RESPONSE ANALYSIS

Following the finite element formulation, the equation of motion for a structure is given by:

$$[M]\{\ddot{u}(t)\}+[C]\{\dot{u}(t)\}+[K]\{u(t)\}=\{F(t)\} \tag{1}$$

where $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices respectively. $\{u(t)\}$, $\{\dot{u}(t)\}$ and $\{\ddot{u}(t)\}$ are displacement, velocity and acceleration vectors respectively. $\{F(t)\}$ is the load force vector.

Suppose that there are m elements in the truss structure under consideration. $[K]$ and $[M]$ of a truss structure in global coordinates can be respectively expressed as:

$$[K]=\sum_{e=1}^{m}[K_e]=\sum_{e=1}^{m}[T_e]^T\frac{E_eA_e}{l_e}[G][T_e] \tag{2}$$

$$[M]=\sum_{e=1}^{m}[M_e]=\sum_{e=1}^{m}\frac{1}{2}\rho_eA_el_e[I] \tag{3}$$

where $[K_e]$ and $[M_e]$ are respectively the stiffness and mass matrices of the e^{th} element. E_e, ρ_e, l_e and A_e are respectively the Young's modulus, density, length and cross-sectional area of the e^{th} element. $[I]$ is a 6^{th} order identity matrix, $[G]$ is 6×6 matrix, where $g_{11}=g_{44}=1$ and $g_{14}=g_{41}=-1$, other elements are zero [12]. $[T_e]$ is a transformation matrix that translates the local coordinates of the e^{th} element to global coordinates and $[T_e]^T$ is its transpose. By means of the mode superposition method [13], the structural displacement response can be expressed as:

$$\{u(t)\}=\sum_{i=1}^{n}\{\phi_i\}z_i(t) \tag{4}$$

where the displacement response of j^{th} degree of freedom $u_j(t)$ is:

$$u_j(t) = \sum_{i=1}^{n} \phi_{ji}\, z_i(t) \qquad (j = 1,2,...,n) \quad (5)$$

$$z_i(t) = \frac{1}{\omega_i'} \int_0^t \{\phi_i\}^T \{F(\tau)\} \exp[-\zeta_i \omega_i (t-\tau)] \sin \omega_i'(t-\tau) d\tau \qquad (i = 1,2,...,n) \quad (6)$$

where $z_i(t)$ is the displacement response of i^{th} degree of freedom in principal coordinates. ω_i, $\{\phi_i\}$ and ζ_i are i^{th} order natural frequency, mode shape and modal damping of structure, respectively. $\omega_i' = \omega_i (1-\zeta_i^2)^{1/2}$ is the damped natural frequency. Using the relationship between node displacement and element stress, the stress response of the e^{th} element in the truss structure can be expressed as

$$\{\sigma_e(t)\} = E_e [B] \{u_e(t)\} \qquad (e = 1,2,...,m) \quad (7)$$

where $\{u_e(t)\}$ is the displacement of the nodal point of the e^{th} element, $\{\sigma_e(t)\}$ is the stress response of the e^{th} element. $[B]$ is the element's strain matrix.

3. INTERVAL NATURAL FREQUENCY AND MODE SHAPE ANALYSIS USING THE INTERVAL FACTOR METHOD

The following structural physical parameters (E_e, ρ_e) and the geometric dimensions (l_e, A_e) are all considered interval variables. As such the Young's modulus E_e^I, mass density ρ_e^I, length L_e^I and cross-sectional area A_e^I can be respectively expressed as $E_e^I = E_F^I E_e^c$, $\rho_e^I = \rho_F^I \rho_e^c$, $l_e^I = l_F^I l_e^c$, $A_e^I = A_F^I A_e^c$ ($e = 1,...,m$). E_F^I, ρ_F^I, L_F^I and A_F^I are interval factors of E_e^I, ρ_e^I, L_e^I and, A_e^I, respectively. E_e^c, ρ_e^c, l_e^c and A_e^c are mean values (or midpoint values) of E_e^I, ρ_e^I, L_e^I and A_e^I. Furthermore, the interval change ratio values of Young's modulus, density, cross-sectional area and length can be expressed as:

$$\Delta E_F = \Delta E_e / E_e^c,\ \Delta\rho_F = \Delta\rho_e / \rho_e^c,\ \Delta l_F = \Delta l_e / l_e^c,\ \Delta A_F = \Delta A_e / A_e^c \quad (8)$$

where ΔE_F, $\Delta\rho_F$, ΔA_F and ΔL_F are interval change ratio values of E_e^I, ρ_e^I, L_e^I and, A_e^I.

From Eqs. (2) and (3) the interval variables of the structural mass and stiffness matrices can be obtained in terms of the interval variables of the parameters. Consequently, the structural natural frequencies and mode shapes are now also expressed as interval variables. The mean value ω_i^c and maximum width $\Delta\omega_i$ of the natural frequency interval, and the mean value $\{\phi_i^c\}$ and maximum width $\{\Delta\phi_i\}$ of mode shape interval can be obtained from the following expressions:

$$\omega_i^c = \frac{1}{2}\left[\left(\frac{1+\Delta E_F}{(1-\Delta\rho_F)\cdot(1-\Delta L_F)\cdot(1-\Delta L_F)}\right)^{1/2} + \left(\frac{1-\Delta E_F}{(1+\Delta\rho_F)\cdot(1+\Delta L_F)\cdot(1+\Delta L_F)}\right)^{1/2} \right] \cdot \omega_i^* \quad (9)$$

$$\Delta\omega_i = \frac{1}{2}\left[\left(\frac{1+\Delta E_F}{(1-\Delta\rho_F)\cdot(1-\Delta L_F)\cdot(1-\Delta L_F)}\right)^{1/2} - \left(\frac{1-\Delta E_F}{(1+\Delta\rho_F)\cdot(1+\Delta L_F)\cdot(1+\Delta L_F)}\right)^{1/2} \right]\cdot\omega_i^* \quad (10)$$

$$\{\phi_i^c\} = \frac{1}{2}\left\{\left[1/[(1-\Delta\rho_F)(1-\Delta A_F)(1-\Delta L_F)]\right]^{1/2} + \left[1/[(1+\Delta\rho_F)(1+\Delta A_F)(1+\Delta L_F)]\right]^{1/2}\right\}\{\phi_i^*\} \quad (11)$$

$$\{\Delta\phi_i\} = \frac{1}{2}\left\{\left[1/[(1-\Delta\rho_F)(1-\Delta A_F)(1-\Delta L_F)]\right]^{1/2} - \left[1/[(1+\Delta\rho_F)(1+\Delta A_F)(1+\Delta L_F)]\right]^{1/2}\right\}\{\phi_i^*\} \quad (12)$$

where ω_i^* and $\{\phi_i^*\}$ are respectively the deterministic values of ω_i^I and $\{\phi_i^I\}$, which can be obtained from the conventional finite element model when $E_e^I = E_e^c$, $\rho_e^I = \rho_e^c$, $l_e^I = l_e^c$, $A_e^I = A_e^c$ ($e = 1,\dots,m$).

4. INTERVAL DYNAMIC RESPONSE ANALYSIS

In this study the structural damping and the applied forces are also considered as interval variables. Using the interval factor method, applied forces $\{F^I(\tau)\}$ can be expressed as

$$\{F^I(\tau)\} = \{F^c(\tau)\} + \{\Delta F(\tau)\}e_\Delta = F_F^I\{F^c(\tau)\} = \{F^c(\tau)\} + \Delta F_F\{F^c(\tau)\}e_\Delta \quad (13)$$

where $e_\Delta = [-1,1]$. $\{F^c(\tau)\}$, $\{\Delta F(\tau)\}$, F_F^I and ΔF_F are respectively the mean value, maximum width, interval factor and interval change ratio of interval loads $\{F^I(\tau)\}$. The structural displacements and stress responses can also be considered to be interval variables. Equations (4), (6) and (7) can be rewritten as:

$$\{u^I(t)\} = \sum_{i=1}^{n}\{\phi_i^I\}z_i^I(t) \quad (14)$$

$$z_i^I(t) = \frac{1}{\omega_i^I}\int_0^t \{\phi_i^I\}^T F_F^I\{F^c(\tau)\}\exp[-\zeta_{iF}^I\zeta_i^c\omega_i^I(t-\tau)]\sin\omega_i^I(t-\tau)d\tau \qquad (i = 1,2,\dots,n) \quad (15)$$

$$\{\sigma_e^I(t)\} = E_e^I[B]\{u_e^I(t)\} \qquad (e = 1,2,\dots,m) \quad (16)$$

Since ω_i^I and $\{\phi_i^I\}$ are functions of E_F^I, ρ_F^I, L_F^I and A_F^I, structural displacement and stress are functions of the interval factors of structural parameters, damping and excitation. From Eq. (15), the mean value $z_i^c(t)$ and maximum width $\Delta z_i(t)$ of $z_i^I(t)$ can be obtained by means of interval operations.

$$z_i^c(t) = \frac{1}{\omega_i^c}\int_0^t \{\phi_i^c\}^T\{F^c(\tau)\}\exp[-\zeta_i\omega_i^c(t-\tau)]\sin\omega_i^c(t-\tau)d\tau \qquad (i = 1,2,\dots,n) \quad (17)$$

$$\Delta z_i(t) = \left|\frac{\Delta\omega_i}{\omega_i^{c2}}\int_0^t \{\phi_i^c\}^T\{F^c(\tau)\}\exp[-\zeta_i\omega_i^c(t-\tau)]\sin\omega_i^c(t-\tau)d\tau\right|$$

$$+\left|\frac{1}{\omega_i^c}\int_0^t \{\Delta\phi_i\}^T\{F^c(\tau)\}\exp[-\zeta_i\omega_i^c(t-\tau)]\sin\omega_i^c(t-\tau)d\tau\right|$$

$$+\left|\frac{1}{\omega_i^c}\int_0^t \{\phi_i^c\}^T\Delta F_F\left\{F^c(\tau)\right\}\exp[-\zeta_i\omega_i^c(t-\tau)]\sin\omega_i^c(t-\tau)d\tau\right|$$

$$+\left|\frac{1}{\omega_i^c}\int_0^t \{\phi_i^c\}^T\{F^c(\tau)\}\zeta_i\Delta\omega_i\,(t-\tau)\exp[-\zeta_i\omega_i^c(t-\tau)]\sin\omega_i^c(t-\tau)d\tau\right|$$

$$+\left|\frac{1}{\omega_i^c}\int_0^t \{\phi_i^c\}^T\{F^c(\tau)\}\exp[-\zeta_i\omega_i^c(t-\tau)]\Delta\omega_i\,(t-\tau)\cos\omega_i^c(t-\tau)d\tau\right| \qquad (i=1,2,...,n) \quad (18)$$

From Eq. (14), the mean value $\left\{u^c(t)\right\}$ and maximum width $\{\Delta u(t)\}$ of interval structural displacement response can be obtained:

$$\left\{u^c(t)\right\}=\sum_{i=1}^{n}\left\{\phi_i^c\right\}z_i^c(t) \qquad (19)$$

$$\{\Delta u(t)\}=\left|\sum_{i=1}^{n}\left\{\Delta\phi_i\right\}z_i^c(t)\right|+\left|\sum_{i=1}^{n}\left\{\phi_i^c\right\}\Delta z_i\,(t)\right| \qquad (20)$$

Then, the lower bound $\left\{\underline{u(t)}\right\}$ and upper bound $\left\{\overline{u(t)}\right\}$ of interval structural displacement response can be obtained:

$$\left\{\underline{u(t)}\right\}=\left\{u^c(t)\right\}-\{\Delta u(t)\}=\sum_{i=1}^{n}\left\{\phi_i^c\right\}z_i^c(t)-\left|\sum_{i=1}^{n}\left\{\Delta\phi_i\right\}z_i^c(t)\right|-\left|\sum_{i=1}^{n}\left\{\phi_i^c\right\}\Delta z_i\,(t)\right| \qquad (21)$$

$$\left\{\overline{u(t)}\right\}=\left\{u^c(t)\right\}+\{\Delta u(t)\}=\sum_{i=1}^{n}\left\{\phi_i^c\right\}z_i^c(t)+\left|\sum_{i=1}^{n}\left\{\Delta\phi_i\right\}z_i^c(t)\right|+\left|\sum_{i=1}^{n}\left\{\phi_i^c\right\}\Delta z_i\,(t)\right| \qquad (22)$$

From Eq. (16), the mean value $\left\{\sigma_e^c(t)\right\}$ and maximum width $\left\{\Delta\sigma_e(t)\right\}$ of interval structural stress response can be obtained:

$$\left\{\sigma_e^c(t)\right\}=E_e^c[B]\left\{u_e^c(t)\right\} \qquad (e=1,2,...,m) \quad (23)$$

$$\{\Delta\sigma(t)\}=\left|\Delta E_e\,[B]\left\{u_e^c(t)\right\}\right|+\left|E_e^c[B]\left\{\Delta u_e(t)\right\}\right| \qquad (e=1,2,...,m) \quad (24)$$

Furthermore, the lower bound $\left\{\underline{\sigma_e(t)}\right\}$ and upper bound $\left\{\overline{\sigma_e(t)}\right\}$ of interval structural stress response can be obtained:

$$\left\{\underline{\sigma_e(t)}\right\}=\{\sigma_e(t)\}-\{\Delta\sigma_e(t)\}=E_e^c[B]\left\{u_e^c(t)\right\}-\left|\Delta E_e\,[B]\left\{u_e^c(t)\right\}\right|+\left|E_e^c[B]\left\{\Delta u_e(t)\right\}\right| \qquad (e=1,2,...,m) \quad (25)$$

$$\left\{\overline{\sigma_e(t)}\right\} = \{\sigma_e(t)\} + \{\Delta\sigma_e(t)\} = E_e^c[B]\{u_e^c(t)\} + \left|\Delta E_e[B]\{u_e^c(t)\}\right| + \left|E_e^c[B]\{\Delta u_e(t)\}\right| \quad (e = 1,2,\dots,m) \quad (26)$$

5. NUMERICAL EXAMPLE

To illustrate the method, a 25-bar space truss structure shown in Figure 1 is used. The mean values of the Young's modulus, density and cross-sectional area are respectively $E_e^c = 2.058\times10^5(\text{MPa})$, $\rho_e^c = 7.65\times10^3(\text{kg}/\text{m}^3)$ and $A_e^c = 3.0\times10^{-4}\text{m}^2)$. The mean values of bars' length can be seen in Figure 1. A step load is acting in the positive Y direction at node 1. The mean value of the load is $F^c(t) = 6\times10^3(N)$.

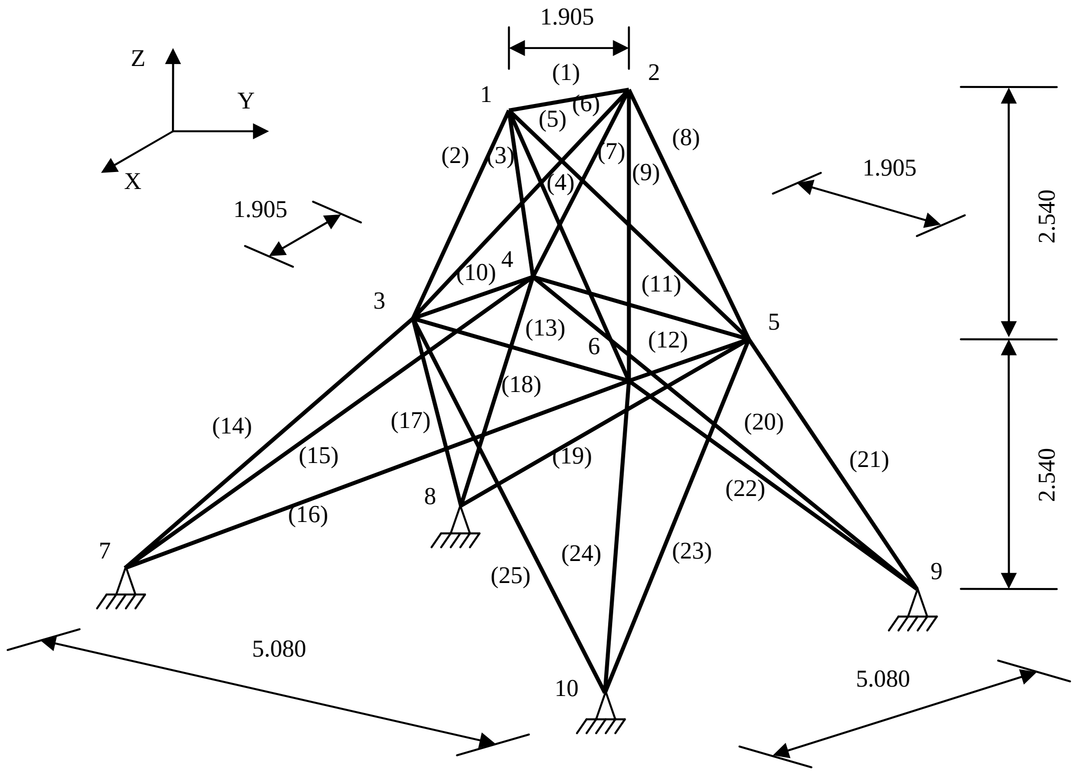

Figure 1. 25-bar space truss structure (unit: m).

In order to investigate the effect that changing the Young's modulus, density, length, cross-sectional area and applied forces has on the structural dynamic responses, the values of interval change ratio ΔE_F , $\Delta\rho_F$, Δl_F , ΔA_F and ΔF_F of interval structural parameters are taken as different groups. Mean value $U_{\max}^c$, maximum width $\Delta U_{\max}$, lower bound $\underline{U_{\max}}$ and upper bound $\overline{U_{\max}}$ of the structural maximum displacement response are given in Table 1. Mean value $\sigma_{\max}^c$, maximum width $\Delta\sigma_{\max}$, lower bound $\underline{\sigma_{\max}}$ and upper bound $\overline{\sigma_{\max}}$ of the structural maximum displacement response are given in Table 2. In addition, in order to verify the method presented in this paper, the structural dynamic displacement and stress responses are obtained using the interval perturbation method (see references [9,10]). These are also presented in Tables 1 and 2.

Table 1. Computational results of displacement (IPM*) (unit: mm).

Model	$U^c_{\max}$	$\Delta U_{\max}$	$\underline{U}_{\max}$	$\overline{U}_{\max}$
Deterministic model $\Delta E_F = \Delta\rho_F = \Delta l_F = \Delta A_F = \Delta F_F = 0$	13.092	0	13.092	13.092
$\Delta E_F = 0.03 \quad \Delta\rho_F = \Delta l_F = \Delta A_F = \Delta F_F = 0$	13.093	0.19638	12.896	13.289
$\Delta\rho_F = 0.03 \quad \Delta E_F = \Delta l_F = \Delta A_F = \Delta F_F = 0$	13.095	0.58914	12.506	13.686
$\Delta E_F = \Delta\rho_F = 0.03 \quad \Delta l_F = \Delta A_F = \Delta F_F = 0$	13.094	0.78552	12.309	13.881
$\Delta l_F = 0.03 \quad \Delta E_F = \Delta\rho_F = \Delta A_F = \Delta F_F = 0$	13.089	0.78552	12.303	13.874
$\Delta A_F = 0.03 \quad \Delta E_F = \Delta\rho_F = \Delta l_F = \Delta F_F = 0$	13.101	0.39276	12.707	13.493
$\Delta l_F = \Delta A_F = 0.03 \quad \Delta E_F = \Delta\rho_F = \Delta F_F = 0$	13.103	1.1795	11.924	14.283
$\Delta F_F = 0.03 \quad \Delta E_F = \Delta\rho_F = \Delta l_F = \Delta A_F = 0$	13.092	0.39276	12.699	13.484
$\Delta E_F = \Delta\rho_F = \Delta l_F = \Delta A_F = \Delta F_F = 0.03$	13.106	2.3604	10.747	15.466
$\Delta E_F = \Delta\rho_F = \Delta l_F = \Delta A_F = \Delta F_F = 0.05$	13.132	3.9433	9.1905	17.076
	13.092*	3.8752*	9.2168*	16.967*

Table 2. Computational results of stress (IPM*) (unit: MPa).

Model	$\sigma^c_{\max}$	$\Delta\sigma_{\max}$	$\underline{\sigma}_{\max}$	$\overline{\sigma}_{\max}$
Deterministic model $\Delta E_F = \Delta\rho_F = \Delta l_F = \Delta A_F = \Delta F_F = 0$	123.65	0	123.65	123.65
$\Delta E_F = 0.03 \quad \Delta\rho_F = \Delta l_F = \Delta A_F = \Delta F_F = 0$	123.66	5.5642	118.09	129.22
$\Delta\rho_F = 0.03 \quad \Delta E_F = \Delta l_F = \Delta A_F = \Delta F_F = 0$	123.68	5.5642	118.12	129.26
$\Delta E_F = \Delta\rho_F = 0.03 \quad \Delta l_F = \Delta A_F = \Delta F_F = 0$	123.67	11.128	112.54	134.81
$\Delta l_F = 0.03 \quad \Delta E_F = \Delta\rho_F = \Delta A_F = \Delta F_F = 0$	123.62	7.4190	116.20	131.04
$\Delta A_F = 0.03 \quad \Delta E_F = \Delta\rho_F = \Delta l_F = \Delta F_F = 0$	123.73	3.7095	120.01	127.44
$\Delta l_F = \Delta A_F = 0.03 \quad \Delta E_F = \Delta\rho_F = \Delta F_F = 0$	123.76	11.141	112.62	134.90
$\Delta F_F = 0.03 \quad \Delta E_F = \Delta\rho_F = \Delta l_F = \Delta A_F = 0$	123.65	3.7095	119.94	127.35
$\Delta E_F = \Delta\rho_F = \Delta l_F = \Delta A_F = \Delta F_F = 0.03$	123.78	26.003	97.782	149.78
$\Delta E_F = \Delta\rho_F = \Delta l_F = \Delta A_F = \Delta F_F = 0.05$	124.03	43.438	80.595	167.48
	123.65*	42.569*	81.081*	166.219*

From Tables 1 and 2, it can be seen that:

(1) The dynamic displacement and stress obtained by the method proposed in this paper are in agreement with that of the structural responses analyzed by interval perturbation method. This confirms the validity of the method.

(2)The effect of the uncertainty of the Young's modulus, density, length, cross-sectional area and applied forces on the uncertainty of the structural dynamic displacement and stress response are different. A change in the bar's length produced the greatest effect on the structural displacement and stress response, however a change in the Young's modulus and applied forces caused the smallest effect.

(3)When the interval change ratio of physical parameters is equal to that of geometric dimensions, the uncertainty of geometric dimensions will produce greater effect on the uncertainty of structural displacement response, however, the uncertainty of physical parameters will produce greater effect on the uncertainty of structural stress response. Along with the increase of the interval change ratio (uncertainty) of structural parameters and applied forces, the dispersal degree of structural dynamic responses will notably increase.

6. CONCLUSIONS

In this paper the effect of the uncertainty of material parameters, structural dimensions and applied forces on the structural dynamic response is presented using the interval factor method. The mean value, maximum width, lower and upper bounds of displacement and stress response of a truss structure have been obtained. The benefit of this method is that only one finite element solution needs to be found in order to determine the uncertainty of the response, this significantly reduces the computational time required. This method will also be applied to the interval dynamic response analysis of further types of structures with interval parameters.

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