ICSV14
Cairns • Australia
9-12 July, 2007

14th International Congress on Sound & Vibration

AN INVESTIGATION INTO THE LINEARITY OF THE SYDNEY OLYMPIC STADIUM

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Abstract

Modal analysis is based on the assumption that the structure under test can be modelled as a linear, time-invariant system. Many structures exhibit non-linearity across their operating range however, which adds uncertainty to the applicability of results obtained through modal tests with contrived excitations. Operational Modal Analysis (OMA) seeks to overcome this limitation by using the in-service forcing functions to excite the structure. Some structures however, such as large stadia, experience in-service excitations across a wide range of amplitudes.

This paper investigates the linearity of such a structure, the Sydney Olympic Stadium, using measurements recorded during a concert. The excitation, mainly crowd induced, differed significantly throughout the concert, corresponding with seated and standing / dancing concert-goers. Separate operational modal analyses were performed with low and high levels of crowd induced excitation, and the modal properties of the structure at these times were compared. This provided an insight into the linearity of the stadium, and the degree to which a single, linear model could represent the structure over its operating range.

1. INTRODUCTION

The modal properties of systems such as stadia are usually obtained through experimental modal analysis, whereby the responses to a known and artificially induced excitation are measured. The frequency response functions so derived are generally curve fitted using a linear system model to estimate the natural frequencies, damping and mode shapes (see e.g. [1, 2]). Many systems exhibit a degree of non-linearity however, exhibited by a non-constant gain (manifested as a change in the scaling of the mode shapes in a linear model), and/or a change in the modal properties, with varying levels of excitation. The latter example is typified by flight flutter testing of aircraft whereby the modal properties of an aircraft are estimated over the full flight envelope to ensure that flutter is not induced due to the merging of natural frequencies of the aircraft and reductions in damping. In a stadium, there may be non-linearities due to human-structure interaction, e.g. during an event the crowd may, at certain times, be seated, standing or even jumping, and therefore influencing the properties of the stadium, especially the damping, in different ways (see e.g. [3]). These non-linearities can also be due to geometric effects, e.g. contact in the structure induced by higher levels of excitation, changes in the damping and stiffness of elements, e.g. non-linear suspension elements employed in vehicles, and can even relate to system variables not included in the model, such as temperature.

In order to quantify such non-linearity, it is required that the gain of the system be measured, i.e. that the excitation for a modal test be known. It is often not possible to represent operational loads using artificial excitation however, because these loads are too complex (as in the flutter testing) or are too large (as in stadia). The modal properties of the system can still be estimated in such situations however, using Operational Modal Analysis (OMA). The disadvantage of OMA however is that the gain of the system is lost, so any non-linearity in the structure would only present as a change in the modal properties.

This paper reports on an investigation into the linearity of the Sydney Olympic stadium, using responses recorded during a concert event. OMA is employed to estimate the modal properties of the stadium and noise cancellation is utilised to lessen the effects of the periodicity in the crowd induced loading.

2. THEORETICAL OVERVIEW

2.1 Methodology

The purpose of this investigation was to compare the modal properties of the stadium at the extremes of its operating range. To this end, two distinct sections of the signals were analysed; the first corresponding to a periods of very low response between songs and hereafter referred to as “low”, and the second corresponding to periods of very high levels of response during particular songs and hereafter referred to as “peak”. The modal properties of the stadium were estimated in separate analyses of the “low” and “peak” signals using the Frequency Domain Decomposition technique.

2.2 Frequency Domain Decomposition

The Frequency Domain Decomposition (FDD) technique for operational modal analysis was introduced by Brincker and his associates at The University of Aalborg [4, 5]. It is very similar to the Complex Mode Indicator Function (CMIF) introduced by Shih et al [6], but applied to a matrix of spectra rather than FRFs. FDD is based on the singular value decomposition (SVD) of this response auto and cross spectral matrix, on the basis that the excitation is both frequentially and spatially white. These assumptions, which are the basis of many frequency domain OMA techniques, allow the expression for the response auto and cross spectral matrix to be simplified from:

$$\begin{aligned}\left[\mathbf{S}_{yy}\right]_{m\times m} &= \lim_{W\to\infty}\frac{1}{W}E\left\{\left\{\boldsymbol{Y}_W\right\}_{m\times 1}\left\{\boldsymbol{Y}_W\right\}^H{}_{1\times m}\right\}\\ &= \lim_{W\to\infty}\frac{1}{W}E\left\{\left[\mathbf{H}\right]_{m\times m}\left\{\boldsymbol{X}_W\right\}_{m\times 1}\left\{\boldsymbol{X}_W\right\}^H{}_{1\times m}\left[\mathbf{H}\right]^H{}_{m\times m}\right\}\end{aligned} \quad (1)$$

To

$$\left[\mathbf{S}_{yy}\right] = [\mathbf{H}][\mathbf{C}][\mathbf{H}]^H \tag{2}$$

where $\left[\mathbf{C}\right] = \lim_{W\to\infty} \frac{1}{W} E\left\{\{\boldsymbol{X}_W\}\{\boldsymbol{X}_W\}^H\right\}$ is an overall scaling constant. It is usually assumed that $\left[\mathbf{C}\right] = \left[\mathbf{I}\right]_{m\times m}$, i.e. spatial and frequential whiteness. This assumption also means that the overall scaling of the system is lost. Founded on this simplification then, the response auto and cross spectra can be expressed as

$$\boldsymbol{G}_{lm}(j\omega) = \sum_{k=1}^{M}\left(\frac{d_k\varphi_{lk}\varphi_{mk}^T}{j\omega - \lambda_k} + \frac{d_k^*\varphi_{lk}^*\varphi_{mk}^H}{j\omega - \lambda_k^*} + \frac{d_k\varphi_{lk}\varphi_{mk}^T}{-j\omega - \lambda_k} + \frac{d_k^*\varphi_{lk}^*\varphi_{mk}^H}{-j\omega - \lambda_k^*}\right) \tag{3}$$

where M is the number of modes in the frequency range of interest, $\boldsymbol{G}$ is the cross power spectral density between responses at locations l and m, d_k are scalar constants related to the modal participation factors, φ_k are the mode shapes, λ_k are the complex resonance frequencies, and * represents complex conjugate. Further, at a particular frequency ω, only a few modes *sub(ω)* will contribute to the response significantly, allowing the cross power spectral density of a lightly damped structure to be reduced to:

$$\boldsymbol{G}_{lm}(j\omega) = \sum_{k=sub(\omega)} \frac{d_k\varphi_{lk}\varphi_{mk}^T}{j\omega - \lambda_k} + \frac{d_k^*\varphi_{lk}^*\varphi_{mk}^H}{j\omega - \lambda_k^*} \tag{4}$$

The matrix of these auto and cross spectral density functions can then be decomposed at each frequency using singular value decomposition to yield:

$$\hat{\mathbf{G}}(j\omega_i) = \mathbf{U}_i\mathbf{S}_i\mathbf{U}_i^H \tag{5}$$

where $\mathbf{U}_i$ is a matrix of singular vectors and $\mathbf{S}_i$ is a diagonal matrix of singular values at frequency ω_i. If only one mode is dominating at a peak in a plot of the largest singular value then only one mode is significant in (5) and the first singular vector is an estimate of the mode shape, i.e. $\hat{\varphi} = \boldsymbol{u}_{1i}$.

On first inspection, the FDD technique seems akin to the peak picking technique [7, 8] which essentially identifies operational deflection shapes. The use of the SVD however, means that the shapes identified are actually estimates of the mode shapes. Further, closely spaced and even coincident modes can be identified, as they are revealed by a peak in the second singular value in addition to the peak in the first singular value.

The FDD technique has been successfully applied to the OMA of bridges [9], buildings [4, 5], a railcar [10], ship structures [11], and stadia as in the current application [12].

2.3 Discrete Random Separation

As mentioned above, one of the principle assumptions underlying OMA is that the excitations are frequentially white, i.e. with a flat auto-spectrum. As noted in [13], crowd induced loading at concert events actually contains significant periodic components, induced when people move to a musical beat. These periodic components manifest as peaks in the auto-spectrum in the frequency range of interest for modal analysis, and confuse the identification of resonance peaks.

A technique for separating periodic and broadband components in a signal was developed by Antoni [14]. This technique forms an H1 type filter between a signal and a delayed version of itself, exploiting the fact that the periodic component has a longer correlation length than the broadband component. This technique of discrete-random separation was employed in the analysis of the "peak" signal to reduce the level of periodic pollution in the measured responses. An example of the separation achieved through this technique is shown in Figure 1. As this figure reveals, the separation is not complete, but sufficient to identify the modes of interest to this investigation.

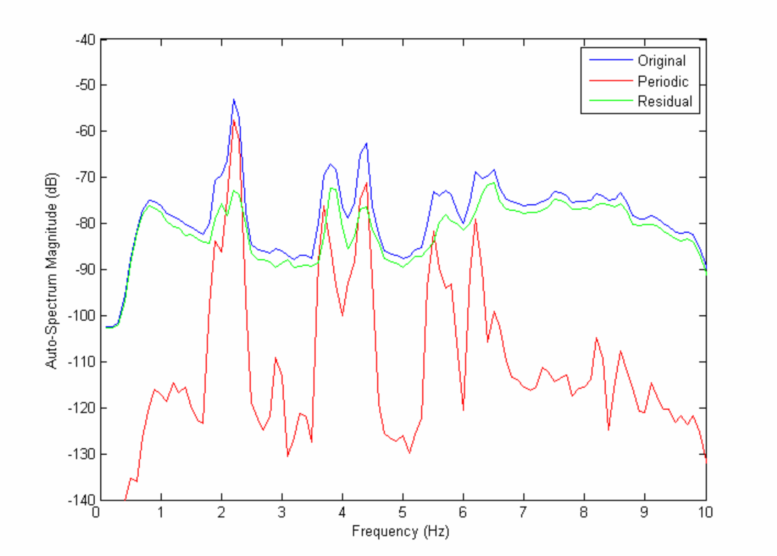

Figure 1 Auto-spectra of a response signal from the stadium (blue) with the periodic (red) and broadband (green) components separated using the technique of Antoni

3. MEASUREMENT OF THE RESPONSE OF THE STADIUM DURING A CONCERT EVENT

3.1 Experiment Setup

The responses of the stadium on the mid-tier cantilever, shown in Figure 2 were measured in a coarse grid, shown in Figure 3. The response of the structure was recorded for the duration of the concert event (a Bee Gees performance lasting approximately 2 hours) using seismic accelerometers and a 16 channel digital audio tape capable of good frequency response to zero Hertz. The data was then resampled using a Somat field computer with a sample rate of 100Hz and bandpass filtered between 0.5Hz and 10Hz to remove signal drift and audio frequencies.

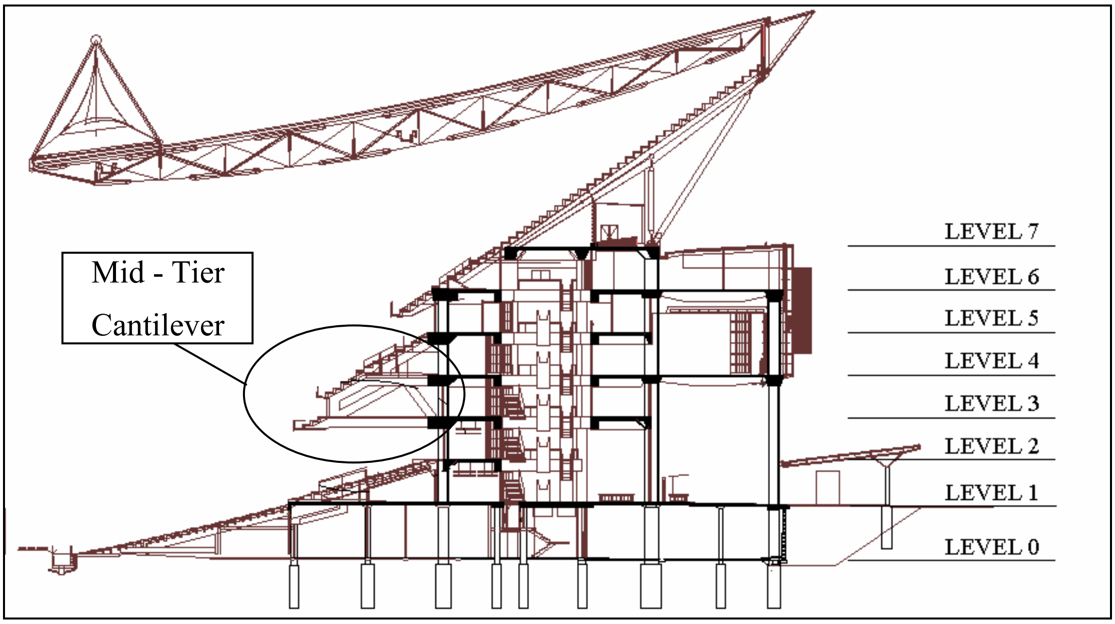

Figure 2 Location of the mid-tier cantilever

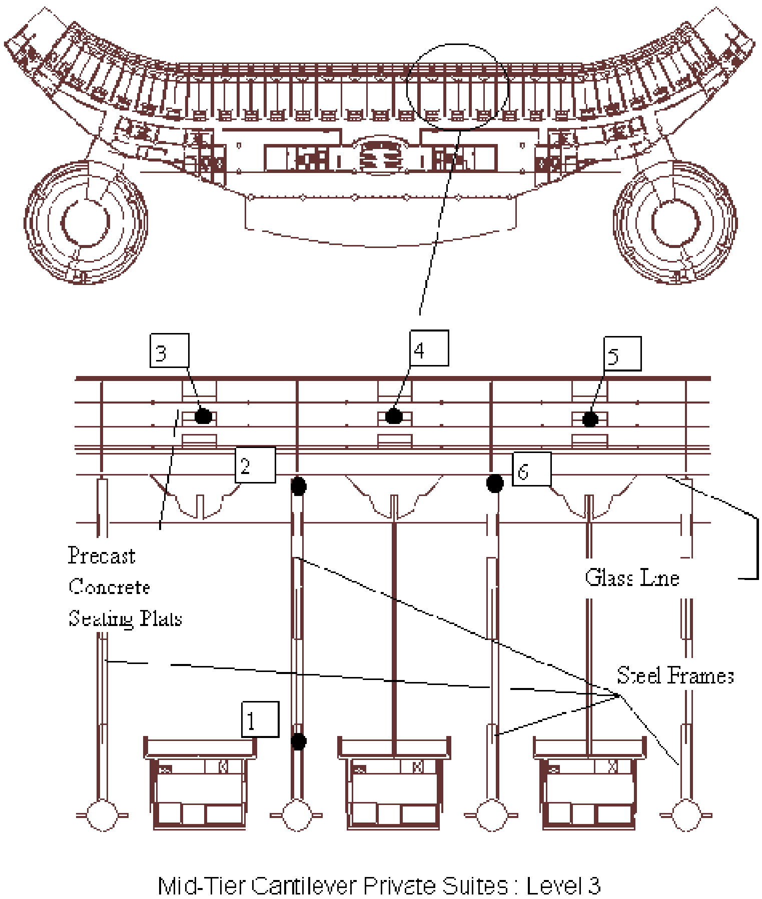

Figure 3 Accelerometer locations on the mid-tier cantilever

4. RESULTS

4.1 Periods of Peak and Low Response

The acceleration response of the structure corresponding to measurement location 3 in Figure 3 is shown in Figure 4. Also highlighted are the sections of signal used to define the "low" and "peak" responses of the structure.

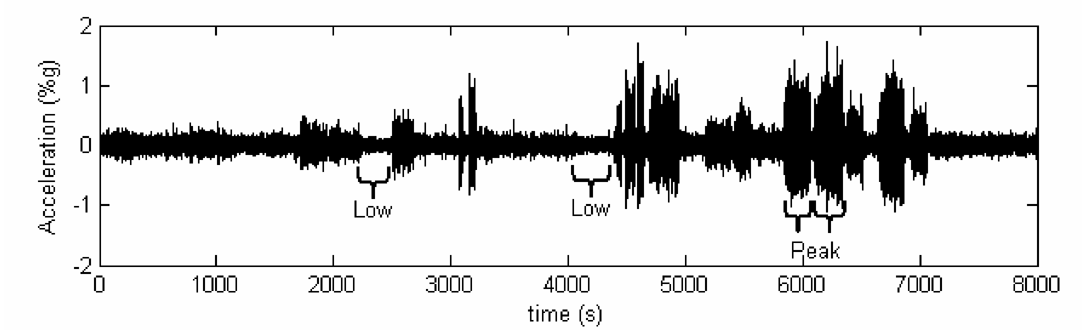

Figure 4 Acceleration response of the structure at location 3

The ratio of rms accelerations of the "peak" and "low" signals was approximately four, corresponding to a significant difference in excitation. Unfortunately, no signals were recorded with the stadium empty, which would provide an even greater contrast to the "peak" signals.

4.2 Operational Modal Analysis

The natural frequencies, damping and modal assurance criteria of the two dominant modes of the mid-tier cantilever for both load cases are contained in Table 1. The natural frequencies and mode shapes were estimated using FDD as described above, and the damping estimates represent the 3dB bandwidth of the resonance peaks in the response auto-spectrum at location 5.

Table 1 Comparison of the modal properties of the stadium with low and peak response

Mode	Low Response		Peak Response		MAC (%)
	Frequency (Hz)	Damping (%)	Frequency (Hz)	Damping (%)	
cantilever bending	6.45	2	6.35	3	95
long wavelength plat	7.03	4	7.32	4	73

The mode shapes corresponding to these resonances are compared in Figure 5 and Figure 6. Note that additional points have been included to facilitate the construction of the geometry, the displacement of which are related to the nearest measured points.

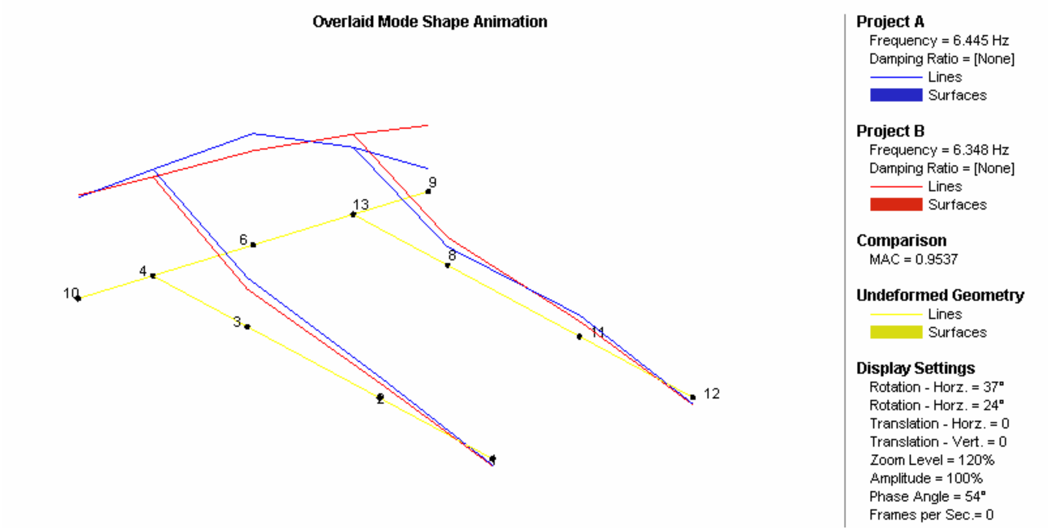

Figure 5 Comparison of the cantilever bending modes

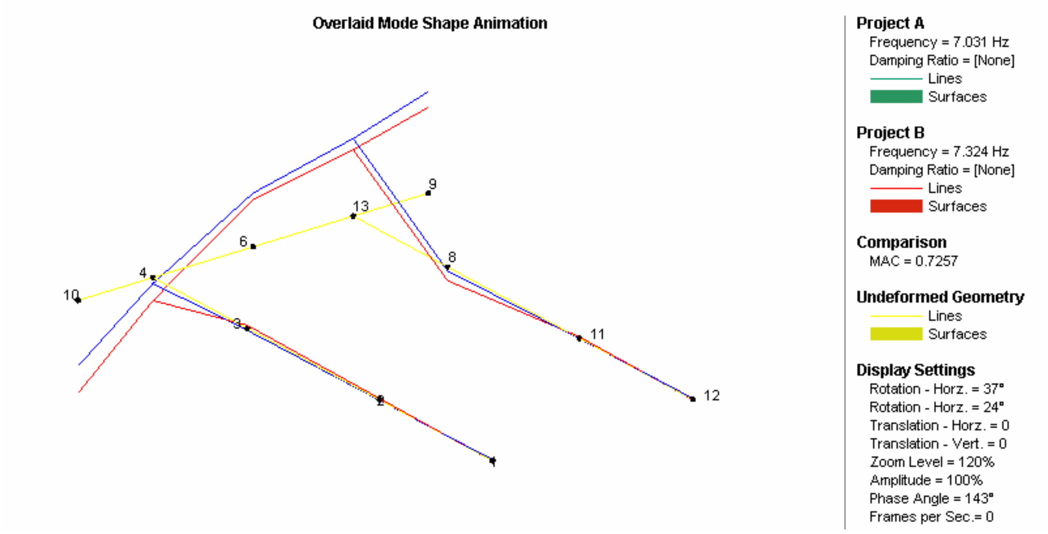

Figure 6 Comparison of the long wavelength plat bending modes

These modeshapes represent the primary cantilever bending mode (Figure 5) and a long wavelength bending mode of the seating plat (Figure 6).

Inspection of Table 1 and Figures 4 and 5 reveal small changes in the natural frequencies, mode shapes of the mid-tier cantilever between the "low" and "peak" load cases. This would suggest that the mid-tier cantilever exhibits a slight non-linearity across its operating range of excitation, but not sufficient to warrant two separate modal models so as to accurately predict the response of the structure in each case. The estimates of damping are insufficiently accurate to inform a prediction of the linearity of the structure although they do correspond with values reported in the literature for similar structures [12].

5. DISCUSSION

This paper described an investigation into the degree of linearity of the Sydney Olympic Stadium using Operational Modal Analysis. Non-linearities in the structure would present as non-constant gain in the system with different levels of excitation, and/or changes in the modal properties such as the natural frequencies, mode shapes and damping. Operational

Modal Analysis uses only the responses of the structure to its in-service excitation, so no assessment of the gain of the system can be made. This investigation therefore attempted to identify changes in the modal properties of the mid-tier cantilever section during a concert event by analysing both sections of low and very high response.

It was found that the stadium exhibited very slight non-linearity, exhibited by a small change in the natural frequencies and mode shapes of the two dominant modes in the frequency range of interest (0 – 10Hz). These changes were not sufficiently large to impact on the validity of a linear system model estimated based on either extreme of the operating range of the stadium.

6. ACKNOWLEDGEMENTS

The authors would like to thank Nader Sawali of The University of New South Wales for his assistance in the preparation of this paper.

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