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DEMONSTRATION OF INADEQUACY OF FFOWCS WILLIAMS AND HAWKINGS EQUATION OF AEROACOUSTICS BY THOUGHT EXPERIMENTS

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Abstract

In 1952, Sir James Lighthill published a theory of sound radiation by a turbulent fluid flow. This theory was extended by Curle in 1955 to a flow with immoveable rigid boundaries. In 1969, Ffowcs Williams and Hawkings extended this theory to a flow with moving rigid objects and derived an equation, which has since become one of the major tools of prediction of the flow induced noise. In 2004, Zinoviev and Bies re-examined Curle's derivation and showed a theoretical inconsistency in his calculations. In this paper, the inadequacy of Ffowcs Williams and Hawkings equation for solving problems of sound radiation by a turbulent flow near solid boundaries is demonstrated by its application to simple thought experiments. First, sound radiation by an oscillating solid sphere is compared with sound radiation by a stationary solid sphere in a variable velocity field (vortex street). It is shown that, contrary to the well-known equivalence of these situations, the Ffowcs Williams and Hawkings equation produces different results in these cases. Second, it is demonstrated that this equation predicts significant acoustic radiation by a light solid object carried by a turbulent flow. Finally, sound radiation by an acoustically small rigid plate vibrating parallel to itself is considered. It is shown that Ffowcs Williams and Hawkings equation predicts significant but physically unjustified dipole acoustic radiation with the dipole axis parallel to the plate.

1 INTRODUCTION

A major goal of the aeroacoustical theory is prediction of sound radiation by turbulent fluid flows. In 1952, Lighthill [1] proposed a theory of sound radiation by a flow without boundaries. In his theory, the mass and momentum conservation equations are transformed to obtain a linear inhomogeneous wave equation for the radiated sound where the non-linear and viscous terms are combined in the right-hand part in the form of quadrupole acoustic sources.

In 1955, Lighthill's theory was extended by Curle [2] to the case of a fluid flow with stationary rigid boundaries. According to Curle, the sound radiated by such a flow is determined, in addition to Lighthill's quadrupole sources, by the distribution of total stresses, including the viscous stresses, over the rigid boundaries. The acoustic sources, determined by the stresses, have dipole characteristics.

Utilising a methodology different from that used by Curle, Ffowcs Williams and Hawkings [3] derived an equation determining the sound radiation with a flow with moving surfaces. In this equation, Lighthill's and Curle's source terms are supplemented with the third term which depends on the normal component of the velocity of the surface with respect to a stationary observer. This term can be interpreted as a distribution of the monopole sources on the surfaces. For a stationary object, Ffowcs Williams and Hawkings (FW-H) equation reduces to Curle's equation.

Since its derivation, the Ffowcs Williams and Hawkings equation has become the foundation for one of the most frequently used methodologies of prediction of sound radiated by a fluid flow near rigid surfaces. A brief list of applications where the FW-H equation is utilised includes the rotating helicopter blades [4], rotating fans [5], and a flow near an airfoil [6]. This equation is also used in the prediction of noise radiated by moving ships and ship propellers. As mentioned by Farassat [7], this equation is the foundation of a helicopter noise prediction code, which is employed extensively by the helicopter industry.

Zinoviev and Bies [8] have conducted a critical analysis of the historically first paper on sound generation by a flow near boundaries [2]. In that analysis, it has been shown that Curle's original account neglects a non-zero integral over the rigid surface. It has been also shown that, if this integral is taken into consideration, Curle's derivation leads to a different equation, where the generated sound is determined by Lighthill's quadrupole term as well as by Kirchhoff's integrals over the rigid surfaces [9]. The differences between Curle's equation and the obtained equation have been discussed.

The goal of this paper is to apply the equation derived by Ffowcs Williams and Hawkings to four cases of sound radiation. It will be demonstrated that Ffowcs Williams and Hawkings equation leads to physically unreasonable results in three of these cases.

2 FFOWCS WILLIAMS AND HAWKINGS EQUATION

According to Ffowcs Williams and Hawkings [3], acoustic pressure fluctuations $p'(\mathbf{x},t) = p(\mathbf{x},t) - p_0$ in the acoustic wave radiated by a fluid flow near a solid boundary, *S*, are determined by

$$p'(\mathbf{x},t) = p'_{quad}(\mathbf{x},t) + p'_{mon}(\mathbf{x},t) + p'_{dip}(\mathbf{x},t) =$$
$$\frac{1}{4\pi}\frac{\partial}{\partial x_i \partial x_j}\iiint_{V_{tot}} \frac{[T_{ij}]}{r} d\mathbf{y} + \frac{1}{4\pi}\frac{\partial}{\partial t}\iint_S \rho_0 \left[\frac{n_j v_j}{r}\right] dS(\mathbf{y}) - \frac{1}{4\pi}\frac{\partial}{\partial x_i}\iint_S \left[\frac{n_j p_{ij}}{r}\right] dS(\mathbf{y}), \tag{1}$$

where $p(\mathbf{x},t)$ is the variable pressure in the fluid, p_0 is the pressure in the fluid at equilibrium, c_0 is the speed of sound in the fluid at rest, v_i is the *i*-th component of the velocity of the boundary, $\mathbf{n}$ is the unity vector normal to the rigid surface in the outward direction, $\mathbf{x} = (x_1, x_2, x_3)$ is the coordinate of the observation point, $\mathbf{y} = (y_1, y_2, y_3)$ is the coordinate of the source point, tensor p_{ij} is the compressive stress tensor, $r = |\mathbf{x} - \mathbf{y}|$, *t* is time, and square brackets indicate the dependence on the retarded time, $\tau = t - |\mathbf{x} - \mathbf{y}|/c_0$. The Lighthill's stress tensor, $T_{ij} = \rho v_i v_j + p_{ij} - c_0^2 \rho \delta_{ij}$, where δ_{ij} is Kronecker's delta function.

The variable V_{tot} is the total volume of the fluid, whereas the surface integrals are taken over the rigid boundary, *S*. Indices repeated in a single term are to be summed from 1 to 3. For example, $\partial v_k / \partial x_k$ must be understood as $\partial v_1/\partial x_1 + \partial v_2/\partial x_2 + \partial v_3/\partial x_3$. Eq. (1) is derived for $|\mathbf{v}| \ll c_0$.

The variable $p'_{quad}(\mathbf{x},t)$ denotes Lighthill's quadrupole acoustic sources distributed in the fluid volume. This term is ignored in the analysis below, as this paper is concerned only with sound generation on the surface of a rigid object. The variables $p'_{mon}(\mathbf{x},t)$ and $p'_{dip}(\mathbf{x},t)$ in Eq. (1) denote sound pressure generated by layers of elementary monopoles and dipoles on the rigid surface respectively. It is important to emphasize that the strength of the dipole layer is determined by the total stress tensor p_{ij}, which includes both viscous and acoustic stresses, whereas the strength of the monopole layer is determined by the velocity **v** with respect to a stationary observer.

3 APPLICATION OF FFOWCS WILLIAMS AND HAWKINGS EQUATION TO PROBLEMS OF SOUND RADIATION

3.1 Equivalence of an oscillating sphere and a stationary sphere in a variable velocity field

3.1.1 An oscillating sphere

Consider a rigid sphere of radius, *R*, vibrating with angular frequency, ω, without changing its form (Figure 1).

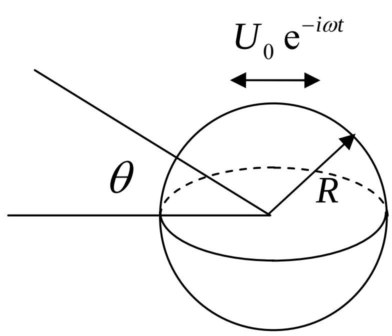

Figure 1. Oscillating sphere.

If the sphere is acoustically small, so that $kR = (\omega/c_0)R \ll 1$, and the amplitude of the velocity of the sphere is U_0, the complex amplitudes of the normal component of the velocity of the sphere, v_n, and the total pressure field on the surface of the sphere, P_{tot}, are known to be determined by [10]

$$v_n(\theta)\big|_{r=R} = U_0 \cos\theta, \tag{2}$$

$$P_{tot}(\theta)\big|_{r=R} = \frac{1}{2}\rho_0 c_0 U_0 ikR \cos\theta. \tag{3}$$

Substitution of Eqs. (2) and (3) into the FW-H Eq. (1) leads to the following equations for the monopole and dipole terms of the acoustic pressure in far field of the oscillating sphere [11]:

$$p'_{OS,mon}\left(\mathbf{x}\right)=\frac{\rho_0 U_0 \omega^2 R^3}{3c_0}\frac{e^{ikr}}{r}\cos\theta, \tag{4}$$

$$p'_{OS,dip}\left(\mathbf{x}\right)=\frac{\rho_0 U_0 \omega^2 R^3}{6c_0}\frac{e^{ikr}}{r}\cos\theta, \tag{5}$$

where *r* is the distance between the observation point and the centre of the sphere. As a result, the total radiated field is determined by

$$p'_{OS}\left(\mathbf{x}\right)=\frac{\rho_0 U_0 \omega^2 R^3}{2c_0}\frac{e^{ikr}}{r}\cos\theta, \tag{6}$$

Since Eq. (6) is well-known [10], the FW-H equation leads to the correct prediction in this case.

3.1.2 A stationary sphere in a variable velocity field

Landau and Lifshitz [10], in Section 78 that is devoted to sound scattering by a stationary object, use the formulae derived previously in this reference for sound radiation by a moving object. As translated from Russian by the present author, they state that "the difference is only in the fact that, instead of motion of an object in a fluid, we now deal with motion of a fluid with respect to an object. These two problems are, obviously, equivalent". This equivalence can be understood as the dependence of the sound wave radiated by a rigid object only on the relative motion of the object and the fluid (if the involved velocities are much smaller than the speed of sound).

Therefore, it can be concluded that the problem of sound radiation by the vibrating sphere is equivalent to the problem of sound radiation by a rigid sphere in a variable velocity field of the same amplitude. This situation can be realised, for example, if the sphere is immersed into a vortex street (Figure 2) that consists of rotating vortices moving towards the sphere with the average flow velocity, **V**.

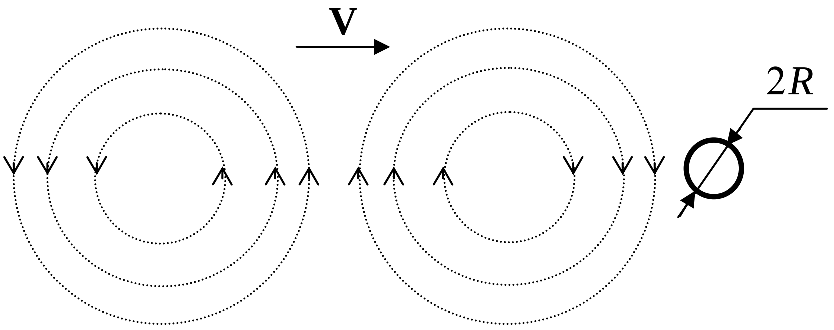

Figure 2. A sphere in a vortex street.

If the velocity fluctuations in the vortex street are much smaller than the sound speed, the pressure fluctuations in such a flow can be neglected. In addition, if the diameter of the sphere is much smaller than the size of the vortex, the incident flow velocity can be considered approximately the same on the whole surface of the sphere at any given moment of time.

Due to the equivalence of this case to sound radiation by an oscillating sphere considered above, the pressure on the surface of the sphere in this case is also determined by Eq. (3), and the dipole component of the acoustic field is equal to the dipole component of the field in the case of the oscillating sphere (Eq. (5)). However, since the monopole term in Eq. (1) is determined by the velocity of the sphere with respect to a stationary observer and the sphere is stationary, the monopole term vanishes, so that the total radiated acoustic field is determined by

$$p'_{SS}(\mathbf{x}) = p'_{SS,dip}(\mathbf{x}) = \frac{\rho_0 U_0 \omega^2 R^3}{6c_0} \frac{e^{ikr}}{r} \cos\theta. \qquad (7)$$

Eqs. (6) and (7) show that, according to the FW-H equation, the amplitude of the sound wave radiated by a stationary sphere in a variable velocity field is three times smaller than the amplitude of the sound wave radiated by an oscillating sphere in a stationary fluid. This directly contradicts the well-known notion that these two cases are equivalent [10].

3.2 A solid object embedded in a turbulent flow

Consider a solid object suspended in a turbulent flow. If the object is light and the force of gravity is neglected, the object will be carried by the flow. If the size of the object is smaller than the scale of velocity fluctuations in the flow, such an object can be said to be embedded in the flow (Figure 3) so that the velocity of the object is equal to the velocity of the flow. It has to be noticed that this object differs from a fluid particle in that respect that it cannot change its shape and its boundary is impenetrable.

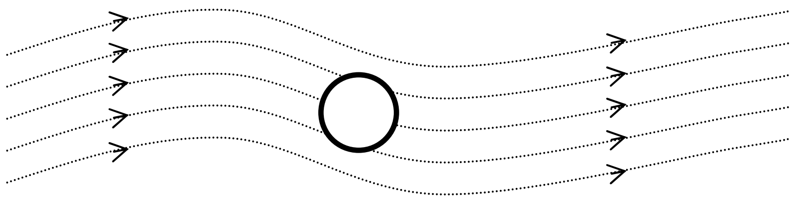

Figure 3. A light solid object embedded in a turbulent flow.

The application of the FW-H equation to this case shows that this equation predicts significant noise radiation from such an object. Indeed, as the velocity of the object is not zero and may experience sharp changes, the monopole term in Eq. (1) determines radiation of acoustic noise of considerable amplitude. At the same time, the force on the object from the flow is negligibly small and the dipole term in Eq. (1) vanishes. Therefore, the total sound radiation from the object is not zero according to the FW-H equation.

The absence of acoustic radiation from an embedded object can be demonstrated in the following way. Since the velocity of the object is equal to the velocity of the flow, the object is not an obstacle for the flow, and there is no force which could compress the fluid on the surface of the object thus leading to radiation of sound.

Therefore, the FW-H equation leads to incorrect predictions for the radiated sound amplitude in the case of a light solid object embedded in a turbulent flow.

3.3 Oscillations of a thin plate in a viscous fluid

3.3.1 Derivation

In this section, the FW-H equation is applied to a thin round rigid plate of radius, R_p, oscillating in a viscous fluid in parallel to itself (Figure 4). It will be shown below that the FW-H equation leads to prediction of dipole acoustic radiation with the axis of the dipole parallel to the plate.

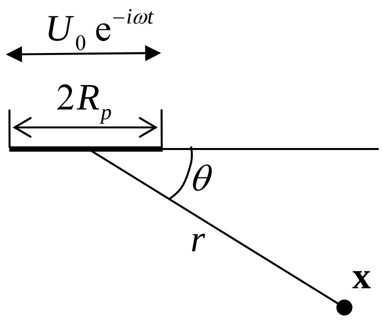

Figure 4. Oscillating thin plate in a viscous fluid.

Since the normal component of the plate velocity is zero, the monopole term in Eq. (1) vanishes. By substituting the harmonic temporal dependence, $\mathrm{e}^{-i\omega t}$, to the FW-H Eq. (1) this equation can be reduced to

$$p'(\mathbf{x}) = -\frac{1}{4\pi}\int_S \mathbf{P}(\mathbf{y})\cdot\nabla\cdot\frac{e^{ikr}}{r}\,dS(\mathbf{y}), \tag{8}$$

where $\mathbf{P}(\mathbf{y})$ is the vector determining the total force per unit area on the surface of the plate, and the operator $\nabla = (\partial/\partial x_1, \partial/\partial x_2, \partial/\partial x_3)$. The plate is considered to be acoustically small, so that

$$kR_p \ll 1. \tag{9}$$

In far field, the following approximation can be made:

$$\nabla\frac{e^{ikr}}{r} = \mathbf{r_0}\left(ik\frac{e^{ikr}}{r} - \frac{e^{ikr}}{r^2}\right) \approx \mathbf{r_0}ik\frac{e^{ikr}}{r}, \tag{10}$$

where $\mathbf{r_0}$ is the unity vector in the direction of the vector $\mathbf{r} = \mathbf{x} - \mathbf{y}$. According to Landau and Lifshitz [10], the force on the vibrating plate is tangential and its value per unit area is determined by

$$P(\mathbf{y}) = -\sqrt{\frac{\omega\eta\rho_0}{2}}(i-1)U_0, \tag{11}$$

where η is the viscosity of the fluid. Therefore, the expression under the integral in Eq. (8) can be reduced to

$$\mathbf{P}(\mathbf{y})\cdot\nabla\cdot\frac{e^{ikr}}{r} = \mathbf{P}(\mathbf{y})\cdot\mathbf{r_0}ik\frac{e^{ikr}}{r} = P(\mathbf{y})ik\frac{e^{ikr}}{r}\cos\theta = U_0\sqrt{\frac{\omega\eta\rho_0}{2}}(i+1)k\frac{\mathrm{e}^{ikr}}{r}\cos\theta. \tag{12}$$

If Eq. (12) is substituted into Eq. (8), the following equation for the sound pressure radiated by the plate is obtained:

$$p'_{PL} = \frac{1}{4}U_0kR_p^2\sqrt{\frac{\omega\eta\rho_0}{2}}(i+1)\frac{e^{ikr}}{r}\cos\theta. \tag{13}$$

3.3.2 Discussion

Equation (13) shows that, according to the FW-H equation, the plate vibrating parallel to itself in a viscous fluid is a dipole source of sound with the dipole axis parallel to the plate. Comparison of Eq. (13) with Eq. (6) for an oscillating sphere shows that the sound radiation by the plate is not negligible. For example, simple calculations show that in the air at normal conditions at the frequency of 100 Hz an oscillating plate 8 cm in diameter should, according to the FW-H equation, radiate sound of the same intensity as a sphere 1 cm in diameter oscillating with the same amplitude.

To confirm that the sound radiated by such a sphere, and consequently by the plate, is significant, the following values have been substituted into Eq. (6):

$$\begin{gathered}\rho_0 = 1.23\,kg/m^3,\quad U_0 = 0.628\,m/s,\quad \omega = 2\pi f \approx 628\,s^{-1},\quad c_0 = 330\,m/s\\ R = 0.005\,m,\quad r = 7\,m,\quad k = \omega/c_0 \approx 1.9\,m^{-1},\quad \theta = 0.\end{gathered} \tag{14}$$

For the above parameters, Eq. (6) gives the amplitude of the sound equal to 21.3 dB re 1μPa. This sound is not negligible and should be audible by a person. Note that the sound will be louder at smaller distances from the sphere. The value of $r = 7\,m$ has been chosen to make sure that the observation point is located in far field ($kr \gg 1$) so that Eq. (6) could be applied.

The actual absence of sound radiation in this case is clear from the fact that stresses existing at the surface of the plate are tangential to the plate. The oscillations of the plate do not create any regions of compression and rarefaction, which could propagate as an acoustic wave.

Therefore, it can be concluded that the Ffowcs Williams and Hawkings equation gives erroneous predictions in the case of a thin plate oscillating in a viscous fluid.

4 CONCLUSIONS

In this paper, the Ffowcs Williams and Hawkings (FW-H) equation has been applied to four cases of sound generation by oscillating rigid objects in a fluid. It has been shown that the FW-H equation leads to incorrect predictions in three cases.

First, sound radiation by an oscillating rigid sphere has been considered. It has been confirmed that the FW-H equation gives correct prediction of radiated sound in this case.

Second, the FW-H equation has been used to calculate sound radiation by a stationary sphere in a variable pressure field. Such a sphere has been modelled as a sphere located in a vortex street. It has been shown that, whereas this case is equivalent to the first one, the FW-H equation leads to sound pressure amplitude three times smaller in this case.

Third, the FW-H equation has been applied to a light rigid object embedded in a turbulent flow. This equation has been shown to predict that this object radiates acoustic noise of significant amplitude. The absence of such noise in reality has been demonstrated by physical considerations.

Finally, oscillations of a rigid thin plate in parallel to itself have been considered. Detailed calculations have shown that, according to the FW-H equation, such oscillations radiate sound of significant amplitude. It has been also demonstrated that, in reality, such oscillations cannot lead to sound generation from the surface of the plate, as they do not produce compression and rarefaction of the fluid.

Overall, it has been shown that the Ffowcs Williams and Hawkings equation is not adequate for solving real-life problems of sound radiation by fluid flows near solid boundaries.

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