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BUCKLING AND VIBRATION ANALYSIS OF MULTIPHASE MAGNETOELECTROELASTIC BEAM

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Abstract

In this paper, linear buckling and vibration behaviour of multiphase magnetoelectroelastic (MEE) cantilever beam is analysed using finite element approach. The constitutive equations for the magnetoelectroelastic materials are used to derive finite element equations involving the mechanical, electrical and magnetic fields for the structure. The multiphase magnetoelectroelastic beam consists of piezomagnetic *($CoFe_2O_4$)* matrix reinforced by piezoelectric *($BaTiO_3$)* material for different volume fraction. The influences of material constants on critical buckling load were studied. Numerical study on multiphase magnetoelectroelastic beam with different volume fraction was attempted.

1. INTRODUCTION

A magnetoelectroelastic structure has gained more importance in recent years due to coupled nature between mechanical, electrical and magnetic fields that is not present in the single phase piezoelectric or piezomagnetic materials. These structures have ability of converting energy one form to the other (among magnetic, electric and mechanical energy) [1-5]. These structures have direct application in sensing and actuating devices to control the vibration in structures. There have been several studies on the electric and mechanical behaviour of piezoelectric laminates. Hui-Shen Shen [6] studied the thermal post buckling of simply supported, shear-deformable laminated plates with piezoelectric actuators subjected to the combined thermal and electrical loads. Buchanan [7] studied the free vibrations of completely coupled magnetoelectroelastic cylinders using finite element method. Dongwei Shu *et al.* [8] investigated the buckling behavior of a two-layered beam with single asymmetric delamination for clamped and simply supported boundary conditions. Metin Aydogdu [9] has studied the buckling analysis of cross ply laminated beams and critical buckling load obtained using the Ritz method where three displacement components are expressed in a series of simple algebraic polynomials. Chen *et al*. [10] derived the two-separated state equations to study the free vibration behavior of non-homogeneous transversely isotropic magneto-electro-elastic plates. Ganesan *et al.* [11] have studied the buckling and vibration behavior of sandwich beam having viscoelastic core under thermal environment using finite element method. Recently, Ramirez *et al*. [12] studied the approximate solution for free vibration problem by combining a discrete layer approach with the Ritz method and compared with the commercial finite element software ABAQUS. Based on the literature survey, there are no studies that deal with buckling and vibration behavior of multiphase magnetoelectroelastic beam under mechanical loading. In the present study, the influence of material properties on critical buckling load and natural frequencies with respect to applied load are analyzed.

2. FINITE ELEMENT FORMULATION

2.1 Constitutive equations

In Cartesian coordinate system $x_i\,(i=1,2,3)$, the coupled constitutive equation for linearly magnetoelectroelastic three-dimensional solid can be written as,

$$\begin{aligned}\sigma_i &= c_{ij}S_j - e_{ik}E_k - q_{ik}H_k\\ D_l &= e_{lj}S_j + \eta_{lk}E_k + m_{lk}H_k\\ B_l &= q_{lj}S_j + m_{lk}E_k + \mu_{lk}H_k\end{aligned} \tag{1}$$

where $i,j=1,2,....,6$ and $l,k=1,2,3$. The reduced notation has been used for each stress tensor representations, $(\sigma_1=\sigma_{11},\sigma_2=\sigma_{22},\sigma_3=\sigma_{33},\sigma_4=\sigma_{23},\sigma_5=\sigma_{31}$ and $\sigma_6=\sigma_{12})$. Where σ_i, D_l and B_l are the components of stress, electric displacement and magnetic induction respectively. c_{ij},η_{lk} and μ_{lk} are the elastic, dielectric and magnetic permeability coefficients respectively. e_{ik},q_{ik} and m_{lk} are the piezoelectric, piezomagnetic and magnetoelectric material coefficients respectively. S_j,E_k and H_k are linear strain tensor, electric field and magnetic field respectively. In the present analysis, the coupled three-dimensional constitutive equations (1) for magnetoelectroelastic solid in x_1-x_3 plane are assumed to be isotropic. The non-zero components of material constants of equation (1) for transversely isotropic magnetoelectroelastic solid can be written in matrix form as,

$$[c]=\begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0\\ & c_{11} & c_{23} & 0 & 0 & 0\\ & & c_{33} & 0 & 0 & 0\\ & & & c_{44} & 0 & 0\\ & Sym & & & c_{44} & 0\\ & & & & & c_{66}\end{bmatrix};\quad [e]=\begin{bmatrix}0&0&e_{31}\\0&0&e_{31}\\0&0&e_{33}\\0&e_{15}&0\\e_{15}&0&0\\0&0&0\end{bmatrix};\quad [q]=\begin{bmatrix}0&0&q_{31}\\0&0&q_{31}\\0&0&q_{33}\\0&q_{15}&0\\q_{15}&0&0\\0&0&0\end{bmatrix}$$

$$[\eta]=\begin{bmatrix}\eta_{11}&0&0\\0&\eta_{11}&0\\0&0&\eta_{33}\end{bmatrix};\quad [\mu]=\begin{bmatrix}\mu_{11}&0&0\\0&\mu_{11}&0\\0&0&\mu_{33}\end{bmatrix};\quad [m]=\begin{bmatrix}m_{11}&0&0\\0&m_{11}&0\\0&0&m_{33}\end{bmatrix} \tag{2}$$

For plane stress problems, the stress components $\sigma_2=\sigma_4=\sigma_6=0$, electric displacement $D_2=0$ and magnetic induction $B_2=0$. The strain-displacement, electric field-electric potential and magnetic field-magnetic potential are used in the finite element analysis along with the constitutive equations (1). The strain filed S_{ij} related to displacement, electric field vector E_i is related to the electric potential ϕ and magnetic field vector H_i is related to the magnetic potential ψ can be written as,

$$S_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}); \quad E_i = -\phi_{,i}; \qquad H_i = -\psi_{,i} \tag{3}$$

The total potential G can be written as,

$$G = \frac{1}{2}S_i^T c_{ik} S_j - \frac{1}{2}E_l^T \eta_{lk} E_k - \frac{1}{2}H_l^T \mu_{lk} H_k - S_i^T e_{ik} E_k - S_i^T q_{ik} H_k - H_k^T m_{lk} E_k \tag{4}$$

2.2 Finite element equations

The magnetoelectroelastic cantilever beam is discretized using four nodded plane stress element having four nodal degrees of freedom *viz*. displacement in x_1, x_3 directions, electric and magnetic potentials. It can be represented by suitable shape functions such as,

$$u = [N_u]\{u_i\}; \phi = [N_\phi]\{\phi\}; \psi = [N_\psi]\{\psi\} \tag{5}$$

where $\{u_i\} = \{u_1 \quad u_3\}^T$, u_1 and u_3 are displacements in x_1 and x_3 directions respectively. Substituting the Equations (1), (3) and (5) in Equation (4), we can get the following coupled finite element equations [after assembling the elemental matrices] as,

$$\begin{aligned}
&[[K_{uu}] - \omega^2[M]]\{u\} + [K_{u\phi}]\{\phi\} + [K_{u\psi}]\{\psi\} = 0 \\
&[K_{u\phi}]^T\{u\} - [K_{\phi\phi}]\{\phi\} - [K_{\phi\psi}]\{\psi\} = 0 \\
&[K_{u\psi}]^T\{u\} - [K_{\phi\psi}]^T\{\phi\} - [K_{\psi\psi}]\{\psi\} = 0
\end{aligned} \tag{6}$$

where different elemental stiffness matrices are

$$\begin{aligned}
&[K_{uu}^e] = \int_v [B_u]^T [\bar{c}][B_u]dv; \quad [K_{u\psi}^e] = \int_v [B_u]^T [\bar{q}][B_\psi]dv \\
&[K_{u\phi}^e] = \int_v [B_u]^T [\bar{e}][B_\phi]dv; \quad [K_{\phi\psi}^e] = \int_v [B_\phi]^T [\bar{m}][B_\psi]dv \\
&[K_{\phi\phi}^e] = \int_v [B_\phi]^T [\bar{\eta}][B_\phi]dv; \quad [K_{\psi\psi}^e] = \int_v [B_\psi]^T [\bar{\mu}][B_\psi]dv \\
&[M^e] = \rho\int_v [N]^T [N]dv
\end{aligned} \tag{7}$$

where $[B_u],[B_\phi]$ and $[B_\psi]$ are derivative of shape function matrix for strain displacement, electric field-electric potential, magnetic field-magnetic potential respectively. The derivative of shape function matrix used in the Equation (7) with respect to four-nodded rectangular element as,

$$[B_u] = \begin{bmatrix} \frac{\partial N_1}{\partial x_1} & 0 & \frac{\partial N_2}{\partial x_1} & 0 & \frac{\partial N_3}{\partial x_1} & 0 & \frac{\partial N_4}{\partial x_1} & 0 \\ 0 & \frac{\partial N_1}{\partial x_3} & 0 & \frac{\partial N_2}{\partial x_3} & 0 & \frac{\partial N_3}{\partial x_3} & 0 & \frac{\partial N_4}{\partial x_3} \\ \frac{\partial N_1}{\partial x_3} & \frac{\partial N_1}{\partial x_1} & \frac{\partial N_2}{\partial x_3} & \frac{\partial N_2}{\partial x_1} & \frac{\partial N_3}{\partial x_3} & \frac{\partial N_3}{\partial x_1} & \frac{\partial N_4}{\partial x_3} & \frac{\partial N_4}{\partial x_1} \end{bmatrix}; \; [B_\phi] = [B_\psi] = \begin{bmatrix} \frac{\partial N_1}{\partial x_1} & \frac{\partial N_2}{\partial x_1} & \frac{\partial N_3}{\partial x_1} & \frac{\partial N_4}{\partial x_1} \\ \frac{\partial N_1}{\partial x_3} & \frac{\partial N_2}{\partial x_3} & \frac{\partial N_3}{\partial x_3} & \frac{\partial N_4}{\partial x_3} \end{bmatrix} \tag{8}$$

ρ is the density of the material. $[\bar{c}],[\bar{q}],[\bar{e}],[\bar{m}],[\bar{\eta}]$, and $[\bar{\mu}]$ are reduced elastic constants matrix, piezomagnetic coefficient matrix, piezoelectric matrix, magnetoelectric coefficient matrix, dielectric coefficient matrix and magnetic permeability matrix respectively [4]. By using standard condensation techniques, the equivalent stiffness matrix is derived by eliminating the electric potential ϕ and magnetic potential ψ from Equation (6). The derived stiffness matrix $[K_{eq}]$ is used to solve for nodal displacements.

$$[K_{eq}]\{u\} = \{F\} \tag{9}$$

where

$$[K_{eq}] = [K_{uu}] + [K_{u\phi}][K_{II}]^{-1}[K_I] + [K_{u\psi}][K_V]^{-1}[K_{IV}] \tag{10}$$

The component matrices for equation (10) are

$$[K_I] = [K_{u\phi}]^T - [K_{\phi\psi}][K_{\psi\psi}]^{-1}[K_{u\psi}]^T \quad ; \quad [K_{II}] = [K_{\phi\phi}] - [K_{\phi\psi}][K_{\psi\psi}]^{-1}[K_{\phi\psi}]^T$$

(14)

$$[K_{IV}] = [K_{u\psi}]^T - [K_{\phi\psi}]^T[K_{\phi\phi}]^{-1}[K_{u\phi}]^T \quad ; \quad [K_V] = [K_{\psi\psi}] - [K_{\phi\psi}]^T[K_{\phi\phi}]^{-1}[K_{\phi\psi}]$$

After evaluating the displacements, the electric potential ϕ and magnetic potential ψ can be computed at each nodal points using the following equations,

$$\phi = \left[K_{II}\right]^{-1}\left[K_I\right]\{u\}\,; \qquad \psi = \left[K_V\right]^{-1}\left[K_{IV}\right]\{u\} \tag{11}$$

In the present analysis, four-point gaussian integration scheme has been adopted to evaluate the integrals involved in different elemental stiffness matrices. The elemental stiffness matrices are assembled to get the global stiffness matrices. The coupled equivalent stiffness matrix of magnetoelectroelastic system has been inverted to evaluate the displacements. After solving the coupled equation, the stresses can be evaluated using the constitutive Equation (1). The element wise stress components are used to formulate the geometric stiffness matrix. For each element, the geometric stiffness matrix can be computed using the relation,

$$[K_g^e] = \int [B_g]^T [\sigma_o][B_g]\, dV \tag{12}$$

where $[\sigma_o]$ is the initial stress matrix and $[B_g]$ is called non-linear strain displacement matrix given by

$$[B_g] = \begin{bmatrix} \frac{\partial N_1}{\partial x_1} & 0 & \frac{\partial N_2}{\partial x_1} & 0 & \frac{\partial N_3}{\partial x_1} & 0 & \frac{\partial N_4}{\partial x_1} & 0 \\ \frac{\partial N_1}{\partial x_3} & 0 & \frac{\partial N_2}{\partial x_3} & 0 & \frac{\partial N_3}{\partial x_3} & 0 & \frac{\partial N_4}{\partial x_3} & 0 \\ 0 & \frac{\partial N_1}{\partial x_1} & 0 & \frac{\partial N_2}{\partial x_1} & 0 & \frac{\partial N_3}{\partial x_1} & 0 & \frac{\partial N_4}{\partial x_1} \\ 0 & \frac{\partial N_1}{\partial x_3} & 0 & \frac{\partial N_2}{\partial x_3} & 0 & \frac{\partial N_3}{\partial x_3} & 0 & \frac{\partial N_4}{\partial x_3} \end{bmatrix} \tag{13}$$

The stress components are found for each element at a specified buckling load and global geometric stiffness matrix $[K_g^G]$ can be formulated. The following eigen value problem has to be solved to evaluate the natural frequencies for prestressed purely elastic and magnetoelectroelastic beam, For purely elastic beam

$$[[K_{uu}^{G}]+[K_{g}^{G}]]\{x_b\}-\omega^2[M^G]\{x_b\}=0 \tag{14}$$

For magneto-electro-elastic beam

$$[[K_{eq}^{G}]+[K_{g}^{G}]]\{x_b\}-\omega^2[M^G]\{x_b\}=0 \tag{15}$$

In the above equation, $[K_{uu}^{G}]$ and $[K_{eq}^{G}]$ are global structural stiffness matrix for purely elastic and magnetoelectroelastic beam respectively. ω is natural frequency and $\{x_b\}$ is the corresponding eigen vector. $[K_{uu}^{G}]$ is global mass matrix of the system.

3. RESULTS AND DISCUSSIONS

Analysis of multiphase magnetoelectroelastic beam is carried out in order to understand the nature of the variation of free vibration frequencies and critical buckling load. The multiphase magneto-electro-elastic beam consists of piezomagnetic *($CoFe_2O_4$)* matrix reinforced by piezoelectric *($BaTiO_3$)* material for different volume fraction. The 100% volume fraction corresponds to piezoelectric *($BaTiO_3$)* material and 0% volume fraction corresponds to piezomagnetic *($CoFe_2O_4$)* material. The geometric details of magneto-electro-elastic beam are: length *(L)* = 0.3 m and thickness *(t)* =0.01m. The mechanical load is applied at the free end of the beam. The material constant for different volume fraction of MEE beam used in the present study is taken from the ref. [4]. The boundary condition incorporated for fixed end is $u_1 = u_3 = \phi = \psi = 0$.

3.1 Validation of the present formulation

The present formulation has been validated with the commercial finite element package ANSYS [13] for the beam considered to be a piezoelectric material. Since the Finite element package ANSYS is unable to directly solve the frequency behavior and critical buckling load for multiphase magnetoelectroelastic materials. The present finite element model is discretized using 840 four nodded elements with 3948 degrees of freedom (i.e. 1974 displacement dof, 987 electric dof and 987 magnetic dof). Table 1, lists the comparison of natural frequency and critical buckling load of piezoelectric elastic beam. From the Table 1, it can be seen that there is a good agreement between the present analysis and ANSYS.

Table. 1 Natural frequency and critical buckling load for Piezoelectric elastic beam

	Natural frequency (Hz)		Buckling load (KN)	
	ANSYS	Present	ANSYS	Present
Piezoelectric elastic beam	85.045	83.079	293.025	280.47

3.2 Influence of magnetoelectroelastic coupling on critical buckling load for multiphase magnetoelectroelastic beam

In order to understand the influence of magnetoelectroelastic coupling on buckling load of multiphase magnetoelectroelastic beam, the analysis was carried out for different volume fraction *viz*. 0.0, 0.2, 0.4, 0.6, 0.8 and 1.0. Table.2 illustrates the critical buckling load for MEE beam with and without the coupling effects. It is observed that the 0% volume fraction corresponds to piezomagnetic material shows the higher buckling load and decreases with volume fraction. This is because of stiffness of the system decreases with volume fraction. The buckling load for 100% volume fraction corresponds to piezoelectric material is higher as compared to 80%. It is noticed that the critical buckling load of MEE elastic beam is lesser as compared to MEE beam. This is because of the piezoelectric, piezomagnetic and magnetoelectric coupling increases the stiffness of the system. This analysis can be used for evaluating the critical buckling load during the design stage and this helps the proper choice of volume fraction of MEE beam based on the applications.

Table 2 Critical buckling load for multiphase magnetoelectroelastic beam

Description	Critical buckling load (KN)					
	V_f=0.0	V_f=0.2	V_f=0.4	V_f=0.6	V_f=0.8	V_f=1.0
without coupling (Elastic)	370.19	311.35	325.18	291.75	248.99	280.21
with coupling (MEE)	370.16	316.72	332.79	301.44	259.07	290.91

3.3 Influence of applied load on natural frequency of multiphase magnetoelectroelastic beam

To understand the variation of natural frequency with respect to applied load, studies have been carried out for multiphase magnetoelectroelastic beam with different volume fraction under clamped – free boundary condition. When the clamped-free magnetoelectroelastic beam is under prestressed condition, the equation (15) has to be solved to obtain the natural frequencies. Table 3 shows the natural frequency of MEE beam with and without magnetoelectroelastic coupling for first five modes and it is noticed that the natural frequency is higher for MEE beam due to coupling effect. The natural frequency decreases with volume fraction increases and higher for 100% as compared to 80%.

Table 3 Natural frequency for multiphase magnetoelectroelastic beam

	Mode	Natural frequency (Hz)					
		V_f=0.0	V_f=0.2	V_f=0.4	V_f=0.6	V_f=0.8	V_f=1.0
Elastic	1	101.52	96.55	91.30	85.71	78.49	82.53
	2	606.59	576.81	569.19	534.57	490.00	514.71
	3	1674.95	1589.39	1580.58	1485.51	1363.59	1430.23
	4	3237.64	3077.35	3061.16	2879.89	2648.85	2772.52
	5	4509.24	4272.86	4132.80	3870.24	3521.29	3734.62
MEE	1	101.74	97.37	92.35	87.11	80.05	84.08
	2	606.63	580.72	575.61	543.20	499.67	524.76
	3	1674.92	1603.77	1598.06	1509.09	1390.17	1459.70
	4	3237.20	3101.49	3094.12	2924.52	2699.57	2833.84
	5	4509.95	4317.35	4188.97	3945.74	3606.05	3817.60

Figure. 1 (a) – (f) exhibits the variation of natural frequency of MEE beam with volume fraction 0%, 20%, 40%, 60%, 80% and 100% for first four modes. From the free vibration studies, it is clear that the natural frequency decreases with increase in applied load. Also it is observed that the natural frequency becomes zero at the critical buckling load. It is noticed that the natural frequencies are higher for lower volume fraction. This is because of percentage of piezomagnetic material increases with volume fraction decreases. The elastic constants for piezomagnetic material are higher as compared to the piezoelectric material. When the volume fraction increases the percentage of piezoelectric material increases which leads to decease the stiffness of the system.

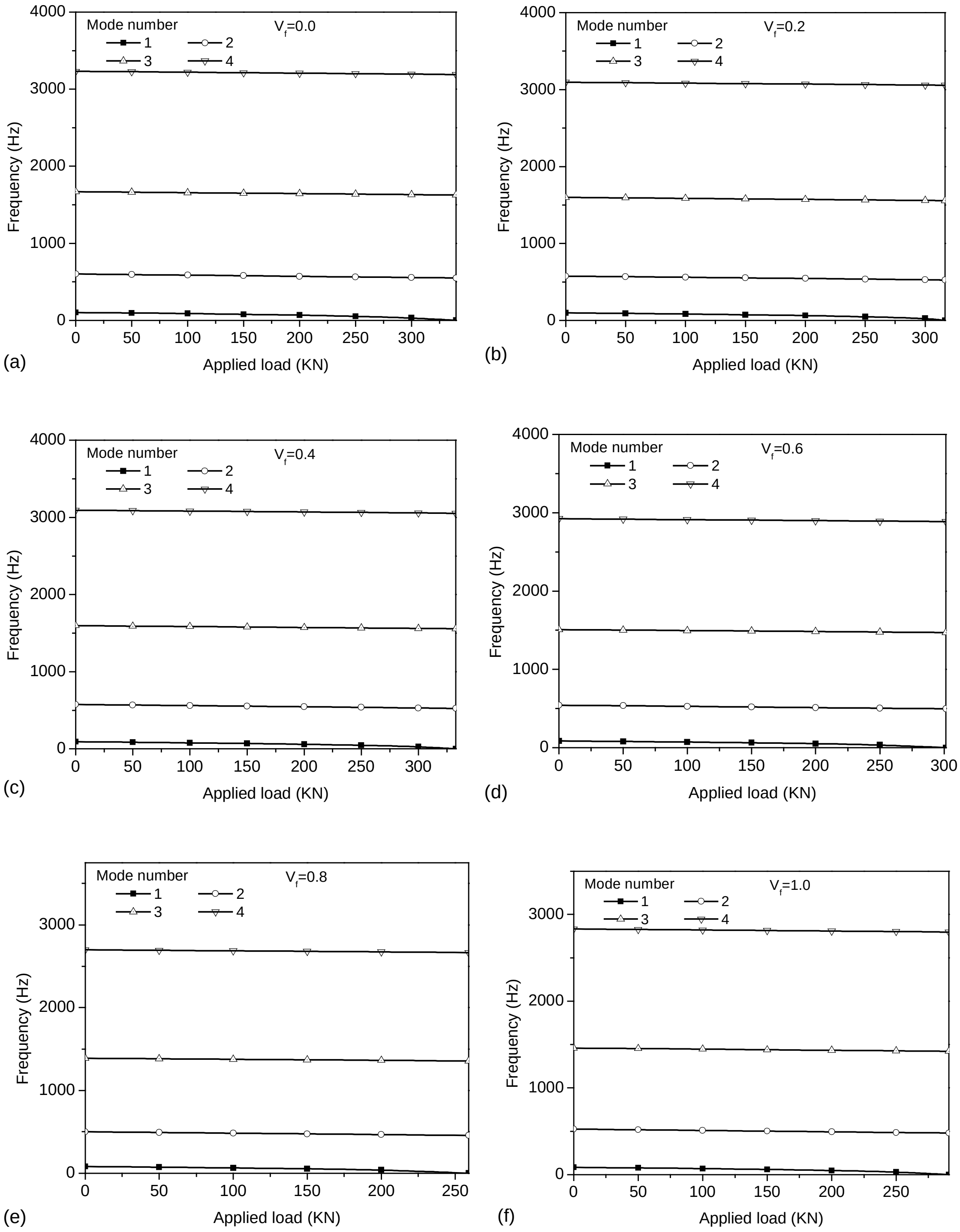

Figure 1. Variation of natural frequency for MEE beam with volume fraction of
(a) 0% (b) 20% (c) 40% (d) 60% (e) 80% (f) 100%

5. CONCLUSIONS

The buckling and free vibration behavior of multiphase magnetoelectroelastic beam is carried out using finite element approach. For the typical beam with material properties used in the present work, the following conclusions were made. The volume fractions alter the critical buckling load and natural frequency of the system. The critical buckling load decreases with volume fraction increases for multiphase magnetoelectroelastic beam. The variation of natural frequency with respect to applied load is smooth, gradual and become zero corresponding to critical buckling load. The present analysis is highly useful in the design stage to achieve the optimum choice of volume fraction of MEE beam.

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