ICSV14

Cairns • Australia

9-12 July, 2007

14th International Congress on Sound & Vibration

STRUCTURAL STRESS ANALYSIS UNDERGOING HARMONIC EXCITATION USING MODAL THEORY

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ABSTRACT

Structural interior stress in operational state depends on its dynamics characters and external excitation. In this paper, the stress state of operational structure is firstly analyzed by using experimental modal analysis technique and structure intension theory. Energy distribution of structural every dominant modal can be discussed by extracting practical dominant modal. Consequently structural dynamic stress is calculated by translating complex static and dynamic coupling spatial stress state into simple stress state of each dominant modal. Furthermore, the simply support beam is modeled, and is carried out numerical simulation calculation and damage experiment. The stress of folding static and dynamic at 1/4L is compared to the stress at 1/2L along the longitudinal beam. The results show that maximum stress occurs at the 1/4L location, where static stress is smaller in still state but dynamic stress is bigger under harmonic excitation.

Keywords: modal theory, dominant modal, coupling static with dynamic stress

1. INTRODUCTION

At present, relating scholars have done a lot of significative work about experimental modal analysis theories and its applications [1~4], which include identification of modal parameters, structural dynamic optimization design, structure dynamic modification, structure damage

detection and finite element model modification etc. Simultaneously, others who applied themselves to structure intension research have also obtained prodigious fruits about theories of structure still intension, structure fatigue damage, stress and strain reliability [5, 6]. In practice, working structure is supposed to not only known steady loads but also unknown random dynamic loads. For the safety of operational structures, both static load stress and dynamic load stress should be considered in intensity calculations. In order to predict operational structural stress state exactly, the paper will connect structural dynamic characters with exterior excitation, combine experiment modal analysis principle with structure intension theory using existing theories, decompose dynamic question into static question on each order modal, unify static and dynamic stress, obtain the maximum stress location of structure, finally provide experience for structure design or structure damage identification.
This paper is organized in five sections. In section 1, a brief introduction of dynamics is given. In section 2, basic principle of modal theory and intensity theory coupling is presented. In section 3, the numerical simulation is performed. And in section 4, experiment on simple support beam structure is carried out. Section 5 gives the conclusion of entire chapter.

2. BASIC PRINCIPLE

It is a premise of structural reliability evaluation that structural practical performance and its stress state can be calculated accurately. Firstly modal participation factors should be determined by computing structural dynamic characters and external excitation. Modal strain energy distribution of each order dominant modal can be calculated by spectrum analysis on measured response of structure. Then the location of the maximum stress on the beam can be obtained by folding static and dynamic under harmonic excitation.
Confirmation of Modal Participation Factors: The modal participating factor is also called dominant modal, which plays the dominating role in structural response. If structures and exterior excitation are confirmed, then structural dominant modal can be gained. Generally, there are several dominant modals according to their relative occupation ratios.
The ordinary differential equation of dynamic system generally is

$$M\ddot{x} + C\dot{x} + Kx = f(t) \tag{1}$$

where M, C, and K are mass, damping, and stiffness matrices, respectively. $f(t)$ is an excitation force vector and x is a displacement vector.

Defining $f(t) = Fe^{j\omega t}$, $x = Xe^{j\omega t}$, transforming coordinate is

$$X = \Phi q \tag{2}$$

where q are modal participating factors; Φ is an eigenvector shape matrix.
Change equation (1) into frequency domain equation, and use the weighted orthogonal conditions for the eigenvector shape. Suppose C is scale-damping matrix, so

$$(-\omega^2 m_r + k_r + j\omega c_r)q = \Phi^T F \tag{3}$$

where m_r, c_r, k_r respectively are modal mass, damping and stiffness matrices, all are diagonal matrices.

From equation (2) and equation (3), response expression of displacements can be obtained

$$X = \Phi Y_r \Phi^T F \tag{4}$$

Modal participation factor is $q = Y_r \Phi^T F$.

Thus r th order modal participation factor can be obtained

$$q_r = \frac{F_r}{K_r - \omega^2 M_r + j\omega C_r} \tag{5}$$

Confirmation of Every Order Modal Strain Energy Distribution: In three dimensions space, equation (4) can be described as

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} \phi_u \\ \phi_v \\ \phi_w \end{bmatrix}_{3N\times m} Y_{r_{m\times m}} \begin{bmatrix} \phi_u^T & \phi_v^T & \phi_w^T \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} \tag{6}$$

According to elastic mechanics theory and Cauchy stress principle, the response expression of strain can be obtained

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial \phi_u}{\partial x} & \frac{\partial \phi_v}{\partial y} & \frac{\partial \phi_w}{\partial z} & \frac{\partial \phi_u}{\partial y} + \frac{\partial \phi_v}{\partial x} & \frac{\partial \phi_v}{\partial z} + \frac{\partial \phi_w}{\partial y} & \frac{\partial \phi_w}{\partial x} + \frac{\partial \phi_u}{\partial z} \end{Bmatrix}^T Y_r \left[\phi_u^T \phi_v^T \phi_W^T \right] \begin{Bmatrix} F_x \\ F_y \\ F_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} \tag{7}$$

Supposing $\frac{\partial \phi_u}{\partial x} = \psi_x$, $\frac{\partial \phi_v}{\partial y} = \psi_y$, $\frac{\partial \phi_w}{\partial z} = \psi_z$,

$$\frac{\partial \phi_u}{\partial y} + \frac{\partial \phi_v}{\partial x} = \psi_{xy}, \quad \frac{\partial \phi_v}{\partial z} + \frac{\partial \phi_w}{\partial y} = \psi_{yz}, \quad \frac{\partial \phi_w}{\partial x} + \frac{\partial \phi_u}{\partial z} = \psi_{zx}$$

Then equation (7) becomes $\varepsilon = \Psi Y_r \Phi^T F$

Where

$$\Psi = \left[\psi_x \psi_y \psi_z \psi_{xy} \psi_{yz} \psi_{zx} \right]^T \tag{8}$$

is called strain modal shape matrix.

So modal strain energy can be expressed as

$$U(x) = \frac{1}{2} \int_0^l EI \left(\frac{\partial^2 \Phi}{\partial x^2} \right)^2 dx \tag{9}$$

Because $\Phi = \sum_{i=1}^{n} \alpha_i \phi_i$, equation (9) can be expressed as

$$U(x)=\frac{1}{2}\int_0^l EI\left(\sum_{i=1}^{n}\alpha_i\frac{\partial^2\phi_i}{\partial x^2}\right)^2 dx \tag{10}$$

i th order modal strain energy

$$U_i(x)=\frac{1}{2}\int_0^l EI\left(\frac{\partial^2\phi_i}{\partial x^2}\right)^2 dx \tag{11}$$

j th element modal strain energy corresponding i th order modal is

$$U_{ij}(x)=\frac{1}{2}\int_{a_j}^{a_{j+1}}(EI)_j\left(\frac{\partial^2\phi_i}{\partial x^2}\right)^2 dx \tag{12}$$

Therefore structural dominant modal and its energy distribution under the certain excitation environment can be obtained by spectrum analysis. Then we can begin to calculate the structural stress state of each independent order modal.

Structural Stress State of Every Independent Modal: Stress will be bigger where structural modal strain is bigger when the frequency of excitation force is close to some order natural frequency. Following dynamic stress can be calculated under the harmonic force.

Natural frequency and orthonormalized modal shape functions respectively are

$$f_i=\frac{(i\pi)^2}{l^2}\sqrt{\frac{EI}{\rho A}}, \quad \phi_i(x)=\sqrt{\frac{2}{\rho A l}}\sin\frac{i\pi x}{l}$$

Vibration displacements caused by external excitation forces can be denoted by using series of

$$y=q_1\phi_1+q_2\phi_2+q_3\phi_3+\cdots \tag{13}$$

where q_1, q_2, q_3 are unknown time functions, ϕ_1, ϕ_2, ϕ_3 are orthonormalized functions, which are location coordinate functions satisfying boundary condition.

As for simple supported beam, after orthonormalized, series (13) can be denoted as

$$y=\sum_{i=1}^{\infty}q_i\sin\frac{i\pi x}{l} \tag{14}$$

According to d'Alembert principle and virtual work principle, differential equation of time function q_i can be expressed as

$$\ddot{q}_i+\frac{i^4\pi^4a^2}{l^4}q_i=\frac{2Pg}{A\rho l}\sin\frac{i\pi c}{l} \tag{15}$$

where $a^2=\dfrac{EIg}{A\rho}$, c is the distance between excitation point and left support point. Its solution is

$$q_i = \frac{2g}{A\rho} P_0 \sin\frac{i\pi c}{l}\left(\frac{l^3}{i^4\pi^4 a^2 - \omega^2 l^4}\sin\omega t - \frac{\omega l^5}{i^2\pi^2 a(i^4\pi^4 a^2 - \omega^2 l^4)}\sin\frac{i^2\pi^2 at}{l^2} \right) \tag{16}$$

where second item of right side denotes free vibration. Without considering free vibration, equation (14) can be written as

$$y = \frac{2gP_0 l}{A\rho}\sum_{i=1}^{\infty}\frac{\sin\frac{i\pi c}{l}\sin\frac{i\pi x}{l}}{i^4\pi^4 a^2 - \omega^2 l^4}\sin\omega t \tag{17}$$

Suppose α is the ratio ω/ω_1 of excitation frequency to first order frequency, so dynamic displacements response is

$$y = \frac{2P_0 l^3 \sin\omega t}{EI\pi^4}\sum_{i=1}^{\infty}\frac{\sin\frac{i\pi c}{l}\sin\frac{i\pi x}{l}}{i^4 - \alpha^2} \tag{18}$$

Dynamic moment function expression along the beam is

$$M(x,t) = EI\cdot\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{2P_0 l^3 \sin\omega t}{\pi^4}\sum_{i=1}^{\infty}\frac{\sin\frac{i\pi c}{l}\sin\frac{i\pi x}{l}}{\alpha^2 - i^4}\left(\frac{i\pi}{l}\right)^2 \tag{19}$$

Therefore the maximum dynamic stress function expression is

$$\sigma = \frac{M}{W} = \frac{12P_0 l^3 \sin\omega t}{\pi^4 bt^2}\sum_{i=1}^{\infty}\frac{\sin\frac{i\pi c}{l}\sin\frac{i\pi x}{l}}{\alpha^2 - i^4}\left(\frac{i\pi}{l}\right)^2 \tag{20}$$

3. NUMERICAL SIMULATION RESULTS

Model Parameters: Figure 1 is the schematic view of the test beam. And finite element model can be seen in Figure 2. Geometry and physical parameters for the beam can be found in the table 1. In the finite element model, eccentric rotor is replaced by the equivalent mass at the position of 3/8L from left end.

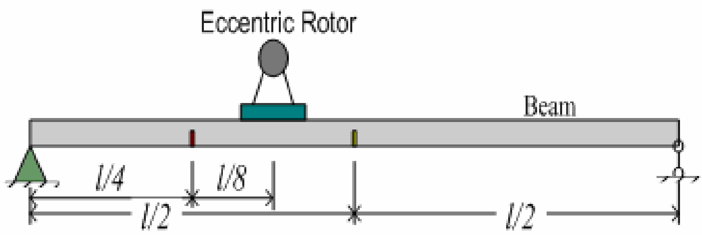

Fig.1 Schematic Views of the test Beam

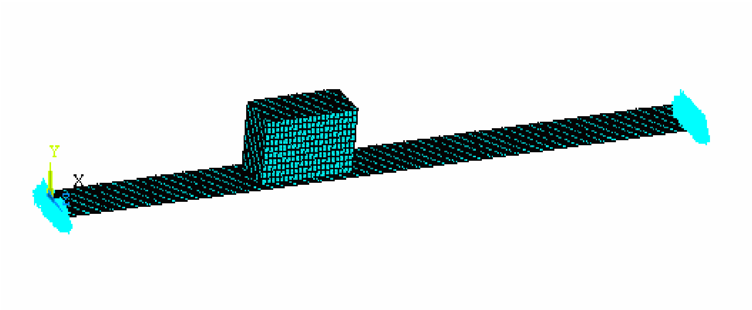

Fig.2 The FE Model of Beam

Table 1. Geometry and physical parameters of the beam with eccentric rotor

Parameter	Value	Parameter	Value
Length	$l = 8.48 \times 10^{-1} m$	Poisson's ratio	$v = 0.30$
Width	$b = 5.0 \times 10^{-2} m$	Mass of eccentric rotor	$M_1 = 3.2kg$
Height	$t = 0.5 \times 10^{-2} m$	Eccentric mass	$m' = 7.4 \times 10^{-2} kg$
Mass density of beam	$\rho = 7.85 \times 10^3 \, kg/m^3$	Eccentric distance	$e = 2.7cm$
Young's modulus	$E = 2.06 \times 10^{11} \, N/m^2$		

By using FEM, first five orders natural frequencies of bending modal of beam with eccentric rotor respectively are 19.446 Hz，78.054 Hz，146.97 Hz，343.179 Hz and 423.836Hz.

Modal Strain-Energy Distribution Rules: i th element modal strain energy corresponding i th-order modal is

$$U_i(x) = \frac{1}{2}\int_0^l EI\left(\frac{\partial^2 \phi_i}{\partial x^2}\right)^2 dx \tag{21}$$

And modal shape is $\phi_i(x) = \sqrt{\frac{2}{\rho Al}} \sin\frac{i\pi x}{l}$，thus element modal strain energy of second order modal is

$$\begin{aligned} U_2(x) &= \frac{1}{2}\int_0^l EI\left(\frac{\partial^2 \phi_2}{\partial x^2}\right)^2 dx = \frac{EI}{2}\int_0^l \frac{2}{\rho Al}\cdot\left(\frac{2\pi}{l}\right)^4 \cdot \sin^2\frac{2\pi x}{l} dx \\ &= \frac{EI}{\rho Al}\cdot\left(\frac{2\pi}{l}\right)^4 \cdot \int_0^l \sin^2\frac{2\pi x}{l} dx = \frac{EI}{\rho Al}\cdot\left(\frac{2\pi}{l}\right)^4 \cdot \left(\frac{1}{2}x - \frac{1}{4\times 2\pi/l}\sin\frac{4\pi}{l}x\right)_0^l \\ &= \frac{EI}{\rho Al}\cdot\left(\frac{2\pi}{l}\right)^4 \cdot \frac{1}{2}l = \frac{8\pi^4 EI}{\rho Al^4} \end{aligned} \tag{22}$$

j th-order element modal strain energy corresponding to i th-order modal is

$$U_{ij} = \frac{1}{2}\int_{a_j}^{a_{j+1}} (EI)_j \left[(\frac{\partial^2 \phi_i}{\partial x^2})^2 dx + (\frac{\partial^2 \phi_i}{\partial x^2})^2 dy + (\frac{\partial^2 \phi_i}{\partial x^2})^2 dz \right] \tag{23}$$

Just considering plane stress state, then

$$\begin{aligned} U_{ij}(x) &= \frac{1}{2}\int_{a_j}^{a_{j+1}} (EI)_j \left(\frac{\partial^2 \phi_i}{\partial x^2}\right)^2 dx = \frac{1}{2}\int_{a_j}^{a_{j+1}} (EI)_j \frac{2}{\rho Al}\left[-\left(\frac{i\pi}{l}\right)^2 \sin\frac{i\pi x}{l}\right]^2 dx \\ &= \frac{i^4\pi^4}{\rho Al^5}\int_{a_j}^{a_{j+1}} (EI)_j \sin^2\frac{i\pi x}{l} dx = \frac{i^4\pi^4 (EI)_j}{\rho Al^5}\left(\frac{1}{2}x - \frac{1}{4i\pi/l}\sin\frac{2i\pi}{l}x\right)_{a_j}^{a_{j+1}} \end{aligned} \tag{24}$$

Therefore Element modal strain energy at 1/4L beam can be calculated as

$$U_{\frac{1}{4}}(x) = \frac{i^4\pi^4 (EI)_j}{\rho Al^5}\left(\frac{1}{2}x - \frac{1}{4i\pi/l}\sin\frac{2i\pi}{l}x\right)_{a_j}^{a_{j+1}} \tag{25}$$

Ignoring the second term in the bracket of equation (25), then element modal strain energy responding to second-order modal is

$$U_{2,\frac{1}{4}}(\frac{1}{4}L) = \frac{2^4\pi^4 EI}{\rho Al^5}\cdot\frac{1}{2}\Delta x \tag{26}$$

where $\Delta x = 1.5mm$. Therefore element modal strain energy of beam at 1/4L and 1/2L responding to second-order modal can be obtained by solving equation (26). Comparison of modal strain-energy curve under effects of static and dynamic are shown in Figure 3.

Static and Dynamic stress: Comparison of nodal stress curve in X-axis under effects of static and dynamic are shown in Figure 4.

From table2, the numerical results show that the stress at 1/4L is more than 1/2L under harmonic excitation, while the static stress of the location at 1/4L is less than that of the location at 1/2L. So the final maximum stress occurs at 1/4L by folding static and dynamic stress.

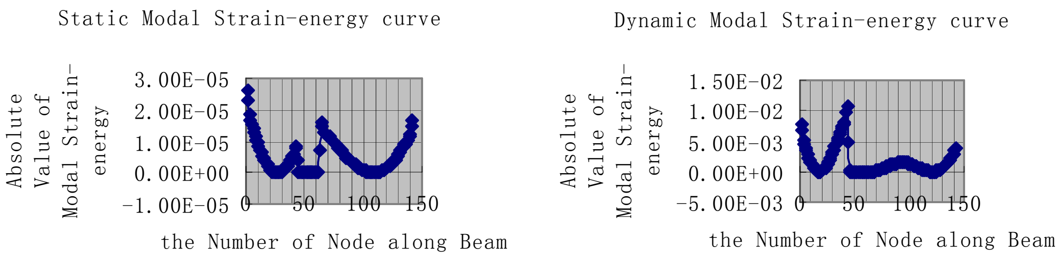

Fig.3 Comparison of Static and Dynamic Nodal Modal Strain-Energy along Beam's Length

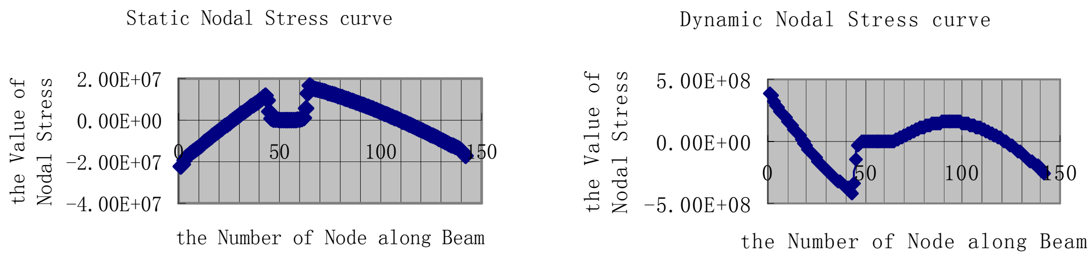

Fig.4 Comparison of Static and Dynamic Nodal Stress along Beam's Longitude

Table 2. Comparison of static, dynamic and folding stress between 1/4L and 1/2L

Slit depth	Static(MPa)		Dynamic stress(Mpa)		Folding stress(MPa)	
	1/4L	1/2L	1/4L	1/2L	1/4L	1/2L
40%	64.17	72.31	128.70	--	192.87	72.31
60%	144.40	162.70	289.57	--	433.97	162.70

** The dynamic stress at 1/2L is close to zero because the excitation frequency almost is equal to the second order natural frequency of the beam. And the final folding stress is calculated in case of invariant area of the cross section.

4. EXPERIMENT VALIDATION

Figure 5 and Figure 6 are photographs of the experiment setup, and the beam is excited by an eccentric rotator. The aim of this experimental design is described as follows: ①Eccentric rotor is parked at 3/8L from left supported-end in order to make static at 1/4L be less than the stress at 1/2L.②Parameters of beam can be seen from table1.③In order to make experimental results easy to observe, structure may be damaged finally, beam is previously slot at the location of 1/4L, 3/8L and 1/2L. The depth of slit is 0.2×10^{-2}.

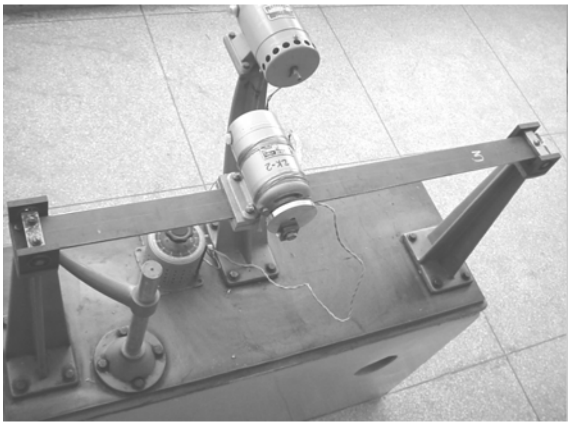

Fig.5 Experimental Setup

Fig.6 Local Damage at 1/4L Position

When the excitation frequency is close to second–order natural frequency, Experimental results show that structure is damaged at 1/4L, but not the static maximum position of 3/8L or not the position of 1/2L. Therefore the rationality and validity of assumption advanced earlier can be illuminated, that is, the stress state of beam not only relies on inherent dynamics characteristics but also undergoing exterior excitation. Complex stress state of static and dynamic coupling forces should be converted to simple stress state of each order modal in order to confirm the maximum stress location range of the beam. Furthermore, potential damage position of structure under dynamic excitation can be predicted, which provides guidance of structure damage location identification and structure optimization design.

5. CONCLUSIONS

From above numerical simulation and experiment study, conclusion can be summed up as follows:

Firstly, the maximum static stress occurs at 3/8L according to traditional static intensity structure design theory. But structure may be damaged at the location of the maximum dynamic stress when excitation frequency is close to the certain order natural frequency, e.g. when excitation frequency equals to the second order natural frequency, the maximum dynamic stress appears at 1/4L.

Secondly, Structural interior stress in operational state depends on its dynamics characters and excitation. The maximum stress location of structure can be obtained by confirming dominant modal and folding static and dynamic stress under the action of harmonic excitation.

Finally, the calculation of structural dynamic stress based on dominant modal obviously is more reasonable. The dominant modal of the same structure alters according to excitation forces, so the maximum stress location should also be different in different operational state.

Therefore, in order to analyze structural dynamic intensity, the complex stress state of folding static and dynamic should be newly measured in different operation. The method has been validated through numerical simulation and experiment, so it has widely practical foreground in the engineering application.

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