

IMPACT MASS ESTIMATION FOR A PLATE TYPE STRUCTURE BY USING SMOOTHED WIGNER-VILLE DISTRIBUTION

Doo-Byung Yoon, Jin-Ho Park, and Young-Chul Choi

Advanced Condition Monitoring & Diagnostics Lab., Korea Atomic Energy Research Institute, 1045 Daedeok-daero, Yuseong-gu, Daejeon, 305-353, Korea yoondb@kaeri.re.kr

Abstract

The LPMS(Loose Parts Monitoring System) is a monitoring system designed to detect, localize, and estimate the loose parts within the primary pressure boundary of a reactor system. When the LPMS gives an alarm signal, it is important to find out not only the impact location but also the mass of the loose part in order to make an accurate evaluation of the structural integrity of the reactor system. Conventionally, center frequency method and frequency ratio(FR) method have been widely used for a mass estimation. However, these methods do not work when the noise level becomes high. Thus a new method to more accurately estimate the impact mass is suggested. The validity of the proposed method is verified through an experiment. The experimental results demonstrate that the proposed method is valid for estimating the center frequency of an impact response signal easily even in a noisy environment. It is expected that the proposed method can be used to enhance the accuracy of the impact mass estimation for a plate type structure.

1. INTRODUCTION

Two typical methods have been used to estimate an impact mass when an elastic object collides with an elastic plate. One is a center frequency method[1,2,3] and the other is a frequency ratio(FR) method[4]. These techniques estimate the impact mass by utilizing the frequency spectra of the impact response signal measured by accelerometers mounted on the plate. These techniques can be successfully applied when the impact response signal has a high SNR(signal-to-noise ratio). However, the acceleration signals obtained from the sensors may contain a background noise and a reflected wave component as well as an impact response signals are normally embedded in the operating noise, such as the noise signal due to a fluid induced vibration of a reactor coolant. In addition, in frequency domain, it is not easy to distinguish between the impact signal and noise component. Therefore, in a noisy environment, the conventional methods may fail to estimate the impact mass.

The objective of this work is to enhance the mass estimation capability for a plate type structure. In order to achieve this objective, a smoothed Wigner-Ville distribution[5] is

proposed to extract the impact response component from the measured acceleration signal. An experiment is also being conducted to verify the usefulness of the proposed technique.

2. CONVENTIONAL METHOD FOR IMPACT MASS ESTIMATION

Figure 1 shows the conventional method for impact mass estimation: center frequency method and frequency ratio(FR) method.



(a) Center frequency method

(b) Frequency ration method

Figure 1. Conventional mass estimation technique.

The center frequency f_c for a measured signal s(t) is defined as[2]

$$f_c = \sum_i R_{fi} f_{ci} / \sum_i R_{fi}$$
(1)

where, *i* is the number of frequency intervals, *f* is a frequency, f_{ci} is a center frequency at the *i*th interval, and R_{fi} is a ratio of S(f) to $S_n(f)$ at the *i*th frequency interval. S(f) is an auto-power spectral density of a measured impact response signal s(t), and $S_n(f)$ is an auto-power spectral density of a background noise.

Based on the center frequency of the measured impact response signal from Equation (1), the impact mass can be estimated by the Hertz's impact theory[3] as follows:

$$T_{d} = \pi \cdot k_{h} \cdot m^{0.4} \cdot V_{0}^{-0.2} \cdot R^{-0.2}$$
(2)

where, $T_d = 1.6/(2 \cdot f_c)$, k_h is a material constant which is a function of the Young's modulus and the Poisson's ratio of the plate and impacting object[3], *m* is a mass of the impacting object, V_0 is the velocity of the impacting object, and *R* is the radius of the curvature at the contact point of the impacting object. Next, the FR(frequency ratio) for a measured signal s(t) is defined as[4]

$$FR = \frac{\int_{\Delta f, low} S(f) df}{\int_{\Delta f, high} S(f) df}$$
(3)

where *f* is a frequency, and S(f) is an auto-power spectral density of a measured signal s(t). The Δf_{high} denotes the higher frequency band, and Δf_{low} denotes the lower frequency band.

Then the FR becomes a function of the impact mass because the signal power in the lower frequency range is dependent on the impact duration which is also subjected to the impact mass. Accordingly, the impact mass can be obtained by using the 'FR versus mass' diagram previously obtained by an experiment[4].

It is noteworthy that both the center frequency and FR methods assess the impact mass by utilizing the frequency spectra of the impact response signal. Thus these techniques are only reliable when the multi path effect of the propagating response signal is negligible and the signal has a high signal-to-noise ratio(SNR). Actually, the measured response signals obtained from operating structures contain a background noise, a resonance component of a sensor mounting and reflected waves, etc. Therefore, in a noisy environment, the conventional methods using one dimensional power spectra may fail to estimate the impact mass. To overcome this, we propose a new technique for an impact mass estimation by using smoothed Wigner-Ville distributions. The next section describes the details of the proposed method.

3. PROPOSED METHOD FOR IMPACT MASS ESTIMATION

Wigner-Ville Distribution(WVD) W(t, f) and its ambiguity function $A(\xi, \tau)$ are defined as[5,6]

$$W(t,f) = \int_{-\infty}^{\infty} z(t+\frac{\tau}{2}) z^* (t-\frac{\tau}{2}) e^{-j2\pi f\tau} d\tau$$
(4)

and

$$A(\xi,\tau) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W(t,f) e^{-j(2\pi\xi t - 2\pi f\tau)} dt df , \qquad (5)$$

respectively. Where, z(t) is the analytic function of a measured signal s(t), τ is a time delay, j means $\sqrt{-1}$ and * denotes a complex conjugate. By applying a two dimensional smoothing function $\Phi(\xi, \tau)$ to the ambiguity function $A(\xi, \tau)$, the general form of a smoothed WVD can be obtained as[5,6]

$$W_{SM}(t,f) = \int_{-\infty-\infty-\infty}^{\infty} \int_{-\infty-\infty-\infty}^{\infty} \Phi(\xi,\tau) z(u+\frac{\tau}{2}) z^*(u-\frac{\tau}{2}) e^{-j2\pi(\zeta t+f\tau-\zeta u)} d\zeta du d\tau .$$
(6)

If we let a smoothing function $\Phi(\xi, \tau) = e^{-\zeta^2 \tau^2 / \sigma}$, then the corresponding smoothed WVD is represented as[5]

$$W_{SM}(t,f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} z \left(u + \frac{\tau}{2} \right) \cdot z^* \left(u - \frac{\tau}{2} \right) \cdot \sqrt{\frac{\pi\sigma}{\tau^2}} \cdot \exp\left[-\frac{\pi^2 \sigma (t-u)^2}{\tau^2} \right] \cdot e^{-j2\pi j\tau} du d\tau \cdot$$
(7)

where, σ is a weighting factor in the ambiguity domain (ξ, τ) : a small value means an increase of the smoothing effect and a large value means a decrease of the smoothing effect.

Figure 2 shows an example of a time-frequency analysis result of dispersive impact response. It has already been reported that the WVD of an impact response signal(the lowest mode of flexural wave) on an elastic plate shows a monotonic curve whose power is always located within the 2nd and 4th quadrants in the ambiguity domain, however, the random noise terms are scattered in every quadrant. Based on this property, we can weight the value of the weighting factor in each quadrant differently to exclude the noise components in the 1st and 3rd quadrants. Figure 3 illustrates an asymmetric exponential window in the ambiguity domain for reducing the noise effect and the cross terms, simultaneously[5].



Figure 2. Wave propagation due to impact: dispersive flexural wave component is represented as a curved line with a negative slope in the time-frequency domain.



Figure 3. An asymmetric exponential window for removing the noise components located inside the 1st and 3rd quadrants in an ambiguity function domain.

Figure 4 summarizes the proposed method for impact mass estimation for a plate type structure. As shown in Figure 4, the smoothed WVD with an asymmetric exponential window can be used not only for reducing the noise effect in the signal, but also for estimating the center frequency of the direct path wave group of the impact response signals. Eventually, this makes it possible to enhance the mass estimation capability.



Figure 4. Proposed method for impact mass estimation.

4. EXPERIMENTS

An experiment has been performed to investigate the validity of the proposed time-frequency analysis technique for estimating an impact mass on a plate. Figure 5 represents the experimental setup and schematic of the data acquisition and signal analysis of the impact response signals for an elastic square plate($2m \times 2m$, 10t, SUS304). Four sensors were used for measuring the acceleration signals generated by the impact of a steel balls(6.5, 9.3, 17.6, 36.8, 67.6, and 131.6 gram). The impact is generated by dropping the steel ball onto the plates' surface as shown in Figure 5.



Figure 5. Experimental setup for impact mass estimation: (a) View of the experimental setup,(b) Schematic of the experimental setup, where the locations of two impact point(p1 and p2) and four accelerometers(S1, S2, S3 and S4) are indicated.

Figure 6 (a) represents a WVD of the measured acceleration signal(S3 sensor) due to an impact with a 36.8 gram steel ball. In Figure 6 (a), one can see that dispersive flexural wave component is represented as a curved line with a negative slope in the time-frequency domain. In addition, one can easily identify the center frequency not only in the time-frequency domain but also in the frequency domain.

A simulation is performed to verify the applicability of the proposed method in a noisy environment. For this purpose, a zero mean Gaussian random noise (σ =1) is artificially mixed with the acceleration signal measured at the third sensor(S3). Figure 6 (b) shows a WVD of the signal containing a Gaussian random noise with a SNR equal to 0.5. The SNR is a variance ratio of a measured signal to Gaussian noise. In Figure 6 (b), it is not easy to find out the impact response component and the center frequency due to the noise. An asymmetrically weighted exponential function used for reducing the noise effect is depicted in Figure 6 (c). As a result, the noise rejection capability of the proposed method(smoothed WVD) is demonstrated in Figure 6 (d). By comparing Figure 6 (d) to Figure 6 (b), one can see that the noise term has been remarkably reduced and therefore it is possible to identify an impact response component in the time-frequency domain.



Figure 6 Time-frequency analysis results that demonstrate the noise rejection capability:
(a) Wigner-Ville distribution of the measured impact signal which does not contain noise,
(b) Wigner-Ville distribution of the impact signal which contains Gaussian random noise with a SNR equal to 0.5, (c) Smoothing function applied in ambiguity domain for reducing noise,
(d) Smoothed Wigner-Ville distribution obtained by applying the smoothing function to the noisy signal shown in Figure 6 (b).

Table 1 summarizes the comparison results of the mass estimation between the conventional center frequency method and the proposed method. In Table 1, it is observed that the errors of mass estimation increase as the SNR decreases. In addition, one can observe that overall estimation error of the smoothed WVD technique is relatively smaller than that of the conventional method. It is noteworthy that the smoothed WVD technique provides a useful means for an impact mass estimation, even in a noisy environment.

True mass (gram)	Estimated center frequency (kHz)				Estimated mass (gram)			
	Conventional method		Proposed method		Conventional method		Proposed method	
	Without noise	With noise SNR=2	Without noise	With noise SNR=2	Without noise (error)	With noise SNR=2 (error)	Without noise (error)	With noise SNR=2 (error)
6.5	16.3	15.1	18.2	18.3	7.9 (22%)	10.0 (54%)	5.8 (11%)	5.7 (12%)
9.3	15.4	14.6	15.6	15.8	9.5 (2%)	11.1 (19%)	9.1 (2%)	8.8 (5%)
17.6	12.4	12.2	12.9	12.8	18.0 (2%)	18.9 (7%)	16.0 (11%)	16.4 (7%)
36.8	10.2	10.5	10.4	10.5	32.3 (12%)	29.6 (20%)	30.5 (17%)	29.6 (20%)
67.6	8.3	9.1	8.1	7.8	59.9 (11%)	45.5 (33%)	64.5 (5%)	72.3 (7%)
131.6	7.2	8.3	6.7	6.8	91.9 (30%)	60.0 (54%)	114.0 (13%)	109.1 (17%)

Table 1 Result of mass estimation by using center frequency method(S1 sensor)

* SNR = variance of measured signal / variance of Gaussian random noise

* Error = |Estimated mass- True mass| / True mass

5. CONCLUSIONS

In order to enhance the impact mass estimation capability for a plate type structure, a time-frequency analysis technique by using a smoothed Wigner-Ville distribution was proposed. Also, experiments were carried out to verify the validity of the proposed method. The results showed that even in a noisy environment, the smoothed Wigner-Ville distribution can provide a more reliable means for estimating an impact mass of a steel ball for a plate type structure. Therefore, it is expected that the reliability of an impact mass estimation could be enhanced when the proposed time-frequency analysis technique is applied to an impact mass estimation problem for a plate type structure.

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