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UPDATING AN ELASTODYNAMIC MODEL OF GEAR PUMPS TO HELICAL GEAR PROTOTYPES

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Abstract

A non-linear kineto-elastodynamic model for the dynamic analysis of external gear pumps for automotive applications has been presented and experimentally assessed in previous works. The model takes into account the most important phenomena involved in the operation of gear pumps with spur gears, there being gear meshing, pressure distribution around the gears and journal bearing behaviour. In this paper this model has been updated for the application to a new virtual pump prototype with helical gears in order to evaluate gear accelerations and dynamic forces before the hard prototype is available. The results of the simulations referring to the new pump are compared with the simulation results of the previous pump in terms of dynamic forces and gear accelerations. Finally, the model is also used in order to determine the causes of discontinuities in the dynamic forces and acceleration peaks.

1. INTRODUCTION

It is well known that the use of assessed mathematical models make it possible to predict the effects of design modifications, so reducing the number of tests required for design improvement. In this context, the authors have presented in previous papers a numerical model in view of studying the dynamic behaviour of an external gear pump for steering systems.

In particular this non-linear lumped-parameter model [1], [2], [3], [4] takes into account the variability of the pressure distribution on gears, the hydrodynamic bearing behaviour, the parametric excitations due to the time-varying meshing stiffness, in the case of spur gears, and the tooth profile errors, the effects of the backlash between meshing teeth, the lubricant squeeze and the possibility of tooth contact or separation on both lines of action; the equations of motion are numerically integrated. In order to reduce the integration time, the average positions of the journal axes into the bearings were previously estimated. Thus, the variable pressure forces on gears can be approximately estimated from this average axis positions, before starting the numerical integration, obtaining an important reduction of the integration time. It was verified that this approach gives accurate results. The model has been implemented in Matlab and Simulink [5]; the implementation of the model can be divided in two parts: in the first part implemented in Matlab, all the data dealing with gears, case and bushes are introduced in order to perform the calculus of the stationary centre position, while in the second part implemented in Simulink, the equations of motion are numerically integrated using the data already calculated in Matlab. The model has been validated [6] by comparison with experimental vibration data, concerning a wide range of operational conditions.

In this work, the assessed model is adapted to a new virtual prototype, namely *4cc/rev* in view of evaluating gear accelerations and dynamic forces before the hard prototype is available. This new pump has helical gears, while the previous one has spur gears. The use of helical gears introduces an axial force that must be taken into account for the definition of the pressure compensation. Moreover, if the helical angle increases, the inlet and outlet chambers will be connected for more time reducing the hydraulic efficiency. As a consequence, the helical angle can not be higher than six degrees for this kind of application. This situation allows us to assume that the axial forces are negligible and therefore the gear pump can be modelled using the planar model of the previous gear pump; the assumption of planar model is also acceptable for the new *4cc/rev* pump due to the helix angle of 4°. Thus, the original model described in [1], [2], [3], [4] can be used in order to estimate the dynamic behaviour of this new pump.

In particular, in Section 2 a brief geometrical description of the new prototype, highlighting the main differences between the previous pump (namely GENB) and the new one is presented. Section 3 shows how the original model has been modified in order to perform the dynamic analysis of the new pump. In Section 4 the results of the simulations are shown, comparing the behaviour of the new pump and the GENB pump, in terms of pressure forces and gear accelerations; moreover the events that determine discontinuities in the dynamic forces and in the acceleration peaks are presented and discussed.

2. PUMP DESCRIPTION

The gear pumps under study have an usual configuration with two twin gears, which are assembled by a couple of lateral floating bushes packed inside a close tolerance case. Table 1 summarizes the main characteristics of the new *4cc/rev* and of the GENB pump. The main difference of the new *4cc/rev* pump compared to the GENB pump refers to gears: in fact the twin gears have 11 teeth with helix angle of 4°.

Table 1. Geometrical characteristic of the pumps.

	Pump *GENB*	Pump *4cc/rev*
Number of teeth	z=12	z=11
Gear module [mm]	$\widehat{m} = 1.15$	$\widehat{m} = 1.7$
Helix angle on the pitch circle	β=0°	β=4°
Face width [mm]	b=12.1	b=14.9
Pressure angle [deg]	α =20	α =22
Pressure angle in working conditions[deg]	α_w =27.727	α_w =24.142
Contact ratio	$\tilde{\varepsilon} = 1.4$	$\tilde{\varepsilon} = 1.7$
Angles defining the ending of the isolated vanes for gear 1 [deg].	$\Phi_{A1,}$ =339.68°	Φ_{A1} =334.82°
Angles defining the beginning of the isolated vanes for gear 1 [deg].	Φ_{B1} =200.32°	Φ_{B1} =205.18°

The operating principle of external gear pumps is very simple. The fluid is carried around the outside of each gear from the intake to the discharge side within the space included between two subsequent gear teeth, the case and the lateral floating bushes. As the gears turn, these isolated spaces increase progressively their pressure up to the high pressure. In the gear meshing area, when two tooth pairs come in contact, a trapped volume arises and could undergo a sudden volume reduction and consequently a violent change in its pressure. To avoid this, the trapped volume is put in communication with the high or low pressure chambers. That is the role of the relief grooves milled in the internal face of lateral bushes whose shape and location are so important in the resulting dynamic behaviour. This pump works with a pressure ranging from 3.5 to 100 bar and angular speed ranging from 1500 to 3400 rpm.

3. MODEL UPDATING

The previous model must be modified in order to study the dynamic behaviour of the new prototype: in particular, three modifications in the original model have to be performed: the geometrical data have to be updated, the meshing stiffness formulation for helical gears has to be introduced, the phase difference between the lower and upper tooth surfaces in contact with bushes has to be taken into account.

Since the code has a pre-processing module for data introduction, the geometrical data updating of the original model to a new prototype becomes simple and fast.

The consequence of the phase difference between the lower and upper tooth surfaces of helical gears can be explained as follows. The beginning of the connection between the trapped volume and the inlet chamber happens at the same instant as in case of spur gears. On the other hand, the end of the connection with the outlet chamber is postponed. Ths delay is due to the helix angle. In fact, a spur gear can be studied as a plane system because its behaviour is the same in all the axial sections of the gear. Otherwise, in helical gears, two different axial sections of the gear are always out of phase and between the tooth upper surface and the tooth lower surface, the phase difference is maximum. Such a phase difference is directly dependent on the helix angle. With reference at Figure 1a, when the contact point on the tooth upper surface intersects the relief grooves, the connection between the trapped volume and the outlet chamber is closed only in the upper surface, but it is still open in the lower one, till the instant when the contact point on the tooth lower surface intersects the relief grooves as shown in Figure 1b. In order to model such a behaviour, i.e in order to postpone the end of the connection with the outlet chamber, the base pitch of the gears has been artificially increased by the quantity ΔP_b:

$$\Delta P_b = b \cdot \tan\left(\beta_b\right) = 14.9 \cdot \tan\left(3.708°\right) = 0.9656 \ [mm] \tag{1}$$

where β_b is the helix angle calculated on the base circle:

$$\beta_b = \tan^{-1}\left(\frac{r_b \cdot \tan\left(\beta\right)}{R_p}\right) \tag{2}$$

where r_b and R_p are the base and pitch radius.

With reference to Figure 2, we can indicate as seal line the segment of the line of action limited by the intersections with the relief grooves ($\overline{CD} = \dfrac{B}{\cos(\alpha_w)}$, where B is the relief grove length); the percentage difference *%diff* between the seal line and the base pitch P_b indicates the percentage of the meshing period in which the inlet and outlet chambers are I communication; for spur gear pumps, this percentage is:

$$\%diff = \frac{\frac{B}{\cos(\alpha_w)} - P_b}{P_b} \cdot 100 \tag{3}$$

For helical gear pumps, the base pitch increment (1) has to be added, for taking into account the phase delay between the gear surfaces. Considering that for the GENB and *4cc/rev* pumps the base pitch are 3.395 and 4.9623 mm respectively, the contemporaneous communication between the inlet and outlet chamber increases itself and becomes:

$$\%diff_{4cc/rev} = \frac{\frac{B}{\cos(\alpha_w)} - (P_b + \Delta P_b)}{P_b} \cdot 100 = \frac{\frac{5.25}{\cos(24.142°)} - (4.9623 + 0.9656)}{4.9623} \cdot 100 = -3.52\% \tag{4}$$

While for the GENB is:

$$\%diff_{GENB} = \frac{\frac{B}{\cos(\alpha_w)} - P_b}{P_b} \cdot 100 = \frac{\frac{2.9}{\cos(27.727°)} - 3.395}{3.395} \cdot 100 = -3.50\% \tag{5}$$

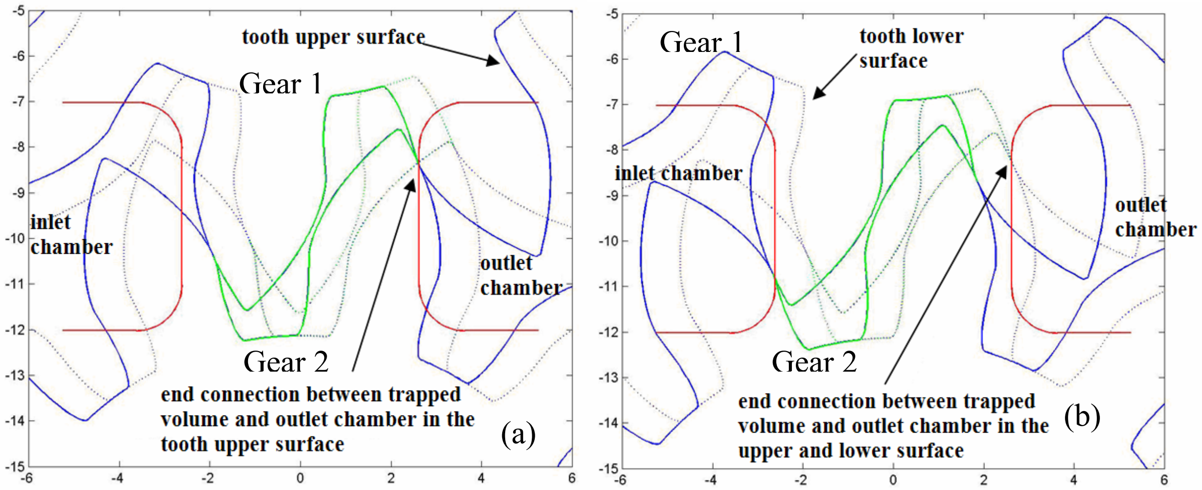

Figure 1. (a)Ending of the connection between trapped volume and outlet chamber in the tooth upper surface, (the connection is still open in the lower surface) (in dotted line) and (b) Ending of the connection between trapped volume and outlet chamber in the tooth upper and lower surfaces.

A similar problem happens when a new vane becomes isolated in the inlet side. For gear 1, at the inlet port, a vane becomes isolated later with respect to a spur gear, due to the phase difference between the upper and lower surfaces of the tooth in helical gears. In fact, taking Figure 2a as a reference, the first vane becomes completely isolated at the inlet side not when the tooth upper surface goes beyond the line identified by the angle Φ_{B1} but only when the tooth lower surface goes beyond this line (see Figure 2b). Therefore, in order to take such a delay at the inlet port into account, the angle Φ_{B1} has been decreased by a quantity equal to the above phase difference. The same happens for gear 2 at the inlet port. So in the model, the angle Φ_{B2} has been decreased by the same quantity as the angle Φ_{B1}. At the output side, the behaviour of the helical gears and spur gears is similar, therefore no shifting in terms of angle has to be applied.

The last modification that has to be taken into account in the model regards the meshing stiffness. Among the stiffness approaches available in literature the Cai's formulation [7] [8] was selected. Such a theory proposes to consider the helical gears as a plane system and introduces the helical effect by a stiffness function smoother than the stiffness function in case of spur gears:

$$\widetilde{K}(t) = \bar{k} \cdot \exp(C_\beta \cdot |\bar{s}(t)|^3) \tag{6}$$

where $\bar{k}$ is the stiffness value at the pitch point [N/m] accounting the gear width , C_β is a constant linearly related to the helix angle β and $\bar{s}(t)$ is a dimensionless time-dependent function that defines the position along the line of action. The complete formulation is specified in [7]. Then, the total stiffness of the tooth pair is obtained considering the Hertzian stiffness as well [1] [2].

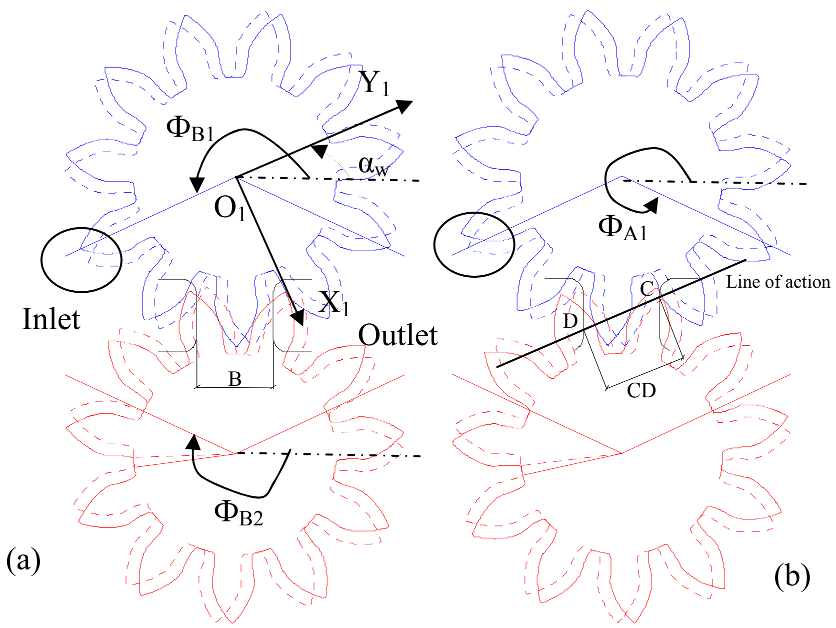

Figure 2. (a) The upper surface of gear 1 (in blue) goes beyond the Φ_{B1} line and (b) the lower surface of gear 1 goes beyond the Φ_{B1} line and the first vane becomes completely isolated (the lower surface is in dotted line and the upper are in solid line.

4. SIMULATION RESULTS AND DISCUSSION

Hereafter the model has been used in order to identify the physical events that determine discontinuities on forces and torques and therefore on the dynamic response of the system (gear accelerations). Obviously, the more relevant events on the dynamic response are those related with discontinuities on forces and torques. The events due to dynamic phenomena are: change in number of meshing teeth (from 1 to 2 and 2 to 1), change in number of isolated vanes carrying fluid for gear 1 and 2 (from 5 to 6 and 6 to 5) and finally beginning and ending of the trapped volume connection with the input and output volume.

Since the hard prototype of the *4cc/rev* is not available, obviously, the profile error trend can not be measured. Therefore hereafter the simulation results are obtained neglecting in the equations of motion [1], [2], [4] the terms related to profile errors. Figure 3 shows the events causing high peacks on gear accelerations at the operational condition of 3350 rpm and 34 bar. It can be noted that the phenomena that produce the most important dynamic effects are the increment in the number of isolated vanes (from 5 to 6) and the events related to the trapped volume. Variable meshing stiffness has a notable effect as well, but not as important as the trapped volume.

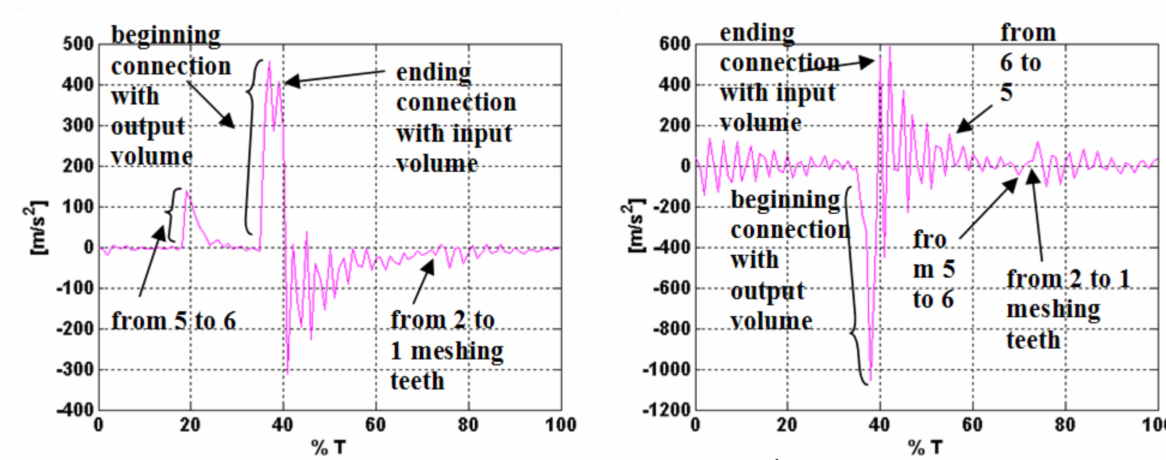

Figure 3. Accelerations of gear 1 (left) and gear 2 (right) in X_1-direction (reference frame of Figure 2), over one meshing period; operational condition of 3350 rpm and 34 bar.

In the following, the simulation results of the *4cc/rev* pump and of the *GENB* pump will be compared in terms gear accelerations and pressure forces (Figure 4 and Figure 5) at the operational condition of 2000 rpm and 90 bar. The comparison in terms of pressure distribution (not shown hereafter) does not highlight meaningful differences while the mean level of the pressure forces (Figure 4) and torques is higher (in absolute value) for the *4cc/rev* pump. Nevertheless, the gear accelerations (Figure 5) have similar peak values even if the maximum level of the GENB accelerations is higher than in the *4cc/rev* pump. It is worth noting that the accelerations trends are different because the events that determine the force discontinuities occur in different instants, due to the different geometry of the new prototype.

Finally, since the helix angle β_b is 3.708° for the *4cc/rev*, the meshing axial force $f_{mg,axial}$ is about 6% of the meshing force along the line of action in the X1-Y1 plane, as computed by the model (f_{mg}) :

$$f_{mg,axial} = f_{mg} \cdot tg(\beta_b) = 0.0648 \cdot f_{mg} \tag{7}$$

where f_{mg} is the meshing force and $f_{mg,tang}$ and $f_{mg,axial}$ are its tangential and axial component, respectively. Furthermore, the driven gear is unloaded in axial direction because the axial pressure force equals in modulus the axial meshing force having opposite sense; on the contrary the driving gear is loaded in axial sense by $2 \cdot f_{mg,axial}$ because the axial pressure force has same sense than the axial meshing force (see details in [1]).

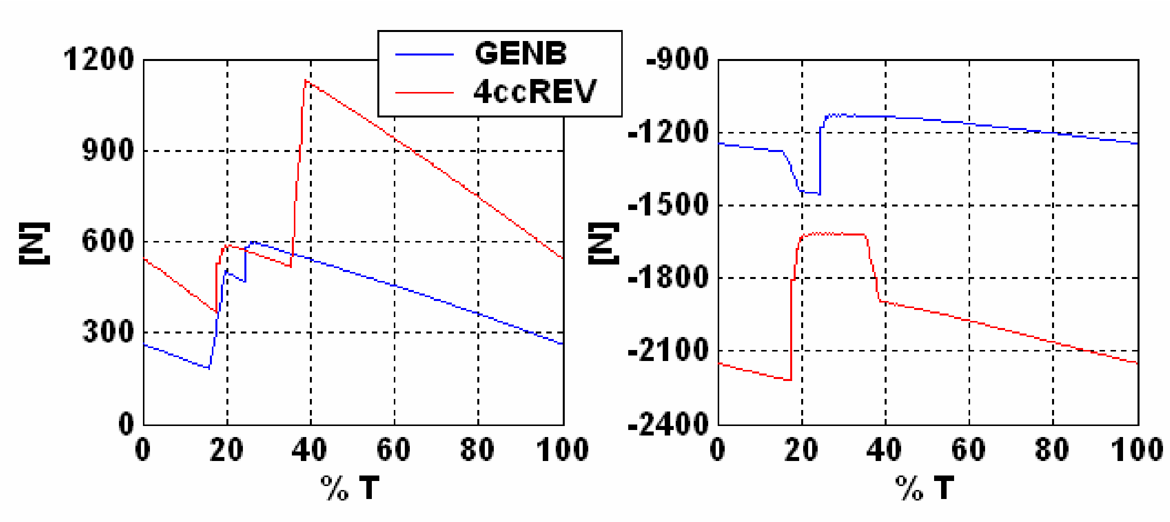

Figure 4. Pressure forces on gear 1 in X_1 (left) and Y_1(right) directions at operational condition of 2000 rpm and 90 bar: in blue line for the GENB pump and in red line for the *4cc/rev* one.

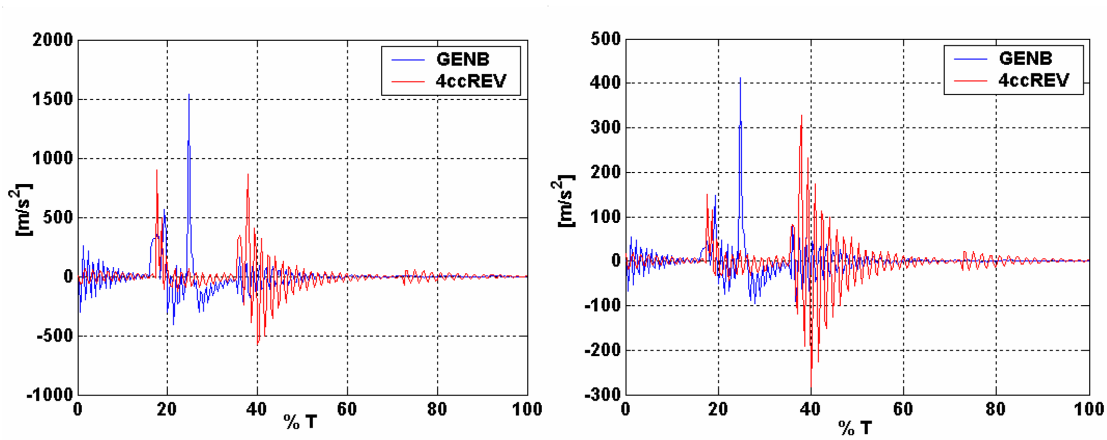

Figure 5. Accelerations on gear 1 in X_1 (left) and Y_1 (right) directions at operational condition of 2000rpm and 90 bar: in blue line for the GENB pump and in red line for the *4cc/rev* one.

4. CONCLUSIONS

An existing lumped parameter model has been updated for the application to a new virtual pump prototype with helical gears (namely *4cc/rev*) in order to evaluate gear accelerations and dynamic forces before the hard prototype is available. The model updating has mainly involved the meshing stiffness formulation and the delay due to the phase difference between the lower and upper surfaces of the gears. Since the model implemented in Matlab/Simulink environment in a modular way, the modification of the code was easy.

The simulation results show that the model makes it possible to estimate dynamic responses and forces taking place in the gear pumps, as a function of working conditions. On the basis of the numerical simulations the variable meshing stiffness has a minor contribution on the discontinuities with respect to the discontinuities due to the change in the number of isolated volumes carrying fluid and to the creation of a trapped volume. Moreover, the mean level of the pressure forces is higher (in absolute value) for the *4cc/rev* pump, however the gear accelerations of the two kinds of pumps have similar peak values. Due to the helical gears, the *4cc/rev* pump is loaded in axial sense, with a load of about 12% of the meshing force for the driving gear.

Thus, the updated model can be a very useful tool in prototype design and in order to develop design improvements for NVH optimization.

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