ACOUSTIC ANTENNA DESIGN GENERAL GUIDELINES FOR 
BEAMFORMING NOISE SOURCE LOCALIZATION
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Abstract

The design criteria of phased microphone array for moving sound sources localization and
tracking by using beam-forming techniques are presented. A detailed analysis on how to
evaluate the main array characteristics (microphones layout, number and spacing, antenna
shape and dimension) from the requested performance (maximum range, angular resolution,
maximum and minimum target sound frequency) has been carried out in order to give all the
elements necessary to properly design the acoustic antenna. A dedicated code has been
developed within MATLAB environment with a user friendly graphic interface. Starting from
the acoustics requirements of the array transducers, the code allows the design of the antenna
and evaluates the main array characteristics, also showed with comprehensive graphical
outputs. Different arrays have been designed and analyzed. The study carried out in this work
is a preliminary investigation necessary to estimate the costs and the reliability of such
methodology for the realization of an acoustic antenna for far field noise source localization.

1. INTRODUCTION

Today microphone arrays are a promising technology in the field of sound source localization.
These systems may be electronically steered to boost a desired source signal while reducing
interfering sources and environmental noise. This paper deals with the general guidelines to
design an array of acoustic sensors of definite features capable of detecting sound source in
far field. The basic principle of microphone is to convert variations in air pressure into
equivalent electrical variations, in current or voltage. Most microphones can be placed in one
of two main groups: omni-directional and directional. The basic directivity patterns, shown in
Fig. 1, include omni-directional (a), cardioid (b), supercardioid (c), hypercardioid (d) and bi-
directional or figure of eight (e). Typically a single transducer has a directivity pattern
relatively wide and provides low values of directivity (gain). Some applications require
acoustic sensor with a high directivity and the need to measure the sound emitted from a
source both in presence of other relevant sources and in a noisy environment. One way to
obtain very directive characteristics and very high gains is to form an assembly of transducers
in a geometrical configuration. This new sensor, formed by multielements, is referred to as an array [1].

![Figure 1: Microphone basic directivity patterns.](image)

A microphone array is made with a number of microphones that work together to produce a defined directivity pattern. These systems may be electronically steered to enhance a desired source signal while attenuating interfering sources and ambient noise. Each array microphone is called array element. The elements can be similar or different; in many practical cases they are identical. This is not necessary, but it is often convenient, simpler and more practical. The array antenna main feature is that varying the element number, distance and geometrical configuration, it is possible to design a definite directivity diagram with a narrow main directivity lobe. Furthermore it is possible to acoustically scan an area electronically steering the array in different directions. From this considerations come out the enormous potential and large application field of microphone array.

### 2. ARRAY DESIGN

The main array parameters to take in account in the array design are:

- array pattern with, Figg. 2a, 2b:
  - the angle between the first two null near the main lobe of the array pattern, known as *angular resolution*;
  - the angle between the two points on the left and on the right of the main lobe, where the measured sound power is half the maximum value, that is the *angular resolution at -3dB*;
  - the Maximum Sidelobe Level (MSL), i.e. the array dynamic range. This dynamic range is given by the difference between the level of the main lobe and the maximum level of the side lobes.
- array size (diameter);
- microphone layout and number
- measurement distance (range);

![Fig. 2a Grid array pattern with angular resolution at -3dB (red arrow); MSL (green arrow) and angular resolution (orange arrow)](image)

![Fig. 2b Grid array pattern with grating lobes.](image)
These quantities are strongly linked to the target sound source characteristics:

- resolution and minimum frequency of target source affect the array diameter;
- maximum frequency of target sound source affects the microphones distance and number;
- array signal to noise ratio affects the maximum measurement distance;
- microphone spatial arrangement affects array patterns and MSL.

3. ARRAY SHAPE

The array geometry optimization is the principal aspect to take in account in order to improve the array performance, because it determines the array pattern and the main parameters, such as the maximum sidelobe level profile, which defines the antenna capability to minimize ghost images. Geometrically, most arrays fall into one of three designs: linear, planar and spherical. Perhaps the most straightforward design is the linear microphone array. In [3], general linear sensor arrays for beamforming are elaborated. A more symmetric and compact configuration is the planar microphone array such as the grid array, Fig. 3(A), with constant microphone spacing d. With grid array at frequencies above fmax, there is the problem of the grating lobes, Fig. 2b, that introduce worrying ghost images in the source map.

Furthermore to design a grid array for high frequencies of analysis it is necessary to decrease the grid spacing and this requires, for a fixed dimension, a relevant number of sensors. A practicable way of constructing a regular array with large working frequency bandwidth is the cross-array, Fig. 3(B), which is a combination of two uniform linear arrays. The cross-array shows high sidelobes only along the sensor directions and good sidelobe attenuation in all other directions. There are specialised beam-forming algorithms [4], which make use of Hanning weighting, to minimize the ghost-image problem caused by the array structure, but this is done with a degradation of the resolution.

The aliasing problem due to the repeated space sampling is the most important limitation of regular arrays. This problem can be avoided using random geometry, Fig. 3(C). The array patterns of irregular arrays have, in general, gradually increasing maximum sidelobe levels. However, the random arrays have a complicated layout that gives problems in the manufacturing and operational process. Furthermore the numerical optimization of random array pattern isn’t easy to perform for the large number of free parameters.

As pointed out formerly, to improve the array resolution, particularly at the lower frequencies it is necessary to increase antenna size. Another way to increase the resolution with a compact microphone array consists in expanding the array in the third dimension. Thus a simple shape to satisfy this condition can be simply to put the microphone on the surface of a sphere. Spherical microphone arrays, Fig. 3(E), are recently becoming the subject of some study as they can capture a 3D soundfield and provide direction invariant beam patterns in all directions [5].
4. ARRAY RESOLUTION AND SIZE

The resolution of a beamformer defines its ability to distinguish waves incident from directions close to each other [2]. In far field, resolution is defined as the smallest angular separation between two plane waves that allows them to be separated. Let’s consider two plane waves with wave number vectors $k_1$ and $k_2$, with a magnitude $k$, incident on an array with array pattern $W$ (see Figure 4). According to the Rayleigh criterion, two directions can be just resolved when the peak of $W(k - k_2)$ falls on the first null of $W(k - k_1)$.

Assuming that the angular separation between $k_1$ and $k_2$ is small, it can be shown [6] that the angular resolution is given by:

$$ R(\alpha) = \frac{A}{\cos^3 \alpha} \frac{z}{D} \lambda $$

(1)

where $D$ is the array diameter and $A$ is a constant related to the shape of the array. From the above relation it can be noticed that resolution is proportional to the wavelength and improves with larger diameters but becomes poor with an increasing array to object distance. The ratio between the on-axis ($\alpha = 0$) and off-axis resolution is given by:

$$ \frac{R(\alpha)}{R_{\text{Axis}}} = \frac{1}{\cos^3 \alpha} $$

(2)

This ratio is depicted in the figure below. It can be derived that for angles of incidence more than 30°, the resolution becomes more (and so, it becomes worse) 50% greater than the on-axis resolution. That’s why the usual maximum opening angle of a beaformer is taken as 30°. In the following we’ll define $\theta_{\text{max}}$ this angle. The first array design parameter is the minimum angular resolution, $\alpha^*$. Thus, the linear resolution evaluated at a distance $z$ from the array is:

$$ R = z \tan(\alpha^*) $$

(3)

In a design condition, we have to evaluate the linear resolution in the worse conditions, that is at the maximum range of the array ($L_{\text{max}}$):

$$ R = L_{\text{max}} \tan(\alpha^*) $$

(4)

If we put $\alpha^* = 5^\circ$ and $L_{\text{max}} = 9000m$, the resolution $R$ is 787m.

By using this value, from eq. (1) it can be evaluated the array diameter:
If we choose an array with a linear aperture \((A=1)\), a maximum signal frequency of \(2500\text{Hz} \) \((\lambda = 14 \text{ cm})\) and \(\alpha = \theta_{\text{max}}=30^\circ\), we obtain an array diameter of \(2.11\text{m}\). So, if we want to resolve two acoustic waves separated by \(5^\circ\) starting from a direction defined by an angle of \(\alpha = \theta_{\text{max}}=30^\circ\) and at a range of \(9000\text{m}\), we have to choose an antenna diameter of \(2.11\text{m}\). In the Figure 6, it’s depicted the array diameter as a function of linear resolution at a fixed frequency \((2500\text{Hz})\) and for different ranges.

As stated previously, in order to have a good (small) resolution, we have to use an antenna with a great size. It is also possible to analyze array diameter as a function of the linear resolution at a fixed range \((9000\text{m})\) and for different frequencies (see Figure 7).

The second array design parameter is the microphone distance. Assuming to have a constant speed of sound, this parameter depends on the maximum frequency \((\text{related to the minimum wavelength by: } f_{\text{max}} = c/\lambda_{\text{min}})\) of the signal to be received. By applying the Nyquist sampling theorem, it can be shown that the minimum distance between the sensors of the array to avoid aliasing problems, is given by:

\[
d = \frac{c}{2f_{\text{max}}}
\]  

At this point, geometry has to be defined. We assume here a square array with a constant microphone distance along the two directions, taking care to avoid aliasing problems: this choice produces array pattern functions that are relatively simple to analyze and permits the use of fast algorithms, such as the Fast Fourier Transform, to compute spatial spectra and array output signals. In this case, the total size of the array is the diagonal of the square, which is related to its side dimension by: \(l = D/\sqrt{2}\). So, the number of microphones to put on each side of the array is the nearest integer that comes out from the relation: \(n = [l/d]_{\text{int}}+1\). Thus, the total number \(N\) of microphones that defines the total extent of the array is: \(N = n^2\).

### 7. MEASUREMENT DISTANCE

The analysis of free field sound propagation in air takes typically into account two important phenomena: sound absorption due to spherical propagation and sound absorption due to the atmosphere. When sound propagation over long distances is considered, losses, which irreversibly convert acoustic energy into heat energy, become significant and must be
accounted for in the overall problem. Apart from the propagation losses, the resulting interference between incident and reflected waves contributes to modify the real path of wave propagation, really measured by an acoustic antenna. By considering, for example, the noise emitted from an aircraft, Fig. 8, if we regard the noise as produced by a point source and knowing the aircraft distance from the acoustic antenna, it is possible to evaluate the sound power attenuation due to the spherical divergence and to estimate the sound power measured by the sensor, in ideal conditions of negligible absorption. This attenuation is about 6 dB every double distance from the noise source. Aircraft orientation can contribute to improve or reduce the reception of the acoustic signal: if the X body axis is perpendicular to the array, the attenuation is about 3 dB; the sound intensity, instead, increases of 3 dB if the aircraft is placed along the array vertical. The specific absorption coefficient \( a_{dB} [dB/m] \) can be experimentally plotted as function of sound signal frequency and evaluated in terms of relative humidity, Fig. 9. The sound propagation is supposed in standard air, (1 atm and 20°C), without turbulence.

Because all these effects, acoustic signals emitted by a noise source have a sound power \( W_{signal} \) strictly different from the power \( P_{signal} \) measured by the microphone array. The sound power \( W_{signal} \) emitted by source decays for effect of spherical divergence with a function:

\[
P_{signal} = W_{signal} \cdot \frac{Q_R Q_T \lambda^2}{(4\pi r)^2}.
\]

Such formula takes into account the sound directivity, that cannot be homogenous along all the sphere directions and the sound may propagate over preferred paths. Thus \( Q_T \), directivity factor of the source, influences the sound power measured by the microphone array: \( W_{signal} Q_T \) where \( Q_T = 1 \) when the source is a point and isotropic. If \( r \) is the array distance, the sound power measured by the array is attenuated by the factor due to the spherical divergence \( 1/(4\pi r^2) \) and, the overall acoustic intensity can be defined as:

\[
J = \frac{W_{signal} Q_T}{4\pi r^2}.
\]

The power measured by the antenna can be obtained by the measured acoustic intensity by knowing the array section of the array: \( S = \lambda^2/4\pi \). Therefore, the sound power measured by the antenna is:
The $Q_R$ factor is an important design parameter, which takes into account the background noise and is equal to the ratio between the sound power measured by an omnidirectional sensor which measures an acoustic intensity equal to the antenna at $r$ distance and the effective sound power measured by the antenna:

$$Q_R = \frac{P_{\text{omnidirectional,signal}}}{P_{\text{antenna,signal}}}$$

Besides, by considering other factors as the atmospheric absorption, air turbulence and aircraft orientation, the sound power will be:

$$P_{\text{signal}} = W_{\text{signal}} \cdot \frac{Q_T Q_R \lambda^2}{(4\pi)^2} \cdot \frac{1}{L_O} \cdot \frac{1}{L_M} \cdot \frac{1}{L_A}$$

where: $L_0$ is the power attenuation due to the aircraft orientation = 3dB; $L_M$ is the power attenuation due to the meteo conditions = 7dB; $L_A$ is the power attenuation due to the atmospheric absorption, dependent of the source distance, the frequency of the signal and the percentage of relative humidity = $\alpha_{dB} r$. Another important array design parameter is the signal to noise ratio $SNR$. It is defined as the ratio between the power of the signal $P_{\text{signal}}$ and the power of the background noise $P_{\text{noise}}$. If $SNR=20dB$, the array must to detect signals with power 20dB higher than the background noise. The array range is the final parameter that contains all these information. It is defined as the maximum distance that allows distinguishing the sound source signal from the background noise. If $r_{max}$ is the range, $L_A = \alpha_{dB} r$, and the aircraft can be considered as a point and isotropic source ($Q_T = 1$):

$$r_{max} = \sqrt{\frac{W_{\text{signal}} \cdot \lambda^2}{4\pi P_{\text{signal}}} \cdot \frac{1}{L_O} \cdot \frac{1}{L_M} \cdot \frac{1}{\alpha_{dB}}}$$

For example, considering values typical for aircraft noise emission, i.e. a $W_{\text{signal}}=140$ dB @2500 Hz and a SNR=10, from eq. (15) we can get a value of $r_{max} \approx 9$km. This distance is well-matched with the dimension of Aerodrome Traffic Zone (ATZ), thus allowing the employment of microphone array as acoustic radar within the airport neighbourhood.

### 8. GRID ARRAY: A TEST CASE

According to the previous considerations, the design of a grid array is here proposed. Following are reported the design parameters, the array main characteristics and, Figg. 10-15, some plots of the main array parameters. The convention for the spherical coordinates is reported in Fig. 10: The azimuth $\phi$ goes from 0 to 360 degrees, the elevation $\theta$ from 0 (North pole) to 180 degrees.

**Design parameters:** $f_{min}=400Hz$; $f_{max} = 2000Hz$; Range = 1000 m; Resolution=500 m

**Grid array main parameters:** Diameter=2.6404 m; Mic distance=0.0858 m; Array side=1.87m; Mic number= 484.
6. CONCLUSIONS

In this paper the general guidelines to design microphone array for moving sound sources localization and tracking by using beam-forming techniques are presented. A detailed analysis on how to design the array to get the requested performance has been carried out. A dedicated code has been developed within MATLAB environment with a user friendly graphic interface. Starting from the acoustics requirements of the array transducers, the code allows the design of a grid array and evaluates the antenna main characteristics. The study carried out in this work is a preliminary investigation necessary to estimate the reliability and the costs of such methodology for the realization of an acoustic antenna for far field noise source localization.

7 REFERENCES