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SIMULTANEOUS OPTIMIZATION OF STRUCTURE AND CONTROL OF SMART TENSEGRITY STRUCTURES

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Abstract

A multidisciplinary design optimization (MDO) of a smart tensegrity structure using genetic algorithm is considered as a mixed discrete - continuous objective programming problem. Robust H_2 and H_∞ controller norms are considered as the objective. Performance of the H_2 and H_∞ controllers for vibration control of the tensegrity structures are compared. A nested optimization strategy is used. The optimal gain matrices of the controller and estimator are found by solution of the corresponding Ricatti equations in the inner iteration as the first step. The optimal structural parameters are found in the outer iteration as the next step. Design variables include twist angle and the locations of the actuator, which are either discrete or continuous. The force generated by the electro-mechanical coupling of the piezoelectric actuator is used in the formulation. The tensegrity structure of class-1 comprising of two modules with piezoelectric actuators is optimized.

1. INTRODUCTION

In many modern structures, active control is used to improve dynamic characteristics such as reducing the transient response and vibration amplitude. In traditional design practice of an actively controlled structure, the structure and its control system are designed separately. First, a nominal structure is designed or optimized to meet some open loop performance requirements. Next a control system is designed for the structural model to satisfy the closed loop specifications. Although such individual designs may be optimal in the separate sense, the integration of the designs may not be optimal in a combined sense. Therefore it is important to consider simultaneously structural optimization with optimal control strategies.

The concept of tensegrity structure is known for many years [1]. However, only limited applications of them are found. Tensegrity structures have been receiving fresh attention in recent years in view of the application to deployable aerospace structures. In this context, tensegrity structures have become suitable candidates for space applications Furuya [2]. Jager and Skelton [3] presented a method for sensor and actuator selection for planar tensegrity structure using H_∞ controller.

In this paper, a multidisciplinary optimization problem is considered. The importance of this paper is that optimal design of active tensegrity structures is considered by optimizing the structure and controller simultaneously. The force generated by the piezoelectric actuators is considered instead of a generic force generating actuators. The shape of the structure is optimized with the twist angle as the design variable. The controller design variables include the location of the actuator also. The control objective function is the norm of H_2 and H_∞ controller. Numerical examples show that the H_2 and H_∞ controller are suitable for the vibration control of tensegrity structures.

2. PROBLEM FORMULATION

Consider the tensegrity structure shown in Figure 1. It has a total of 30 members consisting of 6 struts (compression members) and 24 cables (tension members). In the figure the thick lines represent the struts and the thin lines represent the cables. All the vertical cables are identical made of the same material and having the same length and area of cross section. Similarly diagonal cables, saddle cables, top and the bottom cables are identical within each category.

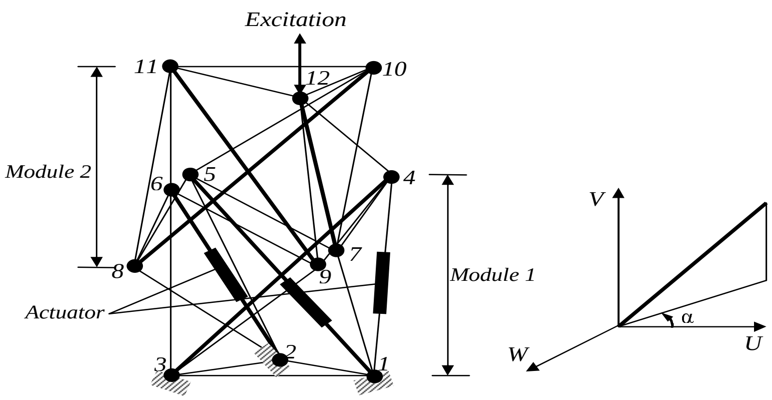


Figure 1. Two stage 3-strut tensegrity structure

In this work the optimal design of an active tensegrity structure is considered by optimizing the structure and controller simultaneously. The control effect by the piezoelectric actuators is minimized with respect to control optimization and the shape of the structure optimized with the twist angle (α) as the design variables. The controller design variable includes the location of the actuator. The control objective function is the norm of H_2 and H_∞ controller. The response of the optimized tensegrity structure due to the impulse type of excitation is sought to be controlled by the use of optimal control theories with the help of piezoelectric actuators embedded in the compressive member of the tensegrity structure. The number of actuators is limited to two. The actuators are considered as an integral part of the structural members. The dynamic analysis of the tensegrity structure including the piezoelectric actuators is carried out using the finite element analysis.

3. FINITE ELEMENT FORMULATION

Piezoelectric actuators in adaptive trusses replace either an entire member of the truss structure or just part of it. The active member is modeled using the finite element method. The finite element idealization of the active member is shown in Figure 2. The piezoelectric active member occupies part of the axial member from node 2 to node 3 as shown in the figure. The axial displacement u at each node is the degree of freedom. The axial displacement within the nodes is obtained by interpolation using a linear polynomial shape function in x defined over the length l. The local nodal displacements for the axial member with the piezoelectric actuator are given by

$$\{\delta\} = \{u_1\, u_2\, u_3\, u_4\}^T \tag{1}$$

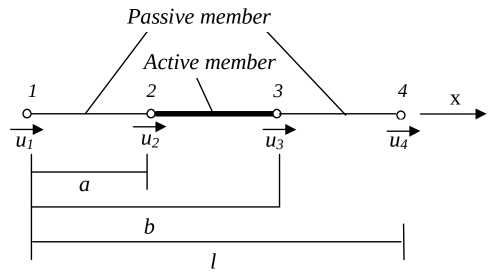


Figure 2. Member with piezoelectric actuator

The constitutive equation of the piezoelectric material coupling the elastic field and the electric field can be written as ,

$$\sigma = E_P\varepsilon - e_{33}E \tag{2}$$

$$D = e_{33}\varepsilon + \epsilon_{33}E \tag{3}$$

where σ, E_p, e_{33}, ε, E, D and ϵ_{33} are the stress, Young's modulus of piezoelectric material, piezoelectric stress coefficient, strain, electric field, electric displacement and the dielectric constant of the piezoelectric material respectively. The piezoelectric actuators are placed on the strut, either on the entire element or as part of the element as shown in Figure 2. However in this paper it is assumed that the piezoelectric actuators are only part of the struts. The stress due to the electric field is given by

$$\sigma = -e_{33}\frac{\phi_a(t) - 0}{(l_a/n)} \tag{4}$$

where $\phi_a(t)$ is the voltage of the actuator, l_a is the length of the actuator, n is the number of stacks and e_{33} is the piezoelectric stress coefficient which relates the stress in the direction 3 to the electric field applied in the direction 3.

The virtual work done by the electric force through the nodal displacement is given by,

$$\triangle W = \int_0^{l_a} \triangle(\varepsilon^T\sigma)dv = -\int_0^{l_a} \triangle(\varepsilon^T)e_{33}Edv \tag{5}$$

By substituting equation (8) into equation (9) we get

$$\triangle W = -\int_0^{l_a} \triangle(\frac{\partial u}{\partial x})e_{33}(\frac{\phi_a(t)}{L_a})dv = -\{\triangle\delta\}^T\frac{A}{L_a}e_{33}\phi_a(t)[1-1]^T = -\{\triangle\delta\}^T\{F_a\}\phi_a(t) \tag{6}$$

where A, $\{\triangle\delta\}^T$, F_a are the area of the actuator, virtual displacement and actuator force respectively, $L_a = (l_a/n)$ is the thickness of single piezoelectric element in the direction 3 and $F_c = \{F_a\}\{\phi_a(t)\}$ is the control force.

Using Hamilton's principle, the equations of motion for the structure can be expressed in matrix form as

$$[M]\{\ddot{\delta}\} + [C]\{\dot{\delta}\} + [K]\{\delta\} = \{F_{ext}\} + [L]\{F_c\} \tag{7}$$

where $[M]$, $[C]$, $[K]$, $\{F_{ext}\}$ and $[L]$ are the mass matrix, damping matrix, stiffness matrix, external force vector and location matrix of the actuator respectively.

In terms of the state vector $\{\xi\} = \{\delta, \dot{\delta}\}^T$ the above equation can be written as

$$\{\dot{\xi}\} = [A]\{\xi\} + [B_2]\{F_c\} + [B_1]\{F_{ext}\} \tag{8}$$

where,

$$[A] = \begin{pmatrix} [0] & [I] \\ -[M]^{-1}[K] & -[M][C] \end{pmatrix}; [B_2] = \begin{pmatrix} [0] \\ [M]^{-1}[L] \end{pmatrix} and$$

$$[B_1] = \begin{pmatrix} [0] \\ [M]^{-1} \end{pmatrix} \tag{9}$$

4. OPTIMIZATION PROBLEM FORMULATION

The optimal design of the tensegrity structure involves the finding of optimal values of the design variables which are the twist angle $'\alpha'$ and the location of the actuator l_o consists of direction cosines which minimizes the performance index J of the H_2 and H_∞ controller respectively. The genetic algorithm is used for the minimization of the controller.

The H_2 optimal control problem consists of finding a causal controller K which stabilizes the plant G and which minimizes the quadratic performance index

$$J_2(K) = \|G(K)\|_2^2 \tag{10}$$

The time domain it is given by

$$J_2(K) = \sum_{k=1}^{m} \int_0^\infty z(t)^T z(t) dt : w = e_k \delta(t) \tag{11}$$

where $\delta(.)$ is the Dirac delta function.

The H_2 norm [4] is defined as

$$J_2(K) = tr([D]_{12}[K]_{opt}[Q][K]_{opt}^T[D]_{12}^T) + tr([B]_1^T[P][B]_1) \tag{12}$$

where $[P]$ and $[Q]$ are the solutions of the controller and the estimator Riccati equations. The structure variable twist angle (α) of the tensegrity structure and the location of the piezoelectric actuator depend on the solutions of the controller and estimator Riccati equation $[P]$ and $[Q]$. The matrices $[P]$ and $[Q]$ of equation (12) in turn depends on the system matrix $[A]$ and the location matrix $[B_2]$.

The H_∞ optimal control problem consists of finding a causal controller $K(s)$ which stabilizes the plant G and which minimizes the quadratic performance index [4]

$$J_\infty(K) = \| G(K) \|_\infty \tag{13}$$

subject to the constraint that no two actuatorts overlap with each other.

5. RESULTS AND DISCUSSION

As an example of the simultaneous structural optimization and control, the tensegrity structure of Figure 1 is considered. The struts and cables are assumed to be of circular cross section made of steel. The diameter of the struts is 10^{-2}m and of the cables 10^{-3}m. The cables are prestressed up to 0.1 MPa. The radius of the circumscribing the base of the tensegrity structure is 0.25m and the height of module is 0.3m. The overlap ratio between the two stages of the tensegrity structure is taken as 0.5. The structural optimization problem is formulated in two different ways. In one formulation, the control is effected through the piezoelectric actuators placed as part of the struts, while in the other formulation the control is assumed to be effected through generic force generating actuators placed as part of the cables. The material properties of the structural members and the piezoelectric actuators are given in Table 1. It is assumed that the

Table 1. Material property of structural member and piezolectric actuator

Poperty name	Structure (steel)	PZT
Young's modulus (E_s)	2.1×10^{11} (N/m^2)	5×10^{10} (N/m^2)
Density (ρ)	7800 (kg/m^3)	7600 (kg/m^3)
Piezoelectric stress coefficient (e_{33})	-	18.6 (C/m^2)

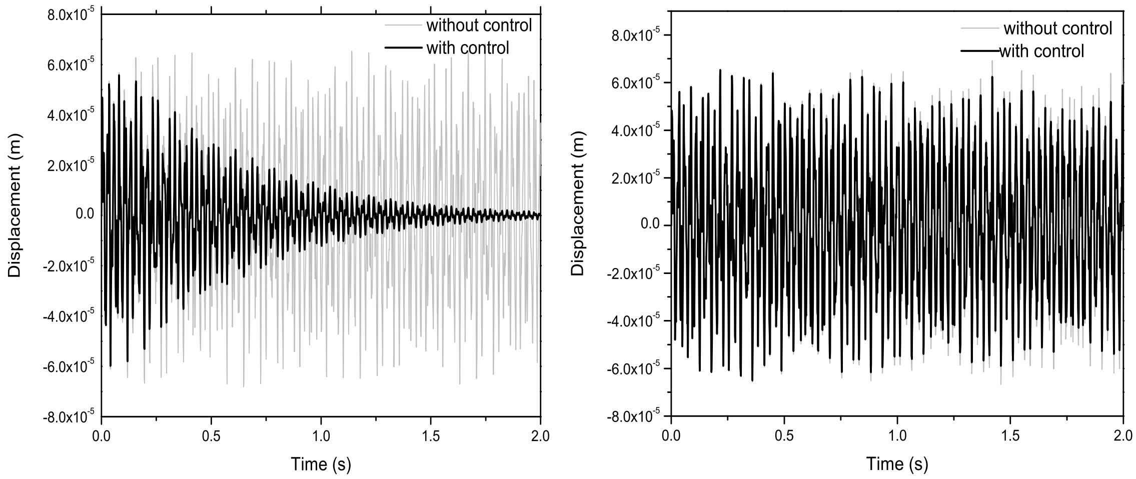
control is effected by two actuators which will be placed in the cables and struts. Since only two actuators are assumed to be available for use the optimization problem also involves the determination as to which of the struts/cables these actuators have to be optimally placed.

The structure and the controller are simultaneously optimized, for three different cases of external excitation acting at the node 12. In the first case an impulse load of magnitude 10 N and for a duration of 1×10^{-3} s is applied along the V direction and with an amplification factor of 100 for the actuator. The output y is measured at the node 10 in the V direction and at the node 11 in the W direction. The performance output z is measured at the disturbance and at the output y. The optimized values of the twist angle, actuator location and the value of the objective function for the cases of the actuators placed on the cables or the struts are presented in Tables 2.

It is evident from the above table that the control is effective for most of the cases when the actuator is placed near the fixed end of the tensegrity structure. The difference in values of the objective function of H_2 and H_∞ control is due to the fact that the objective function of H_2 control is obtained by the direct minimization of the quadratic performance index which leads to the equation (12) whereas in the H_∞ control the direct minimization of the cost $J_\infty(K)$ turns out to be difficult. Hence, it is convenient to take 'γ' as the objective function to make the solution procedure simpler. For the optimized structure, the displacement response with H_2 control and without control at node 12 in the V direction for an external impulse load of

Table 2. Optimized twist angle, actuator location and objective function for actuator placed on cables and struts

Controller	Excitation direction	Actuator placed on cables			Actuator placed on struts		
		Twist angle (radians)	Location of actuator	Objective function	Twist angle (radians)	Location of actuator	Objective function
H_∞	U	0.9756	1-4 and 3-6	157.54	0.1329	2-6 and 3-4	222.91
	V	1.0308	1-4 and 8-6	156.04	0.4203	1-5 and 8-10	221.41
	W	0.9725	4-7 and 8-6	157.54	0.0102	1-5 and 2-6	229.15
H_2	U	0.8866	1-4 and 2-5	4130.30	0.2167	2-6 and 3-4	771.79
	V	0.8283	1-4 and 2-5	4116.40	0.9357	1-5 and 2-6	741.77
	W	0.8068	1-4 and 2-5	4115.90	1.0472	1-5 and 2-6	757.18

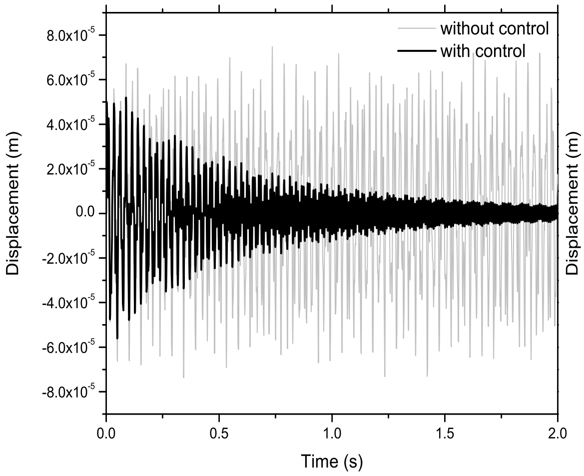


(a) (b)

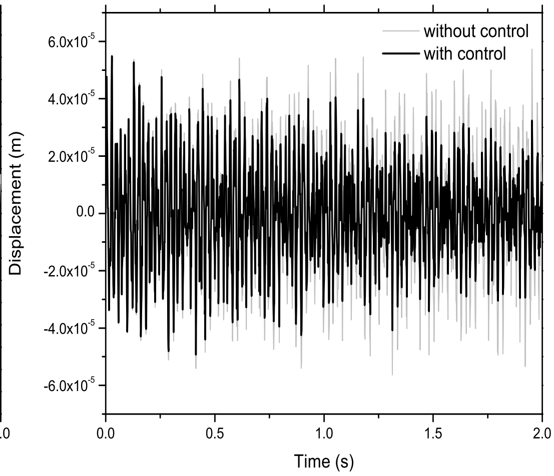
Figure 3. (a) Time response for actuator placed on cable with H_2 controller (b) Time response for actuator placed on strut with H_2 controller

magnitude 10 N and duration 1×10^{-3} s applied at the same position for the actuators placed in cables and struts are shown in Figures 3 (a) and (b) respectively. It is observed from Figure 3 (a) that the control is not effective as compared to Figure 3 (b). The displacement response with H_∞ control and without control for the actuators placed in cables and struts are shown in Figures 4 (a) and (b) respectively. It is evident from these figures that the vibration of the structure is controlled effectively. It is also observed that the actuators placed in the cables are found to be more effective when compared to the actuators placed on the struts. The control can be made more effective for the actuator placed in the strut of the tensegrity structures by increasing the amplification factor as shown in Figures 5 (a) and (b), which in turn leads to larger control effort in terms of the control voltage.

The actuator voltage can be obtained from the relation $\phi_a(t) = F_c/(Ae_{33}/L_a)$. The number of piezoelectric stack elements 'n' are taken to be 300. Figures 10 and 11 show the control voltage for the actuator placed in the struts corresponding to H_2 control and H_∞ control re-

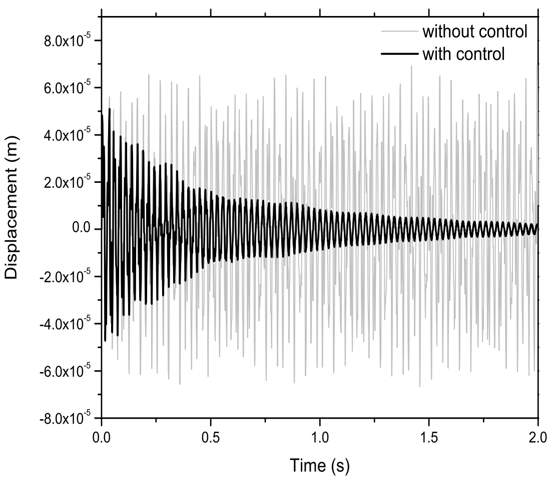


(a)

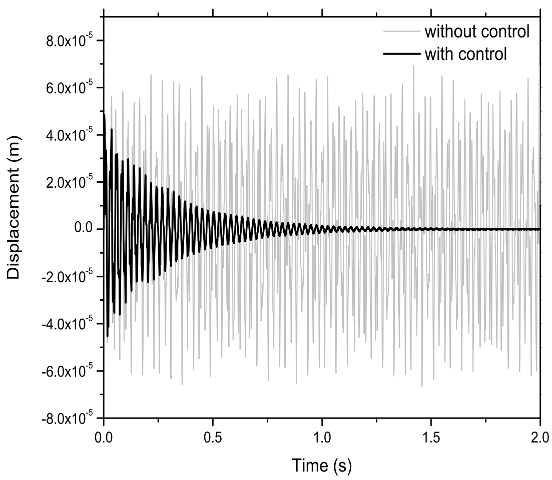


(b)

Figure 4. (a) Time response for actuator placed on cable with H_∞ controller (b) Time response for actuators placed on strut with H_∞ controller

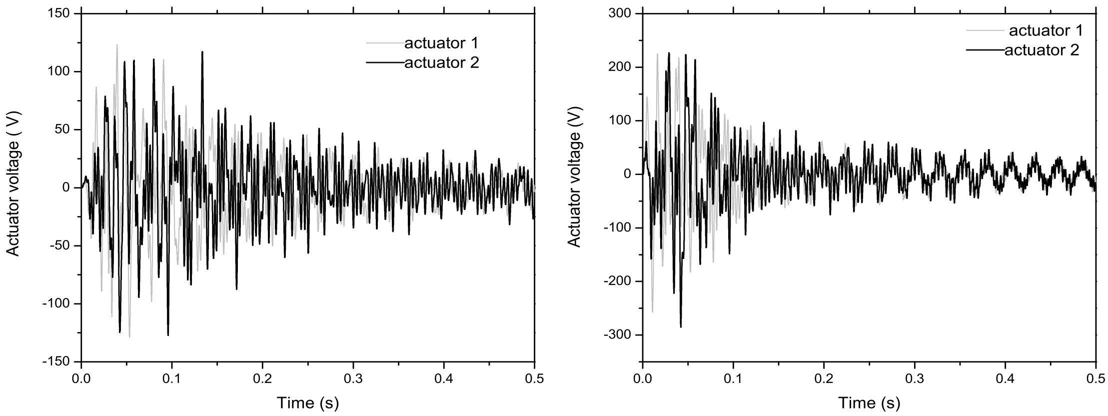


(a)



(b)

Figure 5. (a) Time response for actuator placed on strut with H_2 controller (b)Time response for actuator placed on strut with H_∞ controller



(a) (b)

Figure 6. (a) Actuator voltage corresponding to Figure 5 (a) (b) Actuator voltage corresponding to Figure 5 (b)

spectively. It is observed from the Figures 5 (a) and (b) that the displacement is less for the H_∞ control than the H_2 control. However, the control voltage for the H_∞ control is more than that of the H_2 control as seen in Figures 6 (a) and (b).

6. CONCLUSION

This paper demonstrates the capability of simultaneous optimization of control and structure with respect to a tensegrity structure. While the control strategy uses the concepts of optimal control theory, the simultaneous optimization makes use of the genetic algorithm which combines the optimal control strategy in its fold. The control is effected through piezoelectric actuators which minimize the performance index representative of the total energy of the structure. The following conclusions can be drawn from the study. 1) The control is effective for most of the cases when the actuator is placed near the fixed end of the tensegrity structure for H_2 control. 2) Actuators placed on cables of the tensegrity structure are more effective in controlling the vibration response than actuators placed on the struts. 3) The displacement is less for the H_∞ control than for the H_2 control. However, the control voltage for the H_∞ control is more than that of the H_2 control.

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