ICSV14
Cairns • Australia
9-12 July, 2007

14th International Congress on Sound & Vibration

EFFICIENT WAVE BASED MODELS FOR ACOUSTIC SCATTERING AND TRANSMISSION PROBLEMS USING POINT SOURCE AND PLANE WAVE EXCITATION

Bart Bergen, Bert Van Genechten, Bert Pluymers, Dirk Vandepitte, Wim Desmet

K.U.Leuven - Department of Mechanical Engineering
Celestijnenlaan 300B
B-3001 Leuven, Belgium

bart.bergen@mech.kuleuven.be

Abstract

The wave based method (WBM) is an efficient deterministic prediction method for the analysis of steady-state acoustic problems. It offers an alternative to the finite element and boundary element method, especially in the mid-frequency range, where these element based methods are impaired by their computational burden. This paper discusses the use of new source formulations in WB models for two-dimensional unbounded acoustic problems. These source formulations allow the WBM to tackle general acoustic radiation and transmission problems. Numerical validation examples illustrate the advantageous properties of the considered approach.

1. INTRODUCTION

Both the finite element method (FEM) and the boundary element method (BEM) are well established deterministic tools for the analysis of acoustic problems. The FEM [1] discretizes the problem domain into a large but finite number of small elements. Within these elements, the dynamic response variables are described in terms of simple, polynomial shape functions. The BEM [2] is based on a boundary integral formulation of the problem, relating the response variables to a distribution of acoustic variables on the problem boundary. As a result, only this boundary needs to be discretized. Within the applied boundary elements, the acoustic boundary variables are expressed in terms of polynomial shape functions, similar to the FEM. As only the boundary needs to be discretized, the BEM is well suited for the treatment of unbounded problems. The FEM can only tackle unbounded problem domains by introduction of an artificial boundary: the region inside this boundary is modeled using conventional FE elements, whereas a special technique is used to model the unbounded outside region. Three techniques are currently in use: Absorbing boundary conditions, infinite elements and absorbing layers [1].

As the frequency increases, the spatial resolution of the dynamic response variables is characterised by an ever smaller wavelength. To accurately represent this small wavelengths using the applied simple polynomial shape functions, an ever finer discretisation is needed, raising the computational burden of these methods beyond feasible proportions. As a result, the FEM and BEM are only applicable for low-frequency problems [3].

The wave based method (WBM) [4] is an alternative deterministic technique for the analysis of acoustic problems. The method is based on an indirect Trefftz approach, in that the dynamic response variables are described using wave functions which exactly satisfy the governing differential equation. Boundary conditions are imposed using a weighted residual scheme, resulting is a small system that is solved for the unknown contribution factors in the wave function expansion. Due to the small model size and the enhanced convergence characteristics of the WBM, it has a superior numerical performance as compared to the element based methods. As a result, problems at higher frequencies may be tackled [5]. The computational efficiency of the WBM has been successfully proven for many steady-state interior and exterior (vibro-)acoustic problems [6, 7, 8].

This paper discusses the application of the WBM for two-dimensional (2D) unbounded acoustic calculations. Acoustic scattering and transmission calculations constitute an important class of problems in unbounded domains. Common excitation definitions used in such problems are the acoustic plane wave and the point source (2D equivalent of a three-dimensional line source). This paper reviews the feasibility of these excitation approaches in the WBM. Through the introduction of novel source definitions in the unbounded part of both baffled and unbaffled WB models, the applicability of the method for two dimensional scattering and transmission calculations is greatly enhanced. This paper consists out of three parts. The first part gives a mathematical problem definition of a general acoustic problem. The second part shortly describes the WBM, its application for the analysis of 2D unbounded problems and introduces the new acoustic source definitions. The third part illustrates the efficiency of the novel source definitions for various types of unbounded acoustic problems.

2. PROBLEM DESCRIPTION

Consider the 2D, unbounded acoustic problem shown in figure 1. In the absence of any acoustical source, the steady-state acoustic pressure in this domain is governed by the homogeneous Helmholtz equation

$$\nabla^2 p(\boldsymbol{r}) + k^2 p(\boldsymbol{r}) = 0 \tag{1}$$

with $k = \omega/c$ the acoustic wave number. The acoustic fluid is characterised by the density ρ_0 and the speed of sound c. The problem boundary Γ is constituted of 2 parts: the finite part of the boundary, Γ_b, and the boundary at infinity, Γ_∞. The finite boundary can be further divided

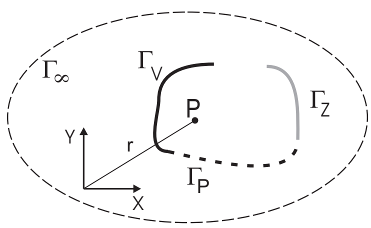


Figure 1. A 2D unbounded acoustic problem

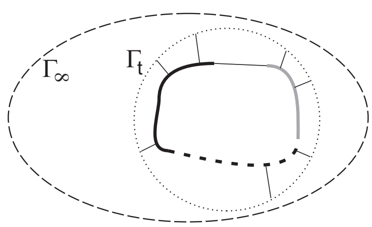


Figure 2. A WB partitioning of the 2D unbounded problem

in three non-overlapping parts: $\Gamma_b = \Gamma_v \cup \Gamma_p \cup \Gamma_Z$, considering the condition imposed

$$\boldsymbol{r} \in \Gamma_v : \quad R_v = \frac{j}{\rho_0 \omega} \frac{\partial p(\boldsymbol{r})}{\partial n} - \overline{v}_n(\boldsymbol{r}) = 0 \ , \tag{2}$$

$$\boldsymbol{r} \in \Gamma_p : \quad R_p = p(\boldsymbol{r}) - \overline{p}(\boldsymbol{r}) = 0 \ , \tag{3}$$

$$\boldsymbol{r} \in \Gamma_Z : \quad R_Z = \frac{j}{\rho_0 \omega} \frac{\partial p(\boldsymbol{r})}{\partial n} - \frac{p(\boldsymbol{r})}{\overline{Z}_n(\boldsymbol{r})} = 0 \ , \tag{4}$$

where $\overline{v}_n$, $\overline{p}$ and $\overline{Z}_n$ are, respectively, the imposed normal velocity, pressure and normal impedance. At the boundary at infinity Γ_∞ the Sommerfeld radiation condition for outgoing waves is applied. This condition ensures that no acoustic energy is reflected at infinity and is expressed as

$$\lim_{|\boldsymbol{r}| \to \infty} \left(\sqrt{\boldsymbol{r}} \Big(\frac{\partial p(\boldsymbol{r})}{\partial |\boldsymbol{r}|} + jkp(\boldsymbol{r}) \Big) \right) = 0 \ . \tag{5}$$

Solution of the Helmholtz equation (1) together with the associated boundary conditions (2), (3), (4) and (5) yields the dynamic pressure field $p(\boldsymbol{r})$.

3. THE WAVE BASED METHOD

3.1. Partitioning into convex subdomains

A sufficient condition for the WB approximations to converge towards the exact solution, is convexity of the considered problem domain [4]. In a general acoustic problem, the acoustic problem domain may be non-convex so that a partitioning into a number of convex subdomains is required. If the WBM is applied for unbounded problems, an initial partitioning of the unbounded domain into a bounded and an unbounded region precedes the partitioning into convex subdomains. Figure 2 illustrates the principle. The unbounded acoustic problem domain is divided into two non-overlapping regions by a truncation curve Γ_t. The unbounded region exterior to Γ_t is considered as one acoustic subdomain.

3.2. Acoustic pressure expansion

The steady-state acoustic pressure field $p^{(\alpha)}(\boldsymbol{r})$ in an acoustic subdomain $\Omega^{(\alpha)}$ is approximated as a solution expansion $\hat{p}^{(\alpha)}(\boldsymbol{r})$

$$p^{(\alpha)}(\boldsymbol{r}) \simeq \hat{p}^{(\alpha)}(\boldsymbol{r}) = \sum_{w=1}^{n_w^{(\alpha)}} p_w{}^{(\alpha)} \Phi_w^{(\alpha)}(\boldsymbol{r}) = \boldsymbol{\Phi}^{(\alpha)}(\boldsymbol{r}) \ \boldsymbol{p_w}{}^{(\alpha)} \ . \tag{6}$$

The degrees of freedom $p_w^{(\alpha)}$ are the contribution factors of the selected wave functions $\Phi_w^{(\alpha)}$. Together they form the vector of degrees of freedom $\boldsymbol{p_w}{}^{(\alpha)}$. The corresponding a priori defined wave functions are collected in the row vector $\boldsymbol{\Phi}^{(\alpha)}$.

The wave functions used in the pressure expansion are chosen to satisfy the Helmholtz equation (1) a priori. For the expansion in the unbounded domain, the functions will also satisfy the Sommerfeld radiation condition. For a detailed description of these functions used to describe the pressure in a bounded or an unbounded domain, the reader is referred to Desmet [4] and Pluymers [9], respectively.

3.2.1. Source descriptions

If an acoustic point source is present in the domain, the Helmholtz equation (1) becomes inhomogeneous:

$$\nabla^2 p(\boldsymbol{r}) + k^2 p(\boldsymbol{r}) = -j\rho_0\omega\delta(\boldsymbol{r}, \boldsymbol{r_q})q \ . \tag{7}$$

In order to cope with the source right-hand side term in the wave based method, an extra term is added to the pressure expansion (6), corresponding to a particular solution of the non-homogeneous Helmholtz equation (7). For a point source at (x_q, y_q), the following solution is proposed:

$$\hat{p}_q^{(\alpha)}(x,y) = \frac{\rho_0\omega q^{(\alpha)}}{4} H_0^{(2)}\left(kr_q^{(\alpha)}\right) \text{ with } r_q^{(\alpha)} = \sqrt{\left(x - x_q^{(\alpha)}\right)^2 + \left(y - y_q^{(\alpha)}\right)^2} \tag{8}$$

Another common type of excitation is the acoustic plane wave. The plane wave satisfies the Helmholtz equation (1), and ensures no reflection occurs at infinity, so the plane wave description can be added to the pressure expansion (6) in the same way as a point source. The following formulation is proposed:

$$\hat{p}_{pw}^{(\alpha)}(x,y) = e^{jk\left(d(\phi)\right)} \ . \tag{9}$$

where $d(\phi)$ is the propagation vector of the plane wave and ϕ the the angle of propagation.

If a baffle is present, a second source term is included to simulate the effect of a rigid plane. This source term corresponds to the mirror image of the source with respect to the baffle plane. In the case of a point source, this mirror is a source of the same amplitude but with mirrored position. In the case of a plane wave both the reference point and the propagation angle are mirrored, as illustrated in figure 3.

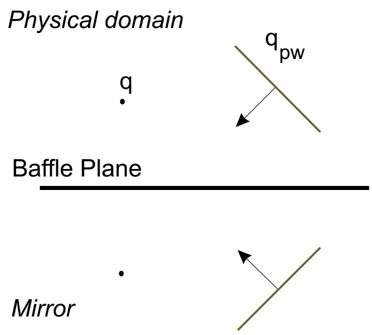


Figure 3. Sources in the physical and mirror domain

3.3. Wave model

The proposed expansions guarantee compliance with the Helmholtz equation and the Sommerfeld radiation condition. The boundary conditions and subdomain continuity are enforced through a weighted residual formulation. Boundary condition residuals are given in (2), (3) and (4), and the subdomain continuity residual can be written as

$$\boldsymbol{r} \in \Gamma_I : \quad R_I^{(\alpha,\beta)} = \left(\frac{j}{\rho_0\omega}\frac{\partial p^{(\alpha)}(\boldsymbol{r})}{\partial n^{(\alpha)}} - \frac{1}{\overline{Z}_{int}}\right) + \left(\frac{j}{\rho_0\omega}\frac{\partial p^{(\beta)}(\boldsymbol{r})}{\partial n^{(\alpha)}} + \frac{1}{\overline{Z}_{int}}\right) \ . \tag{10}$$

For a detailed description of the system matrices, the reader is referred to Pluymers [9].

4. VALIDATION EXAMPLES

4.1. Scattering on a C-shape

A first example examines the acoustic scattering on a C-shape. The WB formulation is compared to a conventional BEM approach, illustrating the accuracy and efficiency of the wave based method. Three different cases are considered: a plane wave source, a point source outside the C-shape (source in unbounded WB domain III) and a point source inside the C-shape (source in bounded WB domain I). The C-shape considered is shown in figure 4, and consists entirely of rigid walls. The C-shape has an outside radius of $1m$ and is 0.25 m wide. The fluid taken yields a speed of sound $c = 340m/s$ and density $\rho_0 = 1.225kg/m^3$.

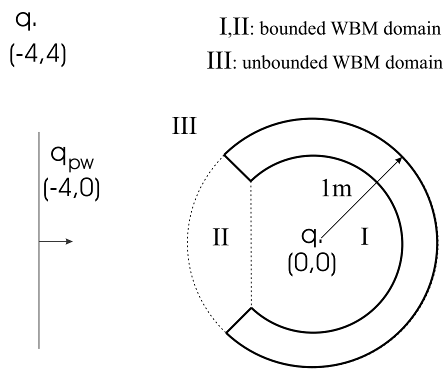


Figure 4. problem definition

4.1.1. *Plane Wave incident on the C-shape*

Several indirect variational BEM meshes are considered, using LMS\Sysnoise rev. 5.6. A coarse mesh is gradually refined until convergence is obtained. The mesh details are given in table 1. The mesh validity is determined using the rule of thumb stating 6 (or 10) linear elements are required per wavelength. The computing time indicated is the total time for calculation of the frequency response function (FRF), consisting of 700 frequency lines between 100 and $1500Hz$. The number of DOF's in the wave model varies with the frequency, and ranges between 29 and 245. The results of the $5mm$ and the $10mm$ models coincide, indicating convergence is reached. The $5mm$ model will therefore serve as a reference.

Table 1. Model information

	element size	♯ DOF	mesh validity: 6 el./ λ [Hz]	mesh validity: 10 el./λ [Hz]	calculation time [s]
BEM	$5mm$	3498	11313	6788	3905
	$10mm$	1748	5652	3391	779
	$30mm$	582	1813	1088	133
	$40mm$	436	1363	815	106
WBM	/	29-245	/	/	105

Figure 5 compares the FRF of the BEM models $5mm$ and $40mm$ and the WB model. The $40mm$ model is only accurate up to roughly $1000Hz$, as predicted by the rule of thumb. The WB model in this comparison is taken to have an equal calculation time to the 40mm BE model. It is clear that the result obtained by the WB model is a close match to the reference, coming at only $1/8$ of the computational cost of the $10mm$ model, the first BE model to coincide with this reference.

Figure 6 shows the pressure field contours at $800Hz$, indicating that the wave based method result matches the BEM reference result.

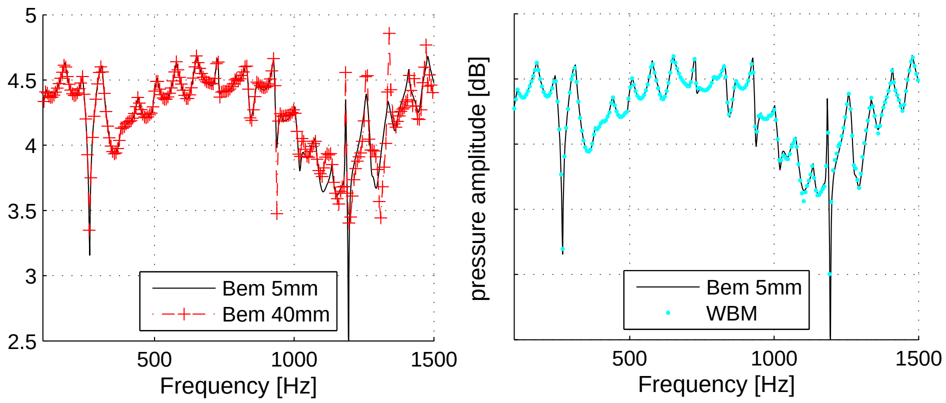


Figure 5. Pressure amplitude FRF [dB re $2e^{-5}$Pa] comparison: BEM and WBM models

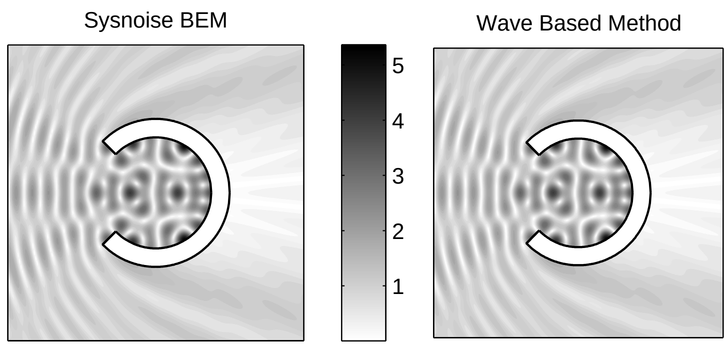


Figure 6. Pressure amplitude contours [Pa] at $800Hz$, plane wave excitation

4.1.2. *Point source outside the C-shape*

The pressure contours in figure 7 compare the result obtained by the BEM reference model with a WBM result, calculated using the source definition as in eq. (8), with the source located in the unbounded domain (fig. 4). It is clear that the WBM solution is very similar to the BEM reference solution.

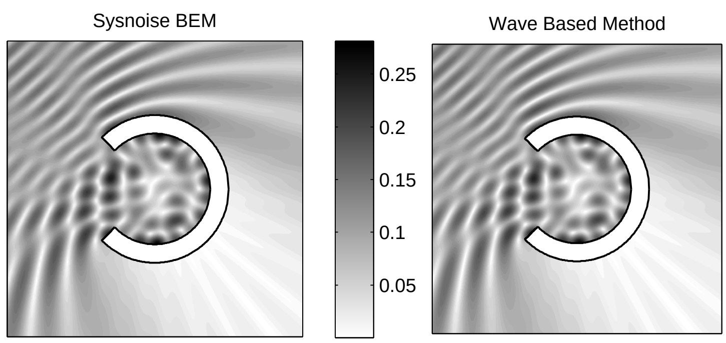


Figure 7. Pressure amplitude contours [Pa] at $800Hz$, point source in unbounded subdomain III

4.1.3. *Point source inside the C-shape*

For a source located in a bounded subdomain (fig. 4), figure 8 compares the BEM and WBM pressure contours, showing that the WBM solution matches the BEM reference result.

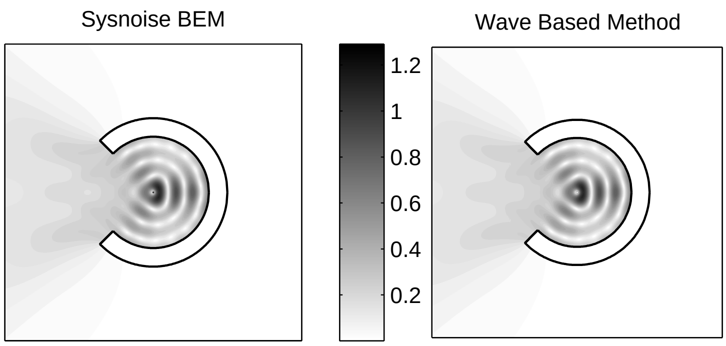


Figure 8. Pressure amplitude contours [Pa] at $800Hz$, point source in bounded subdomain I

4.2. Scattering on a shielded baffle

In a second case the source formulations are validated for a baffled problem, consisting of a baffle shielded by a plate. Figures 9 shows the problem and the WB domain decomposition. The acoustic fluid is the same as defined above. Figures 10(a) and 10(b) show the pressure field and the active intensity, respectivly, in the case of a plane wave incidence. Figure 11(a) and 11(b) show the pressure field and the active intensity resulting from a point source excitation. In both cases, it is clear that the obtained pressure field satisfies the boundary conditions imposed: pressure contours are perpendicular to the rigid walls, and the inter-domain continuity is respected. The rigid baffle, which is introduced by the choice of the wave functions and the source definition, is also correctly represented: pressure contours are perpendicular to it and no intensity is flowing through.

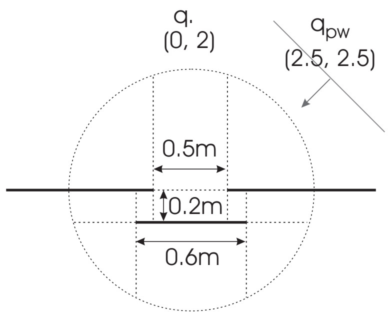
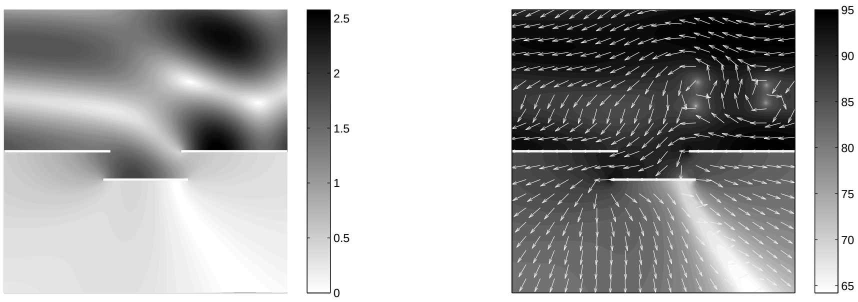


Figure 9. problem definition

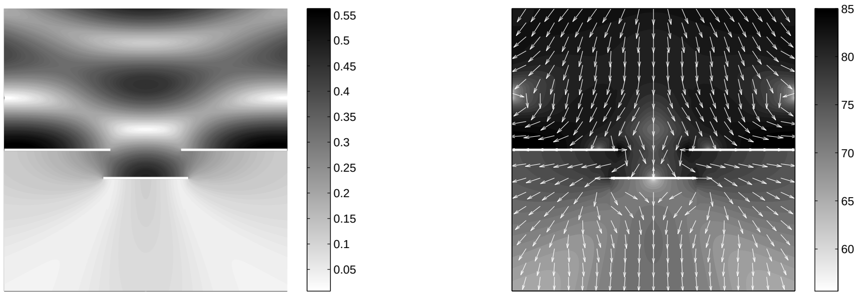


(a) Pressure amplitude [Pa] (b) Active intensity [dB re 1pW]

Figure 10. Baffle results at $300Hz$, plane wave source

5. CONCLUSION

This paper discusses the use of new acoustic source definitions in the WBM for scattering and transmission problems. The focus is on the treatment of unbounded problem domains. In the WBM, acoustic sources are introduced as a particular solution added to the regular expansion of wave functions. A formulation of this particular solution is proposed for a point source, located either in a bounded or unbounded WBM subdomain, and for a plane wave. These formulations



(a) Pressure amplitude [Pa] (b) Active intensity [dB re 1pW]

Figure 11. Baffle results at $300Hz$, point source

are validated both on an unbaffled and a baffled scattering problem. A comparison with the BEM indicates the reduced computational demands of the WBM, while obtaining results of the same accuracy. This reduction in computational load allows the WBM to tackle problems in an extended frequency range, as compared to the BEM.

ACKNOWLEDGEMENTS

The research of Bart Bergen is funded by a Ph.D. grant of the Institute for the Promotion of Innovation through Science and Technology in Flanders (IWT-Vlaanderen) and Bert Van Genechten is a Doctoral Fellow of the Fund for Scientific Research - Flanders (F.W.O.), Belgium.

REFERENCES

[1] I. Harari. A survey of finite element methods for time-harmonic acoustics. *Computer Methods in Applied Mechanics and Engineering*, 195:1594–1607, 2006.

[2] O. Von Estorff. *Boundary Elements in Acoustics: Advances and Applications*. WITpress, 2000.

[3] S. Marburg. Six boundary elements per wavelength: is that enough? *Journal of Computational Acoustics*, 10:25–51, 2002.

[4] W. Desmet. *A wave based prediction technique for coupled vibro-acoustic analysis*. KULeuven, division PMA, PhD. thesis 98D12, `http://people.mech.kuleuven.ac.be/~wdesmet/desmet_phd_thesis.pdf`, 1998.

[5] W. Desmet, P. Sas, and D. Vandepitte. A comparison between an indirect trefftz method and the finite-element method for solving coupled vibro-acoustic problems. (a). *Journal of Acoustic Society of America*, 106:2118, 1999.

[6] W. Desmet, B. Van Hal, P. Sas, and D. Vandepitte. A computationally efficient prediction technique for the steady-state dynamic analysis of coupled vibro-acoustic systems. *Advances in Engineering Software*, 33:527–540, 2002.

[7] B. Pluymers, W. Desmet, D. Vandepitte, and P. Sas. On the Use of a Wave Based Prediction Technique for Steady-State Structural-Acoustic Radiation Analysis. *Journal of Computer Modeling in Engineering & Sciences*, 7(2):173–184, 2005.

[8] B. Pluymers, W. Desmet, D. Vandepitte, and P. Sas. Application of an efficient wave based prediction technique for the analysis of vibro-acoustic radiation problems. *Journal of Computational and Applied Mathematics*, 168:353–364, 2004.

[9] B. Pluymers. *Wave based modelling methods for steady-state vibro-acoustics*. KULeuven, division PMA, PhD. thesis 2006D04, `http://www.mech.kuleuven.be/mod/wbm/phd_dissertations`, 2006.