TRANSFER MATRIX METHOD FOR
DUAL-CHAMBER MUFFLERS

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Abstract

Mufflers are widely used for exhaust noise attenuation in vehicles, machinery and other industrial elements. Modelling procedures for accurate performance prediction had led to the development of new methods for practical muffler components in design. Plane wave based models such as the transfer matrix method (TMM) can offer fast initial prototype solutions for muffler designers. In the present paper the authors present an overview of the principles of TMM for predicting the transmission loss (TL) of a muffler. The predicted results agreed in some limits with the experimental data published in literature.

1. INTRODUCTION

Mufflers are commonly used in a wide variety of applications. Industrial flow ducts as well as internal combustion engines frequently make use of silencing elements to attenuate the noise levels carried by the fluids and radiated to the outside atmosphere by the exhausts. Design of a complete muffler system is, usually, a very complex task because each of its elements is selected by considering its particular acoustic performance and its interaction effects on the entire acoustic system performance. For the frequency analysis of the muffler, as can be seen from the references [1, 3, 5], it is very convenient to use the transfer matrix method. The present paper deals with the fundamentals of the Transfer Matrix Method (TMM) and the method is applied to a specific muffler configuration for the prediction of Transmission Loss.

2. TRANSMISSION LOSS BY TMM FOR DUAL-CHAMBER MUFFLER

Figure 1 shows the geometry of a circular dual-chamber muffler considered in this study where is note the dimensions of radius $R_1$, $R_2$, $R_3$, $R_c$ and length of each section: $l_a$, $l_c$, $l_d$, $l_f$, $l_e$, $l_g$, $l_h$; and $t$ - is the baffle thickness.

In accordance with dimensions on the figure are made the notations
The planar wave propagation model is used in order to evaluate its applicability limits and to assess the need for end corrections. While a brief description of the relevant relationships is given follow-up for a detailed treatment. First, the four-pole matrices of the ducts are obtained. The product of these then yields the four-pole matrix of the entire configuration, which relates the pressure and mass velocity at the inlet and outlet and enables the evaluation of the acoustic attenuation performance by $TL$. The four-pole matrices of the ducts $A$, $E$ and $I$ are, respectively:

$$M_A = \begin{bmatrix} \cos(kl_A) & jY_A \sin(kl_A) \\ j \frac{\sin(kl_A)}{Y_A} & \cos(kl_A) \end{bmatrix},$$

$$M_E = \begin{bmatrix} \cos(kl_E) & jY_E \sin(kl_E) \\ j \frac{\sin(kl_E)}{Y_E} & \cos(kl_E) \end{bmatrix},$$

$$M_I = \begin{bmatrix} \cos(kl_I) & jY_I \sin(kl_I) \\ j \frac{\sin(kl_I)}{Y_I} & \cos(kl_I) \end{bmatrix},$$

where: $Y_A = c_0/S_A$, $S_A = \pi R_1^2$; $Y_E = c_0/S_E$, $S_E = \pi R_2^2$; $Y_I = c_0/S_I$ and $S_I = \pi R_3^2$.

For the four-pole matrices of the two chambers $C$ and $G$:

$$M_C = \begin{bmatrix} \cos(kl_C) & jY_C \sin(kl_C) \\ j \frac{\sin(kl_C)}{Y_C} & \cos(kl_C) \end{bmatrix},$$

$$M_G = \begin{bmatrix} \cos(kl_G) & jY_G \sin(kl_G) \\ j \frac{\sin(kl_G)}{Y_G} & \cos(kl_G) \end{bmatrix}.$$
\[
M_G = \begin{bmatrix}
\cos(kl_G) & jY_G \sin(kl_G) \\
\frac{j}{Y_G} \sin(kl_G) & \cos(kl_G)
\end{bmatrix},
\]

where: \( Y_C = c_0 / S_C \), \( S_C = \pi R_c^2 \), \( Y_G = c_0 / S_G \), \( S_G = \pi R_c^2 \).

For cross-sectional discontinuities, using decreasing element-subscript values with distance from the noise source, the cross-sectional areas upstream and downstream of transition (\( S_1 \), \( S_2 \) and \( S_3 \)) are related through [8]

\[
C_1 S_1 + C_2 S_2 + S_3 = 0,
\]

where the constants \( C_1 \) and \( C_2 \) (Table 1) are selected so as to satisfy the mass conservation equation across the transition. Table 1 also shows the pressure loss coefficient \( K \) for each configuration that accounts for conversion of some mean-flow energy and acoustical field energy into heat at the discontinuities. As indicated, \( K \leq 0.5 \) for area contraction, while \( K \to (S_1 / S_3)^2 \) for area expansions at large values of \( S_1 / S_3 \). The four-pole matrices of the ducts with cross-sectional discontinuities [8] (for Mach number \( M = 0 \)) is given by

\[
M_{\text{discontinuities}} = \begin{bmatrix}
1 & 0 \\
\frac{C_2}{C_1} & 1
\end{bmatrix}.
\]
Finally, the four extended regions are described in accordance with equation (8), by

\[
M_B = \begin{bmatrix}
    \frac{1}{jY_B \cot(kl_B)} & 0 \\
    0 & 1
\end{bmatrix},
\] (9)

\[
M_D = \begin{bmatrix}
    \frac{1}{jY_D \cot(kl_D)} & 0 \\
    0 & 1
\end{bmatrix},
\] (10)

\[
M_F = \begin{bmatrix}
    \frac{1}{jY_F \cot(kl_F)} & 0 \\
    0 & 1
\end{bmatrix},
\] (11)

\[
M_H = \begin{bmatrix}
    \frac{1}{jY_H \cot(kl_H)} & 0 \\
    0 & 1
\end{bmatrix},
\] (12)

with: 
\( Y_B = \frac{c_0}{S_B}, \quad S_B = \pi(R_C^2 - R_i^2) \), 
\( Y_D = \frac{c_0}{S_D}, \quad S_D = \pi(R_C^2 - R_2^2) \), 
\( Y_F = \frac{c_0}{S_F}, \quad S_F = \pi(R_C^2 - R_i^2) \), 
\( Y_H = \frac{c_0}{S_H}, \quad S_H = \pi(R_C^2 - R_1^2) \).

The matrix of the entire configurations relates the pressure \( P \) and mass velocity \( V \) at the inlet and outlet by:

\[
\begin{bmatrix}
    P_{\text{input}} \\
    V_{\text{input}}
\end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\
    T_{21} & T_{22}
\end{bmatrix} \begin{bmatrix}
    P_{\text{output}} \\
    V_{\text{output}}
\end{bmatrix},
\] (13)

where \( T_{11}, T_{12}, T_{21}, \) and \( T_{22} \) are referred to as the four-poles of the acoustical system, which are obtained by multiplying the previous matrices as

\[
\begin{bmatrix} T_{11} & T_{12} \\
    T_{21} & T_{22}
\end{bmatrix} = M_A M_B M_C M_D M_E M_F M_G M_H M_I.
\] (14)

Finally, the transmission loss is obtained by

\[
TL = 20 \log \left( \frac{Y_F}{Y_A} \right) \left( T_{11} + T_{12} + T_{21} + T_{22} + \frac{T_{11} + T_{12} + T_{21} + T_{22}}{2} \right).
\] (15)

### 3. RESULTS AND CONCLUSIONS

For the most part of the considered configurations, the radius of the chamber and the inlet / outlet ducts have the next values: \( R_C = 0.0766 \text{ m} \) and \( R_i = R_1 = 0.0243 \text{ m} \) respectively, and lengths \( l_A = 0.4 \text{ m} \). Is now added a baffle with a thickness \( t = 0.001 \text{ m} \). The chamber and duct lengths, \( l_t \) and the others dimensions are chosen to study different effects including the
partition, the baffle hole radius, and the presence of extended ducts. In figures 2 and 3 is presented the effect of the baffle position, for a total length of the muffler $l_T$ of 0.4 m and 0.28 m. We notice that if the length of the chamber grows up, the number of the arches is also growing as we expected.

In figure 4 is presented comparatively the effect of the extensions at baffle at inlet and outlet ducts $l_B = l_H = 35$ mm also then when $l_B = l_D = l_F = l_H = 25$ mm. It’s remarkable the fact that in case of the extensions at the baffle it’s obtained an arch more larger then in the others two cases. In figure 5 is presented the transmission loss depending on the frequency in cases when the total length of the muffler is changing. In this case the baffle
is center situated and the baffle hole diameter is the same as inlet-outlet of the muffler. We can notes that when the muffler length grows up also grow up the number of the arches.

Next it’s studied again the baffle hole effect with the baffle situated axial. Are taken in consideration three different values of the radius and the obtained results from the analytical model are presented in figure 6, respectively with the total length of the chamber $L = \text{mm}$. Amplitude $T_L$ shows as we expected, an general increase when the hole radius reduces. Also, when the limit is $R_2 = R_C$, the results are similar to those obtained of muffler with one chamber expansion. If the hole radius becomes smaller the amplitude of the first arch reduces.
Fig. 6. The baffle hole effect $R_1 = R_3 = 243$ mm;
$R_C = 766$ mm, $l_B = l_D = l_F = l_H = 30$ mm

REFERENCES