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TRANSFER MATRIX METHOD FOR DUAL-CHAMBER MUFFLERS

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Abstract

Mufflers are widely used for exhaust noise attenuation in vehicles, machinery and other industrial elements. Modelling procedures for accurate performance prediction had led to the development of new methods for practical muffler components in design. Plane wave based models such as the transfer matrix method (TMM) can offer fast initial prototype solutions for muffler designers. In the present paper the authors present an overview of the principles of TMM for predicting the transmission loss (TL) of a muffler. The predicted results agreed in some limits with the experimental data published in literature.

1. INTRODUCTION

Mufflers are commonly used in a wide variety of applications. Industrial flow ducts as well as internal combustion engines frequently make use of silencing elements to attenuate the noise levels carried by the fluids and radiated to the outside atmosphere by the exhausts. Design of a complete muffler system is, usually, a very complex task because each of its elements is selected by considering its particular acoustic performance and its interaction effects on the entire acoustic system performance. For the frequency analysis of the muffler, as can be seen from the references [1, 3, 5], it is very convenient to use the transfer matrix method. The present paper deals with the fundamentals of the Transfer Matrix Method (TMM) and the method is applied to a specific muffler configuration for the prediction of Transmission Loss.

2. TRANSMISSION LOSS BY TMM FOR DUAL-CHAMBER MUFFLER

Figure 1 shows the geometry of a circular dual-chamber muffler considered in this study where is note the dimensions of radius R_1, R_2, R_3, R_C and length of each section: l_B, l_C, l_D, l_F, l_E, l_G, l_H; and t - is the baffle thickness.

In accordance with dimensions on the figure are made the notations

$$l_{T1} = l_B + l_C + l_D\,,$$
$$l_{T2} = l_F + l_G + l_H\,, \tag{1}$$
$$l_T = l_{T1} + l_{T2} + t\,.$$

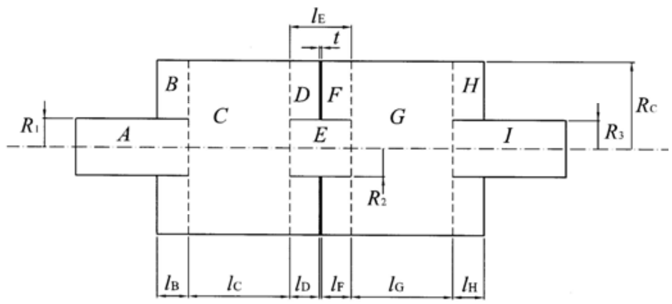


Fig. 1. Circular dual-chamber muffler

The planar wave propagation model is used in order to evaluate its applicability limits and to assess the need for end corrections. While a brief description of the relevant relationships is given follow-up for a detailed treatment. First, the four-pole matrices of the ducts are obtained. The product of these then yields the four-pole matrix of the entire configuration, which relates the pressure and mass velocity at the inlet and outlet and enables the evaluation of the acoustic attenuation performance by *TL*. The four-pole matrices of the ducts *A*, *E* and *I* are, respectively:

$$M_A = \begin{bmatrix} \cos(kl_A) & jY_A \sin(kl_A) \\ \dfrac{j}{Y_A}\sin(kl_A) & \cos(kl_A) \end{bmatrix}, \tag{2}$$

$$M_E = \begin{bmatrix} \cos(kl_E) & jY_E \sin(kl_E) \\ \dfrac{j}{Y_E}\sin(kl_E) & \cos(kl_E) \end{bmatrix}, \tag{3}$$

$$M_I = \begin{bmatrix} \cos(kl_I) & jY_I \sin(kl_I) \\ \dfrac{j}{Y_I}\sin(kl_I) & \cos(kl_I) \end{bmatrix}, \tag{4}$$

where: $Y_A = c_0/S_A$, $S_A = \pi R_1^2$; $Y_E = c_0/S_E$, $S_E = \pi R_2^2$; $Y_I = c_0/S_I$ and $S_I = \pi R_3^2$.

For the four-pole matrices of the two chambers *C* and *G*:

$$M_C = \begin{bmatrix} \cos(kl_C) & jY_C \sin(kl_C) \\ \dfrac{j}{Y_C}\sin(kl_C) & \cos(kl_C) \end{bmatrix}, \tag{5}$$

$$M_G = \begin{bmatrix} \cos(kl_G) & jY_G \sin(kl_G) \\ \dfrac{j}{Y_G}\sin(kl_G) & \cos(kl_G) \end{bmatrix}, \tag{6}$$

where: $Y_C = c_0/S_C$, $S_C = \pi R_C^2$, $Y_G = c_0/S_G$, $S_G = \pi R_C^2$.

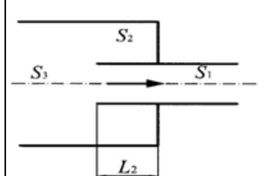
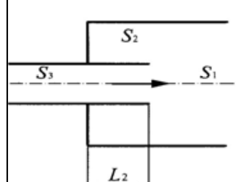
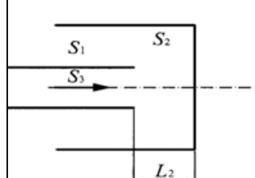
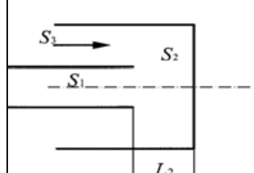
For cross-sectional discontinuities, using decreasing element-subscript values with distance from the noise source, the cross sectional areas upstream and downstream of transition (S_3, S_2 and S_1) are related through [8]

$$C_1 S_1 + C_2 S_2 + S_3 = 0, \tag{7}$$

where the constants C_1 and C_2 (Table 1) are selected so as to satisfy the mass conservation equation across the transition. Table 1 also shows the pressure loss coefficient K for each configuration that accounts for conversion of some mean-flow energy and acoustical field energy into heat at the discontinuities. As indicated, $K \le 0{,}5$ for area contraction, while $K \to (S_1/S_3)^2$ for area expansions at large values of S_1/S_3 . The four-pole matrices of the ducts with cross-sectional discontinuities [8] (for Mach number $M = 0$) is given by

$$M_{\text{discontinuities}} = \begin{bmatrix} 1 & 0 \\ \dfrac{C_2}{C_1\left(-j\dfrac{c_0}{S_2}\cot kl\right)} & 1 \end{bmatrix}. \tag{8}$$

Table 1.Parameter Values of Transition Elements

Element type	C_1	C_2	K
	-1	-1	$\dfrac{\left(1-\dfrac{S_1}{S_3}\right)}{2}$
	-1	1	$\left(\dfrac{S_1}{S_3}-1\right)^2$
	1	-1	$\left(\dfrac{S_1}{S_3}\right)^2$
	1	-1	0,5

Finally, the four extended regions are described in accordance with equation (8), by

$$M_B = \begin{bmatrix} 1 & 0 \\ \dfrac{1}{jY_B \cot(kl_B)} & 1 \end{bmatrix}, \tag{9}$$

$$M_D = \begin{bmatrix} 1 & 0 \\ -\dfrac{1}{jY_D \cot(kl_D)} & 1 \end{bmatrix}, \tag{10}$$

$$M_F = \begin{bmatrix} 1 & 0 \\ \dfrac{1}{jY_F \cot(kl_F)} & 1 \end{bmatrix}, \tag{11}$$

$$M_H = \begin{bmatrix} 1 & 0 \\ -\dfrac{1}{jY_H \cot(kl_H)} & 1 \end{bmatrix}, \tag{12}$$

with: $Y_B = c_0/S_B$, $S_B = \pi\left(R_C^2 - R_1^2\right)$, $Y_D = c_0/S_D$, $S_D = \pi\left(R_C^2 - R_2^2\right)$, $Y_F = c_0/S_F$, $S_F = \pi\left(R_C^2 - R_2^2\right)$, $Y_H = c_0/S_H$, $S_H = \pi\left(R_C^2 - R_3^2\right)$.

The matrix of the entire configurations relates the pressure P and mass velocity V at the inlet and outlet by:

$$\begin{bmatrix} P_{input} \\ V_{input} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \cdot \begin{bmatrix} P_{output} \\ V_{output} \end{bmatrix}, \tag{13}$$

where T_{11}, T_{12}, T_{21} and T_{22} are referred to as the four-poles of the acoustical system, which are obtained by multiplying the previous matrices as

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = M_A M_B M_C M_D M_E M_F M_G M_H M_I . \tag{14}$$

Finally, the transmission loss is obtained by

$$TL = 20\log\left[\left(\frac{Y_I}{Y_A}\right)^{1/2}\left|\frac{T_{11} + T_{12}/Y_1 + T_{21}Y_A + T_{22}\left(Y_A/Y_I\right)}{2}\right|\right]. \tag{15}$$

3. RESULTS AND CONCLUSIONS

For the most part of the considerated configurations, the radius of the chamber and the inlet / outlet ducts have the next values $R_C = 0.0766\,\text{m}$ şi $R_1 = R_3 = 0.0243\,\text{m}$ and respectively lengths $l_A = l_I = 0.4$ m. Is now added a baffle with a thickness $t = 0{,}001\,\text{m}$. The total chamber length, l_T and the others dimensions are chosen to study different effects including the partition, the baffle hole radius, and the presence of extended ducts. In figures 2 and 3 is presented the effect of the baffle position, for a total length of the muffler l_T of 0.4 m and 0.28 m. We notice that if the length of the chamber grows up, the number of the arches is also growing as we aspected.

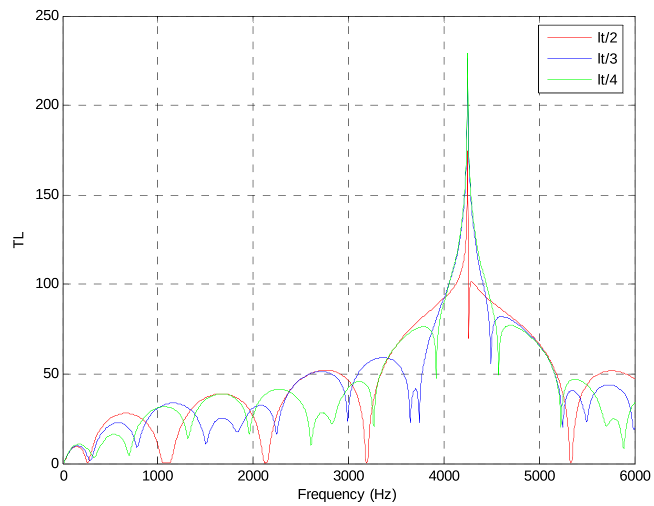


Fig. 2. The partitions effect, when the baffle is on $l_t/2$, $l_t/3$, $l_t/4$; $l_B = l_D = l_F = l_H = 20$ mm; $t = 1$ mm, $R_2 = 243$ mm

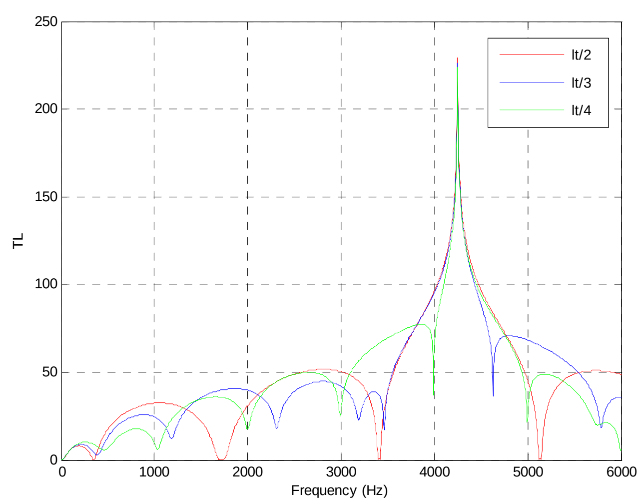


Fig. 3. The effect of the partitions, when the baffle is on $l_t/2$, $l_t/3$, $l_t/4$; $l_B = l_D = l_F = l_H = 20$ mm; $t = 1$ mm, $R_2 = 243$ mm

In figure 4 is presented comparatively the effect of the extensions at baffle $l_D = l_F = 35$ mm, at inlet and outlet ducts $l_B = l_H = 35$ mm also then when $l_B = l_D = l_F = l_H = 25$ mm. It's remarquable the fact that in case of the extensions at the baffle it's obtained an arch more larger then in the others two cases. In figure 5 is presented the transmission loss depending on the frequency in cases when the total length of the muffler is changing. In this case the baffle is center situated and the baffle hole diameter is the same as inlet-oulet of the muffler. We can notes that when the muffler length grows up also grow up the number of the arches.

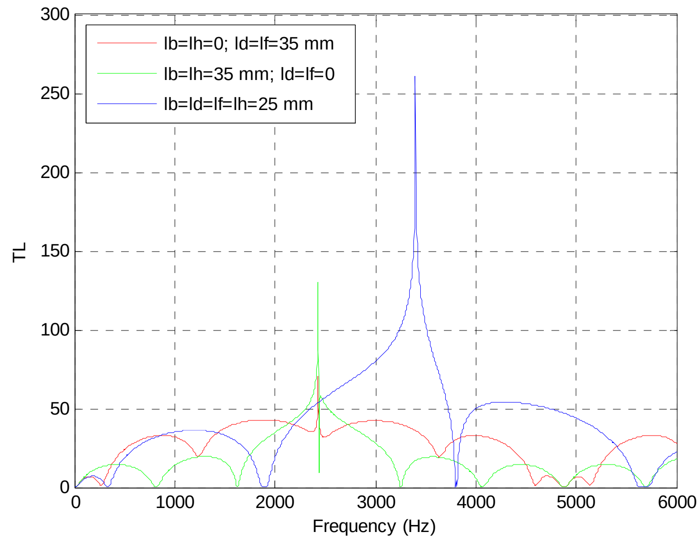


Fig. 4. The effect due to the presence of the extensions at the baffle center situated at the inlet and outlet ducts

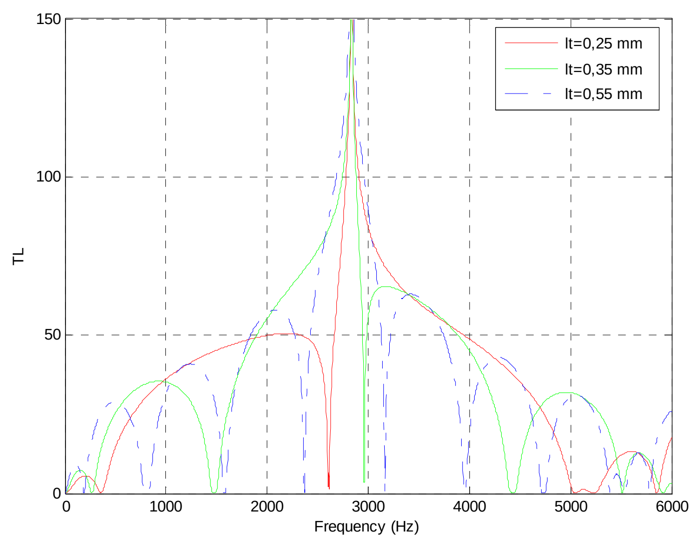


Fig. 5. The effect due to total length of the muffler for dimensions $R_1 = R_2 = R_3 = 243$ mm; $R_C = 766$ mm, $l_B = l_D = l_F = l_H = 30$ mm

Next it's studied again the baffle hole effect with the baffle situated axial. Are taken in consideraion three different values of the radius and the obtained results from the analitical model are presented in figure 6, respectively with the total length of the chamber $l_T = 0.4$ m. Amplitude *TL* shows as we aspected, an general increase when the hole radius reduces. Also, when the limit is $R_2 = R_C$, the results are similar to those obtained of muffler with one chamber expansion. If the hole radius becomes smaller the amplitude of the first arch reduces.

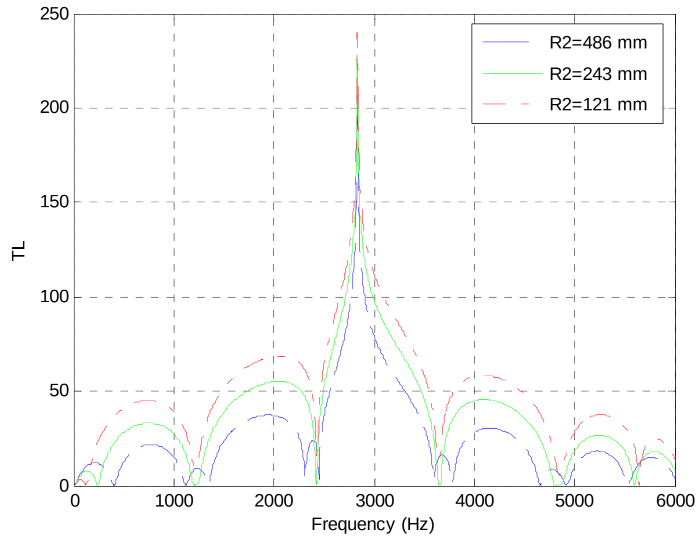


Fig. 6. The baffle hole effect $R_1 = R_3 = 243$ mm;
$R_C = 766$ mm, $l_B = l_D = l_F = l_H = 30$ mm

REFERENCES

[1] M. Bugaru, O. Vasile, "Transfer matrix method for single chamber mufflers", *Proceedings of the 11th WSEAS International Conference on Applied Mathematics*, 22-24 March 2007, Dallas, TX, USA, pp. 47-50.

[2] J.Y. Chung, and D.A. Blaser, 1980, "Transfer Function Method of Measuring In-Duct Acoustic Properties: I. Theory and II. Experiment", *Journal of the Acoustical Society of America*, Vol. **68**, No. 3, pp. 907-921.

[3] P.O.A.L. Davies, 1993,"Realistic Models for Predicting Sound Propagation in Flow Duct Systems", *Noise Control Engineering Journal*, Vol. **40**, pp. 135-141.

[4] S.N.Y. Gerges, F.A. Thieme, R. Jordan, J.L.B. Coelho, J.P. Arenas, 2000, "Muffler Modeling by Transfer Matrix Method and Experimental Verification", *Journal of The Brazilian Mechanical Sciences*, Vol.**31**.

[5] F.P. Mechel, *Formulas of Acoustics*, Springer-Verlag, Berlin, 1175 p, 2002.

[6] M.L. Munjal, *Acoustics of Ducts and Mufflers*. 1st Ed., John Wiley and Sons, New York, 328 p, 1987.

[7] M.L. Munjal, "Plane Wave Analysis of Side Inlet/Outlet Chamber Mufflers with Mean Flow", *Applied Acoustics*, Vol. **52**, pp. 165-175, 1997.

[8] L.L. Beranek, L. Istvan, *Noise and Vibration Control Engineering: Principles and Applications*, John Wiley & Sons, Inc, ISBN 0-471-61751-2, 1992.