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# RECTIFICATIONS OF PARAMETRIC INSTABILITY DUE TO ASYMMETRY OF ROTATING SHAFT

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**Abstract**

In this work, the asymmetric stiffness based parametric instability problem of rotating shaft system is analytically analyzed and its rectifications are presented. The term responsible for the problem in the governing equation is explored to find out the parameters having the problem elimination or minimization potential. Two such analytical findings have been practically implemented and the problem has successfully been rectified.

Although both the strategies qualified, yet one of them has an edge over the other in implementation, risk elimination and qualitative-quantitative results. The two strategies attack the problem from two different visions, one pre and the other post problem symptoms observation. The post problem symptoms observation strategy has constraints and can be implemented only to the rotating shafts which had survived in the initial testing while the pre problem sensing approach is independent of any problem symptom, much easy in implementation and unlimited survival rate. Hence the last one, the pre-emptive measure is to be preferred over the post.

## 1. INTRODUCTION

Since the start of era of high-speed rotating machines like turbines, multistage pumps, compressors, turbo generators etc, the instability problems are on the top of the scene posing critical threat to stability of rotating systems. There are various causes and categories of these instabilities and some of them are extremely dangerous. A number of researchers have analyzed the problem mostly within their problem domain.

Ehrich [1] categorized various instabilities based upon their behaviour and possible causes. Ehrich [2] and Barrister [3] presented a mathematical treatise to elaborate the mechanics of the shaft whirl due to material internal damping. Bentley *et al*. [4] and Kimball [5] described the mechanics of instability due to rotor internal friction. Childs [6] covered the instabilities due to non symmetric clearance effect, while Ehrich [7] modelled the sub-harmonics response of bearing clearance non linearity with low damping, whereas Vance [8] covered the torque whirl failure due to over hanged discs or wheels. Zvolanek [9] discussed the stability of unsymmetric rotor on an unsymmetric support. Black *et al*. [10], Crandall *et al*. [11] , Brosens *et al*. [12], Foote *et al*. [13], Yamamoto *et al*. [14], Peters *et al*. [15] and Smith [16] analyzed the instability problem due to asymmetry of the rotors.

The rotating journals under study of the present authors were generally okay from the balancing criterion point of view, but still they were very critical while traversing up toward or down from the operational speed. By changing the combinations of rotor shaft, bearings, holding structure etc, it was assessed that problem is with the rotor shaft. The literature was surveyed and various causes of instabilities were reviewed in the light of rotor shaft based problems. It was thus assessed that rotor journal perhaps has the stiffness asymmetry problem. It was also assessed that the production process and human error are responsible for this limited inherent asymmetry. The authors concern was to rectify the problem rather than analytical or computerized simulation results analysis, discussions and results based rectifications' suggestions. Thus the authors adopted mathematical analysis route which to the authors' knowledge, none of the researcher opted so far. The mathematical model of the system developed by Yamamoto *et al*. [14] was analyzed within this scenario for rectification of the problem through possible handling of the responsible parameters and it seems to be more direct and simplest approach leading to the desired goal. The adopted approach is discussed below.

## 2. MATHEMATICAL SCENARIO OF THE PROBLEM

Crandall *et al*. [11] and Brosens *et al*. [12] , Black *et al*. [10] and Smith [16] discussed rotor instability as a result of its stiffness asymmetry based upon the Euler angles and kinematical equations [17]

$$\begin{aligned} \omega_X &= \dot{\varphi} \sin\theta \sin\psi + \dot{\theta} \cos\psi \\ \omega_Y &= \dot{\varphi} \sin\theta \cos\psi - \dot{\theta} \sin\psi \\ \omega_z &= \dot{\psi} + \dot{\varphi} \cos\theta \,. \end{aligned} \tag{1}$$

Here $\varphi$, $\theta$ & $\psi$ are Euler angles and $\omega_x$, $\omega_Y$ & $\omega_z$ or only $\omega$ are angular velocities about the three axes. Using following dimensionless quantities

$$\begin{aligned} &\mathrm{I_p}/\mathrm{I} = \mathrm{i_p}, && \Delta\mathrm{I}\ /\mathrm{I} = \Delta, && \mathrm{x}/(\mathrm{I/m})^{1/2} = x', \\ &\mathrm{y}/(\mathrm{I/m})^{1/2} = y', && \omega/(\alpha/\mathrm{m})^{1/2} = \omega', && (m/\mathrm{I})^{1/2}\,\gamma/\alpha = \gamma', \\ &(\alpha/\mathrm{m})^{1/2}\, t = t', && m\delta/(\alpha\,\mathrm{I}) = \delta', \end{aligned} \tag{2}$$

Here *m*, mass of the rotor shaft; Ip, polar moment of inertia; Ix and Iy moment of the inertia about the lateral axes; $2\mathrm{I} = \mathrm{I_x} + \mathrm{I_y}$; $2\Delta\mathrm{I} = \mathrm{I_x} - \mathrm{I_y}$ and $\alpha$, $\gamma$ & $\delta$ are spring constants of the shaft. Yamamoto *et al*. [14] arrived at their following normalized form after deriving rotor equation of motion with the help of Lagrange's equation developed from kinetic and potential energy equations and dropping the prime in the final expression:

$$\begin{aligned} \ddot{\mathrm{x}} + \mathrm{x} + \gamma\theta_{\mathrm{x}} &= 0 \\ \ddot{\mathrm{y}} + \mathrm{y} + \gamma\theta_{\mathrm{y}} &= 0 \end{aligned} \tag{3}$$

pertaining to the linear motions and

$$\ddot{\theta}_x + i_p\, \omega \dot{\theta}_y + \Delta\left\{ 2\omega(\dot{\theta}_x \sin 2\omega t - \dot{\theta}_y \cos 2\omega t) - (\ddot{\theta}_x \cos 2\,\omega\, \mathrm{t} + \ddot{\theta}_y \sin 2\,\omega\, \mathrm{t}) \right\}$$
$$+ \gamma \mathrm{x} + \delta\theta_{\mathrm{x}} = 0$$

$$\ddot{\theta}_y - i_p\, \omega \dot{\theta}_x - \Delta\left\{ 2\omega(\dot{\theta}_x \cos 2\omega t + \dot{\theta}_y \sin 2\omega t) + (\ddot{\theta}_x \sin 2\,\omega\, \mathrm{t} - \ddot{\theta}_y \cos 2\,\omega\, \mathrm{t}) \right\}$$
$$+ \gamma \mathrm{y} + \delta\theta_{\mathrm{y}} = 0 \qquad (4)$$

pertaining to the angular motions.

Here $\omega t = \theta_Z + \pi/2$ and the shaft angular velocity $\omega = \dot{\theta}_Z$

The particular form of equation (4), if mathematically generalized becomes

$$\ddot{\theta}_x + i_p\, \omega \dot{\theta}_y + \Delta\left\{ 2n\omega(\dot{\theta}_x \sin 2n\omega t - \dot{\theta}_y \cos 2n\omega t) - (\ddot{\theta}_x \cos 2n\omega \mathrm{t} + \ddot{\theta}_y \sin 2n\omega \mathrm{t}) \right\}$$
$$+ \gamma \mathrm{x} + \delta\theta_{\mathrm{x}} = 0$$

$$\ddot{\theta}_y - i_p\, \omega \dot{\theta}_x - \Delta\left\{ 2n\omega(\dot{\theta}_x \cos 2n\omega t + \dot{\theta}_y \sin 2n\omega t) + (\ddot{\theta}_x \sin 2n\omega \mathrm{t} - \ddot{\theta}_y \cos 2n\omega \mathrm{t}) \right\}$$
$$+ \gamma \mathrm{y} + \delta\theta_{\mathrm{y}} = 0 \qquad (5)$$

The coefficient ‘2n’ of ω means, that these equations hold for all integer values of ‘n’. This means that instability can occur only when natural frequency of the system is an even integer multiple of the rotor speed and never for odds i.e. these functions have instabilities only at all integer values of *n*. The lowest value of *n* i.e. *n* =1 pertains to sub-synchronous while higher values pertains to super–synchronous instabilities. In generalizing the equation (4), Smith [16] and Peters *et al*. [15] treated coefficient ‘2n’ of ω of equation (5) as ‘n’ and thus for an explicit even integer, they unknowingly introduced odd values too and then had to assign some thing for its non occurring, which had explicit mathematical denial.

Further to this is, that if either sum or the difference of frequencies of any two components of a system [15] or only that of a component all alone [16], is an even integer multiple of the rotor speed, then this may give rise to resonance and hence the instability problem.

## 3. PROBLEM IDENTIFICATION AND RECTIFICATION

As discussed above, various instabilities and their root causes and symptoms discussed in literature were reviewed in the light of available information and observations and this review process guided towards the asymmetry of the rotor. With stiffness asymmetry in view, the generalized form of Yamamoto *et al*. [14] normalized rotor equation of motion (Eq. 5 above) was found to be more explicit and straight forward, hence it was analyzed for possible rectification of the problem. The 3rd term in these equations as shown below pertains to the instability under discussion.

$$\Delta\left\{2n\omega(\dot{\theta}_x \sin 2n\omega\, t - \dot{\theta}_y \cos 2n\omega\, t) - (\ddot{\theta}_x \cos 2n\omega\, \mathrm{t} + \ddot{\theta}_y \sin 2n\omega\, \mathrm{t})\right\}$$

and

$$-\Delta\left\{2n\omega(\dot{\theta}_x \cos 2n\omega\, t + \dot{\theta}_y \sin 2n\omega\, t) + (\ddot{\theta}_x \sin 2n\omega\, \mathrm{t} - \ddot{\theta}_y \cos 2n\omega\, \mathrm{t})\right\}$$

The 2nd term pertaining to gyroscopic precession most probably may excite too by the angular motion resulting from consequences of the 3rd term, yet it is not the root cause of this problem and is not considered any more in this and will be addressed in separate work.

For solution of this instability problem, either the influence of these expressions be counter balanced through damping or minimized through manipulations of parameters of these expressions. The damping can be increased to some extent and has limitations. An overview of these expressions show that all other parameters are out of reach and cannot be manipulated except their coefficient $\Delta$ given by equations (2) above and reproduced as follows.

$$\Delta = \Delta \mathrm{I/I} = \frac{\mathrm{I}_1 - \mathrm{I}_2}{\mathrm{I}_1 + \mathrm{I}_2} \qquad (6)$$

The smaller is the $\Delta$, the better will be the instability control.

The overview of its expression shows that there can be two approaches, (i) reduce the numerator, (ii) increase the denominator. Both these strategies exploit this manageable parameter $\Delta$ for solution of the problem as follows.

### 3.1 Numerator Handling Strategy

In the first strategy, the numerator is decreased either through decrease in $\mathrm{I}_1$ or increase in $\mathrm{I}_2$, both being proportional to stiffness [18] and this decreases the value of coefficient $\Delta$, thus decreasing the amplitude of the excitation. The changes are achieved through relative differential increase or decrease in dimensions under centrifugal action of additional correction weights.

The decrease in numerator is, however, not independent of denominator. Within this method, the 2nd approach, i. e. increase in $\mathrm{I}_2$ is both, mathematically and practically more effective, because it decreases the numerator along with increasing the denominator and these both contribute positively in reducing the coefficient $\Delta$, whereas decreasing $\mathrm{I}_1$, decreases both the numerator and the denominator and decrease in denominator has negative effect, hence their ultimate role is thus slower.

The 2nd option was hence adopted and after a number of trials, solution to the problem was arrived through disturbing the phase combinations of residual unbalance masses by applying the small disturbing masses. The strategy worked to some extent, but was much laborious and with very limited benefits as it could not be applied to all the rotors as most of them had not this problem and applying this strategy to them was to add undue unbalance. It was thus applied only to critical rotors which could survive in the initial testing. The method is both mathematically and practically less effective being a hit and trial approach.

### 3.2 Denominator Handling Strategy

In the second strategy, the denominator is increased though equal increase in both $I_1$ and $I_2$. In this way, the denominator is increased without changing the numerator. Thus coefficient $\Delta$ and hence the excitation is decreased without any change to the balancing criterion of the main rotor. And since the balancing criterion is not changed, the strategy can be applied to all the rotating journals irrespective of their imbedded instability without any risk generation. The achievement of this can be through any component of a rotating journal for example the driving shaft. This method is mathematically and practically more effective as compared to the first one being a direct approach without any constraint.

### 3.3 Damping Solution Strategy

The damping term in equations (3) and (5) has not been included, but if damping is increased, the amplitude of vibration will not increase rapidly in the critical zones.

### 3.4 Cruising Through Critical Zone at Faster Pace

The critical zones of a rotating journal are generally known by experience or can be analytically computed. The machine or journal while cruising towards or from the operational speed at a faster pace will have little time for resonance excitation build up and thus may pass through the dangerous zone well below the dangerous limit and hence the system will remain safe and this strategy is generally used in normal practice.

## 4. CONCLUSIONS

The problem due to un-symmetry of stiffness with respect transverse axes of the rotating journal has an important handle able node mathematically defined by

$$\Delta = \Delta I/I = \frac{I_1 - I_2}{I_1 + I_2}$$

The greater is the value of this $\Delta$ , the critical is the problem. There are two approaches to minimize its value. One is to minimize the numerator, the other is to symmetrically increase the denominator, and these two conclude as follows.

1. The denominator handling strategy i.e. increased overall stiffness of rotating journal is more effective, easy, rugged and has risk free application over the entire production range as compared to that of numerator handling strategy.

2. The numerator handling strategy has further two options i.e. decreasing the stiffness difference through increasing the smaller quantity or through decreasing the larger quantity. The option of increasing the smaller stiffness is relatively more effective both mathematically and experimentally.

3. Both the solution strategies including their additional options are both scientifically and theoretically logical.

4. The degree of suitability and effectiveness of both the strategies and their additional options are mathematically well supported.

5. Damping and cruising through critical zones at a faster rate are logical and have supporting behaviour.

6. Peters *et al*. [15] statement based on Smith [16] prediction and his own practical experience that "**an instability occurs when natural frequency of the system is an even integer multiple of the rotor speed and does not for odds**" is mathematically supported and confirmed as the equations (5) hold only for integer '*n*', enforced to even by the coefficient 2 as shown in equation (5). It rather ratifies the first part of their statement and negates altogether the existence of any form of their later option for which they had to give explanation of its miniature-ness.

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