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ON REDUCING THE SHOCK RESPONSE OF A SINGLE DEGREE OF FREEDOM VEHICLE MODEL BY SWITCHING THE DAMPING COEFFICIENT

Tim Waters, [1], Youngjin Hyun [2] and Michael Brennan[1]

[1] Institute of Sound and Vibration Research, University of Southampton, Southampton SO17 1BJ, UK

[2] Vehicle dynamic lab, R&D centre, Hyundai motors

tpw@isvr.soton.ac.uk

Abstract

This paper concerns the acceleration response of a vehicle traversing a road bump. A linear viscously damped single degree of freedom model is adopted initially from which closed form expressions for the time response are obtained. By assuming the impulse to be of short duration relative to the natural period, these expressions simplify to show that the peak acceleration response is approximately proportional to the damping ratio.

Numerical simulations are then presented for two alternative damping models: a piece-wise linear model, in which the damping ratio is different in the jounce and rebound directions; and a switchable damper that switches to a lower value during the impulse. The piecewise linear damper delivers modest reductions in peak acceleration when, as is common practice in automotive dampers, the damping is less in jounce than rebound. Larger reductions are achievable from the switchable damper and a simple approximate expression is given for the maximum possible reduction.

1. INTRODUCTION

A delicate payload must often be isolated from the base that supports its weight in order to reduce transmitted vibration. In the case of road vehicles isolation usually takes the form of separate springs and dampers. The inputs to the base of the suspension are often largely random in nature and can be characterised statistically by power spectral densities of the road roughness [1]. If linearity of the mechanical system can be assumed then the response is easily estimated from random vibration theory provided the transfer functions between the contact patch and receiver are known. The response of nonlinear systems, on the other hand, can be computed by Monte Carlo simulation [2]. Road profiles, however, are not entirely random and feature transient events due to bumps, hollows, ramps, expansion gaps and, increasingly, speed humps [3]. The mean square response per unit frequency that random vibration theory predicts is not always the quantity of concern so much as the peak acceleration. This is especially true in the case of isolating vehicle occupants since human discomfort from shock inputs is disproportionate [4].

Acceleration shock response has been considered by many researchers, most notably for single degree of freedom (SDOF) systems in much earlier literature where closed form solutions were more readily sought. Snowdon used Laplace transforms to solve for the acceleration response due to rounded step-like and pulse-like displacements with finite rise times [5]. The shock spectra for a range of other input shapes are presented in [6], although only for the undamped case.

This paper is concerned with the acceleration response of a road vehicle to a symmetric bump-shaped transient displacement input and adopts the Laplace transform approach as in [5]. The principle aim is to gain further understanding of the effect of damping on peak acceleration levels. A viscously damped SDOF system is chosen which, although only representative below the frequency of the wheel-hop mode, enables an insightful closed form solution to be obtained.

Automotive dampers are most commonly passive hydraulic devices whose characteristics are chosen “once and for all” so as to achieve an appropriate compromise between ride comfort and other performance criteria such as road holding, handling and control. In practice, this leads to a distinctly non-linear force-velocity curve exhibiting higher damping in extension than compression. A ratio of about 3:1 is often purported on the basis of numerical optimisation [2] or empirical grounds [5,7]. A more favourable compromise can be achieved by semi-active damping [8,9] in which the damping coefficient is modulated either gradually or in real time according to the observed vehicle response. Cost, weight and reliability issues count against such systems except where significant performance gains can be demonstrated.

In the first part of the paper it is shown that the shock response of a vehicle is predominantly due to the damper force during the impulse. For transient inputs at least this suggests that the peak acceleration may be reduced by switching the damper to a lower value during an impulse. The second part of the paper investigates the potential benefits of a switchable damper of this type. Numerical simulations are presented for a SDOF system to illustrate the improvement over a linear damper, and a simple expression is obtained in closed form for the maximum possible reduction in peak acceleration. The results are also contrasted with those of a piece-wise linear damper model which is used elsewhere in the literature to represent the dual-rate characteristic of the typical automotive damper [2,8,10].

2. SDOF VEHICLE MODEL TRAVERSING A VERSED SINE BUMP

Consider the linear viscously damped SDOF quarter car model shown in Fig. 1. When the system traverses a road bump a displacement y is applied to the base of the spring and damper causing a displacement x of the mass.

Both displacements are written as functions of non-dimensional time,

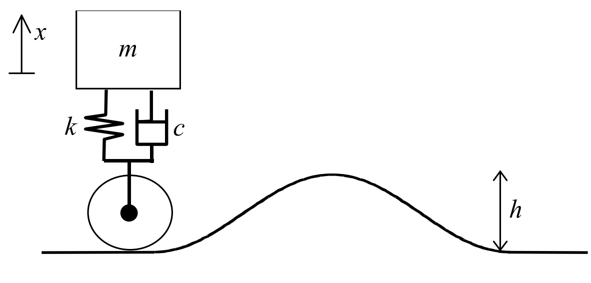


Figure 1. A damped SDOF system subject to a versed sine-shaped base displacement

$$\hat{t} = \frac{t}{\tau} \tag{1}$$

where τ is the duration of the impulse, and the displacements are themselves non-dimensionalised by the peak input displacement h as follows:

$$\hat{x} = \frac{x}{h} \qquad \hat{y} = \frac{y}{h} \tag{2a,b}$$

Then the equation of motion for the mass can be written as

$$\ddot{\hat{z}} + 4\pi\zeta \frac{1}{r}\dot{\hat{z}} + 4\pi^2 \frac{1}{r^2}\hat{z} = -\ddot{\hat{y}} \tag{3}$$

where $\hat{z} = \hat{x} - \hat{y}$ is the non-dimensional relative displacement, $\zeta = c/2\sqrt{mk}$ is the damping ratio and

$$r = \frac{T}{\tau} \tag{4}$$

is the ratio of natural period T to impulse duration. Of primary interest in this study are impulses of relatively short duration such that $1 < r < 10$. When $r > 10$, the wheel-hop mode is typically important and the SDOF model is invalid.

The choice of pulse input to represent a road bump is to some extent arbitrary. A versed sine impulse is chosen here,

$$\hat{y}(\hat{t}) = \begin{cases} \frac{1}{2}\left(1 - \cos 2\pi\hat{t}\right) & 0 \le \hat{t} \le 1 \\ 0 & \hat{t} > 1 \end{cases} \tag{5}$$

whose derivative is continuous thus ensuring a finite acceleration input for all time. (The response of an *undamped* SDOF system to a versed sine input is given in [3]). Substituting for $\hat{y}(\hat{t})$ into Eq. (3) gives the equation of motion of the system due to a versed sine base displacement as

$$\ddot{\hat{z}} + 4\pi\zeta \frac{1}{r}\dot{\hat{z}} + 4\pi^2 \frac{1}{r^2}\hat{z} = \begin{cases} -2\pi^2 \cos 2\pi\hat{t}, & 0 \le \hat{t} \le 1 \\ 0 & \hat{t} > 1 \end{cases} \tag{6}$$

Herein, it is assumed that a point contact is maintained between the wheel and the road at all times. Second order ordinary differential equations of the form of Eq. (6) are easily solved by numerical methods. In this case, an analytical solution is possible by application of Laplace transforms which yields rather more physical insight into the system.

The peak acceleration *during* the impulse ($0 \le \hat{t} \le 1$) is the focus of this paper, and is given by

$$\ddot{x}(\hat{t}) = \frac{h}{\tau^2}\ddot{\hat{x}}(\hat{t}) \tag{7}$$

where

$$\begin{aligned} \ddot{\hat{x}}(\hat{t}) = a(r)\Bigg\{ & b(r)\cos 2\pi\hat{t} + c(r)\sin 2\pi\hat{t} + d(r)e^{-2\pi\zeta\frac{1}{r}\hat{t}}\cos\left(2\pi\sqrt{1-\zeta^2}\,\frac{1}{r}\hat{t}\right) \\ & + e(r)e^{-2\pi\zeta\frac{1}{r}\hat{t}}\sin\left(2\pi\sqrt{1-\zeta^2}\,\frac{1}{r}\hat{t}\right)\Bigg\} \end{aligned} \tag{8}$$

and

$$a(r) = 2\pi^2 \frac{1-r^2}{(2\zeta r)^2 + \left(1-r^2\right)^2} \qquad b(r) = 1 + \frac{(2\zeta r)^2}{1-r^2} \qquad c(r) = \frac{2\zeta r^3}{1-r^2}$$

$$d(r) = -\left(1 + \frac{(2\zeta r)^2}{1-r^2}\right) \qquad e(r) = \frac{1+r^2}{1-r^2}\cdot\frac{\zeta\left(2\zeta^2-1\right)}{\sqrt{1-\zeta^2}} + 2\zeta\left(1-\zeta^2\right)^2 \tag{9}$$

Other quantities of interest such as absolute and relative displacement during and after the impulse are readily obtainable in closed form. However, for brevity, their expressions are omitted and an illustrative example presented instead.

Fig. 2(a) to (d) show the displacement input along with the resulting displacement, relative displacement and acceleration time responses for a system with a damping ratio of 0.25 subjected to an impulse of short duration ($r = 5$). The results are indistinguishable from those obtained using Runge-Kutta direct numerical integration (not shown). Figure 2(c) shows that the road input is largely absorbed by the spring, but significant residual displacement is apparent in Fig. 2(b) after the impulse. Conversely, the acceleration is largest during the impulse, Fig. 2(d).

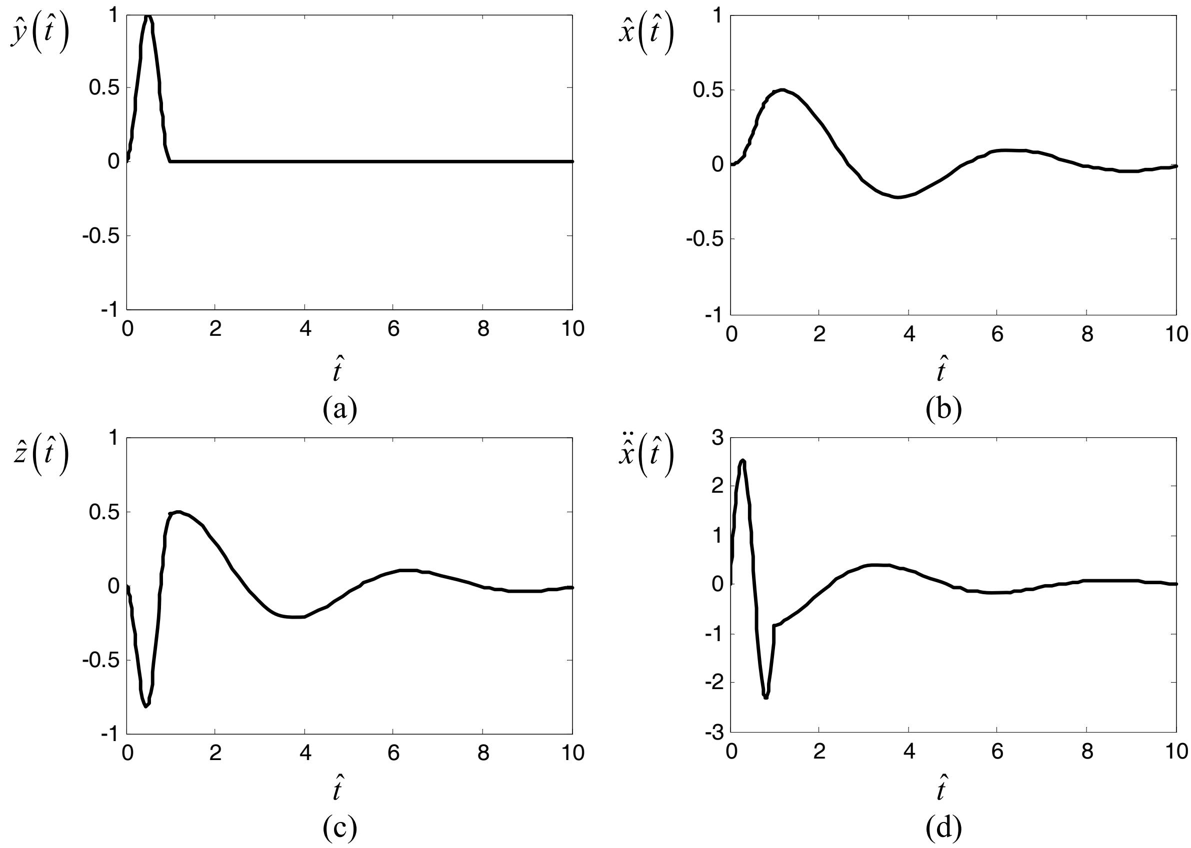


Figure 2. Transient response of a SDOF system with a natural period of 5 times the impulse duration ($r = 5$) and damping ratio of 0.25 to a versed sine base displacement input. (a) displacement input; (b) displacement response; (c) relative displacement response; and (d) acceleration response.

3. APPROXIMATE SOLUTIONS FOR SHORT IMPULSES

The closed form solution presented in the previous section for the response of a SDOF system to a versed sine shaped displacement input is too complicated to exhibit any simple relationships between the acceleration response and the parameters of the system and excitation such as damping ratio, natural period and impulse duration. However, useful approximations may be obtained in the case when the natural period is much longer than the impulse ($r \gg 1$). Expanding Eq. (8) and (9) for small $1/r$ and rearranging gives

$$\ddot{x}(t) \approx \frac{2\pi^2}{r^2}\left(\sqrt{\left(1-4\zeta^2\right)^2+4\zeta^2 r^2}\,\sin\left(2\pi\hat{t}-\varphi\right)+\left(1-4\zeta^2\right)\right)+O\left(\frac{1}{r^3}\right) \tag{10}$$

where

$$\varphi = \arctan\frac{1-4\zeta^2}{2\zeta r} \tag{11}$$

This is a phase shifted sinusoid with an offset, from which it is clear that the maximum and minimum values of acceleration occur when

$$\hat{t} = \frac{1}{4}+\frac{\varphi}{2\pi} \quad \text{and} \quad \hat{t} = \frac{3}{4}+\frac{\varphi}{2\pi} \tag{12a,b}$$

respectively, and are given by

$$\ddot{x}(t)\Big|_{\substack{\max\\ \min}} = \frac{2\pi^2}{r^2}\left(\pm\sqrt{\left(1-4\zeta^2\right)^2+4\zeta^2 r^2}+\left(1-4\zeta^2\right)\right) \tag{13}$$

If $\zeta < \frac{1}{2}$ then the peak accelerations (maximum and minimum) occur slightly later than ¼ and ¾ of the way through the impulse and it is the maximum acceleration that is largest in magnitude. If $\zeta > \frac{1}{2}$ then the peak accelerations occur slightly earlier than ¼ and ¾ of the way through the impulse and it is the minimum acceleration that is largest in magnitude.

Figure 3 shows the peak acceleration as a function of *r*, as given by the second order approximation in $1/r$ from Eq. (13). The damping ratio of the system is 0.25. These approximate solutions are compared with numerically obtained peaks from the exact analytical time responses.

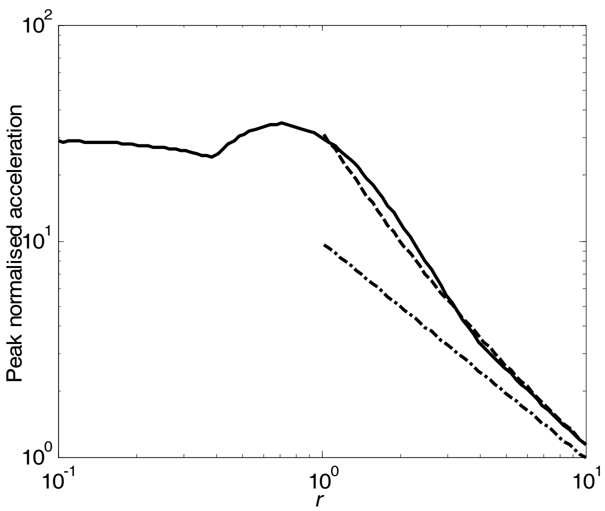


Figure 3. Peak normalised acceleration as a function of the ratio of the natural period to the impulse duration for a damping ratio of 0.25. – exact solution; -- second order approximation (Eq. 17); ·- · first order approximation (Eq. 19)

Also shown is a first order approximation which is obtained by making the additional assumption on the natural period that $2\zeta r \gg \left|1-4\zeta^2\right|$. The acceleration time response becomes

$$\ddot{x}(t) \approx \frac{4\pi^2\zeta}{r}\sin 2\pi\hat{t} \tag{14}$$

and the peak acceleration values are given simply by

$$\ddot{x}(t)\Big|_{\substack{\max\\ \min}} \approx \pm\frac{4\pi^2\zeta}{r} \tag{15}$$

From Fig. 3 it can be seen that when $\zeta = 0.25$, the second order approximation given in Eq. (13) is in close agreement for the range of interest, $1 \le r \le 10$. The first order approximation from Eq. (15) is qualitatively correct from which it is apparent that the peak acceleration is approximately proportional to damping ratio and inversely proportional to natural period.

Unsurprisingly, the peak acceleration is observed to be highest when the natural period is comparable to the impulse duration ($r \sim 1$). When the natural period is much shorter than the impulse duration, the peak acceleration is observed to be largely insensitive to natural period. In the limiting situation of a rigid suspension, the acceleration of the mass is the same as that of the road input, which is given by the right hand side of Eq. (6). It follows that the peak normalised acceleration in this case is given by

$$\frac{\ddot{x}_{\max}}{h/\tau^2} = 2\pi^2 \qquad (16)$$

For impulses of long duration compared to the natural period the peak acceleration is approximately proportional to the square of vehicle speed and only weakly dependent on stiffness and damping.

4. SHOCK RESPONSE OF A SDOF SYSTEM WITH SWITCHABLE DAMPER

For shocks of short duration compared with the natural period, the peak acceleration occurs during the bump and, by Eq. (15), its magnitude is approximately proportional to the damping ratio. It may be argued that, for a single transient displacement input, the peak acceleration may be reduced by switching from a high damping ratio Ξ to a low damping ratio, α times Ξ, during the input to minimise transmitted forces to the mass, i.e.

$$\zeta\left(\hat{t}\right) = \begin{cases} \alpha\Xi & 0 < \hat{t} < 1 \\ \Xi & \hat{t} > 1 \end{cases} \qquad (17)$$

Here, α is a constant and will be referred to as the damping reduction factor. Issues regarding feasibility and practicability of implementing such a damper in a semi-active manner are acknowledged by the authors but not addressed in this paper. Instead, the objective is to assess the maximum potential benefit of such an ideal damper in reducing peak acceleration to inform judgement as to whether implementation of such a damper is worth pursuing further.

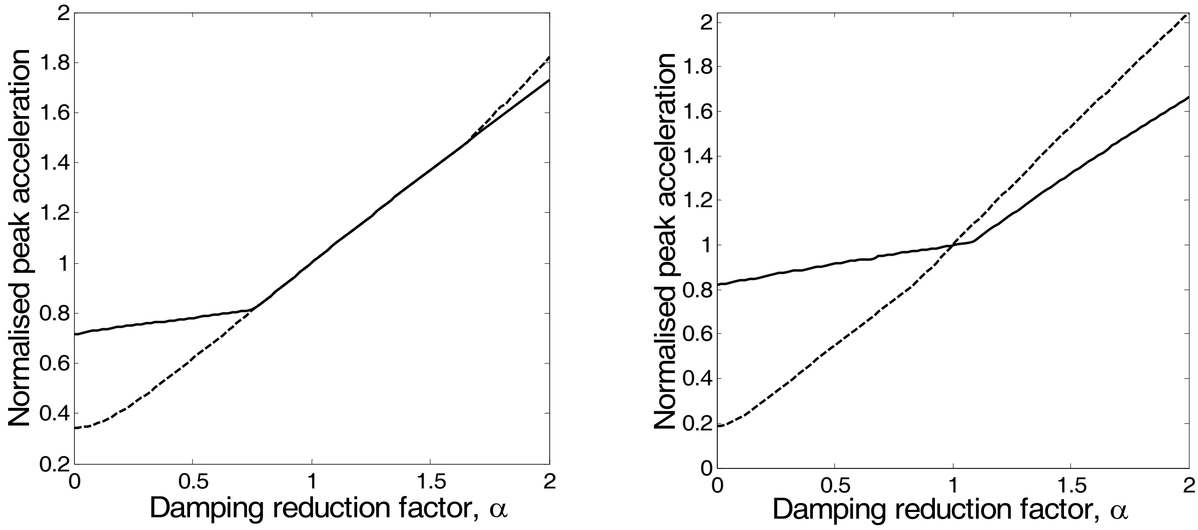
The performance of this switchable damper is compared with that of a piecewise linear damping model which is a commonly adopted to represent a conventional automotive damper [2,8,10]. The damping ratio is assumed to switch instantaneously between high and low values according to the sign of the relative velocity across the damper, i.e.

$$\zeta\left(t\right) = \begin{cases} \alpha\Xi & \dot{x} - \dot{y} < 0 \\ \Xi & \dot{x} - \dot{y} > 0 \end{cases} \qquad (18)$$

The SDOF model described in Section 2 was extended to include each of these damping models in turn, and the acceleration response due to a versed sine base displacement input was calculated by direct numerical integration of the equation of motion.

Figure 4 shows graphs of the peak acceleration as a function of damping reduction factor, α, for both the piece-wise linear damping model and the switchable damper. The results have been normalised by the case of a linear damper ($\alpha = 1$) set to the unreduced damping ratio, Ξ, which is 0.25 and 0.50 in Fig. 4(a) and 4(b) respectively. In both cases, the piecewise linear damper provides a modest reduction in peak acceleration compared with a linear damper; about 20% for example for a typical damping reduction factor in the region of $\alpha = 0.3$. The performance of the switchable damper is almost identical in the case of low damping (Fig. 4(a)) unless $\alpha < 0.6$, in which case some additional benefit is apparent. For

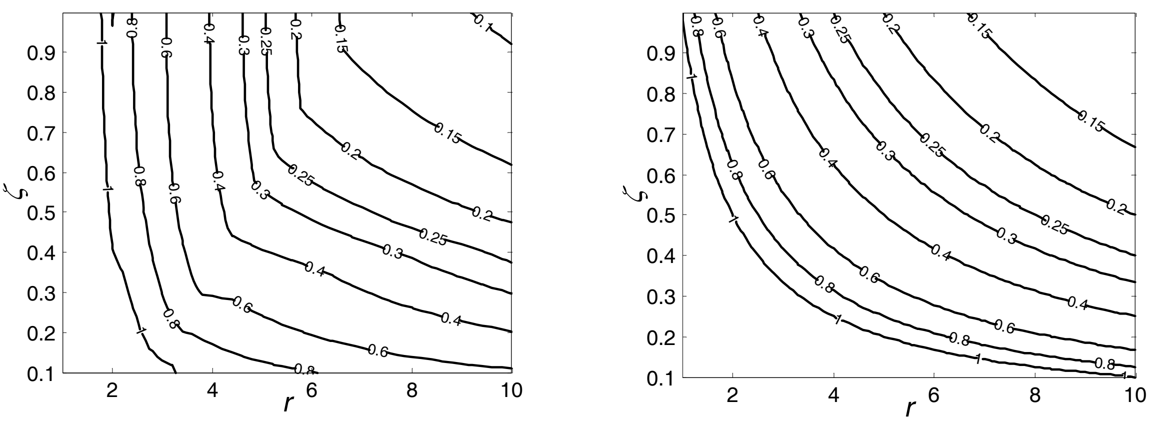
$\Xi = 0.5$ in Fig. 4(b), the switchable damper is beneficial provided that $\alpha < 1$, i.e. that the damping is reduced rather than increased during the impulse. The maximum benefit is obtained by switching off the damper altogether during the impulse ($\alpha = 0$). Whilst this is unfeasible in reality due to other performance constraints, it provides a useful upper bound on what might be achievable.



(a) Unreduced damping ratio, $\Xi = 0.25$ (b) Unreduced damping ratio, $\Xi = 0.50$

Figure 4. Peak acceleration as a function of damping reduction factor for a SDOF system with a natural period such that $r = 10$ (– piecewise linear damper; -- switchable damper).

The contour plot shown in Fig. 5(a) depicts the factor by which the peak acceleration may be reduced by switching off the damper during the impulse ($\alpha = 0$) compared with a linear damper. The benefit is significant only for impulses of short duration, $r > 5$ say, and improves as damping increases.



(a) (b)

Figure 5. Reduction factor in peak acceleration by switching off the damper during the impulse compared with a linear damper. (a) numerical integration; (b) simple approximation, $1/\zeta r$

A corresponding closed form approximation can be obtained by noting that, for large *r*, the peak acceleration occurs during the impulse. Consequently, the peak acceleration for a switchable “off-on” damper is the same as that of an undamped system. Setting $\zeta = 0$ in Eq. (10) gives the acceleration response during a short impulse as

$$\ddot{x}(t) \approx \frac{2\pi^2}{r^2}\left(1-\cos 2\pi\hat{t}\right) \tag{19}$$

which features a peak acceleration of magnitude $4\pi^2/r^2$. The peak acceleration due to a linear damper, by comparison, is approximately given by Eq. (15) to be $4\pi^2\zeta/r$. Then the factor by which the peak acceleration may be reduced is the ratio of these quantities, i.e. $1/\zeta r$. The corresponding contour plot of this estimate to the reduction factor is shown in Fig. 5(b). The results are qualitatively similar to those obtained by numerical integration in Fig. 5(a) and quantitatively meaningful for larger values of ζr.

5. CONCLUDING REMARKS

The response of a road vehicle to a transient input such as a bump is critically dependent on the characteristics of the dampers. In this paper, the effect of a linear damper has been investigated in the first instance. A single degree of freedom system has been adopted and closed form solutions presented for its time response due to a versed sine shaped base input. By assuming that the input is short compared to the natural period of the system simple expressions have been obtained for the peak acceleration in terms of the damping ratio and the ratio of natural period to impulse duration. Next, numerical simulations have been presented for the shock response of the system when the damper is switched to a lower value during the impulse. Such a damping mechanism is seen to reduce the peak acceleration when compared with either a linear or piecewise linear damper, and is most beneficial for vehicles with a large damping ratio and impulses of short duration.

Further work is required to consider the practicality of implementing a semi-active control system of this nature, and in assessing its impact on other performance criteria such as road handling and limited rattle space.

6. ACKNOWLEDGEMENTS

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