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SIMULATION OF MECHANICAL SYSTEMS IN LOADING WITH BROADBAND RANDOM AXITATION

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Abstract
In the report the modelling by a Galerckin method of dynamics of a console beam is submitted at cinematic loading by broadband random vibration. The loading spectrum extends with the certain interval, i.e. the operation similar to experimental definition of frequency characteristics is carried out. At scope by a loading spectrum "n" of own frequencies of a beam vibrospeed and tension in dangerous section accept extreme meanings.

INTRODUCTION

Vibrotests are a necessary condition to increase the reliability and qualities of designs and, first of all, designs in transport. For tests on real broadband random loading, which, basically, is broadband random vibration, it is required the expensive equipment and significant expenses of energy and time. The basic lack of similar tests is low reproducibility of results of the tests, which have been carried out in various laboratories. Here is shown, how, using dynamics of object of tests we can solve a reliability problem of results of tests and to find hardest vibroloading mode for using it in the accelerated tests, consuming besides minimum quantity of energy at the given frequency range of vibroloading.
Plenty of details in transport systems and their engines, in that number turbine details, are fixed as console and are activated in kinematical way. For reproducibility of vibrating and fatigue tests results on these details, it is necessary to standard test

spectra of loading [1], and for the accelerated tests - to find a hardest mode of loading on durability.

SIMULATING APPROACH

With the purpose of decision of these tasks for the console fixed beam were investigated by digital simulation the dependences of average square of vibrospeed meaning < $\dot{S}^2$ > of object in dangerous section at given average square meaning of vibrotensions < σ^2 > from width of a vibroaction spectrum and also average square meaning vibrotensions at constant average square meaning of vibrospeed. The block diagram of imitating modeling is shown in a fig. 1.

As a generator of random excitations it was used a standard subroutine. For formation of a loading spectrum of “white noise” type the filters giving signals with a uniform spectrum on displacement <S> were applied; f_{1bd} bottom boundary frequency of a spectrum, f_{2bd} - top boundary frequency of a spectrum.

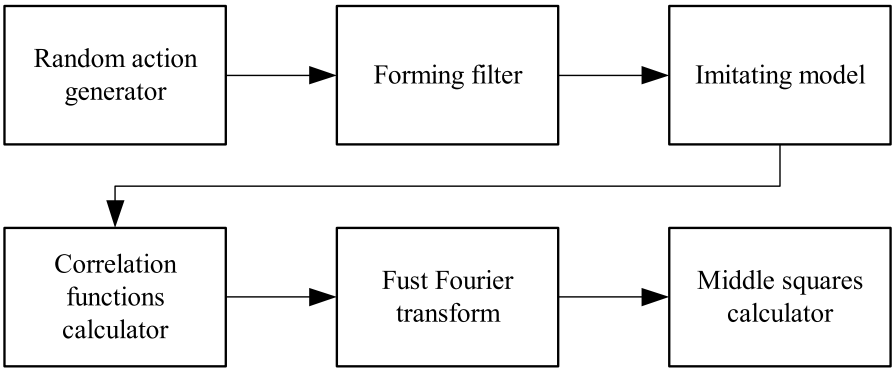

Figure 1. Block diagram of imitating modeling

The real experiment on durability was carried out to a cinematically excited beam, console fixed on shaking table, where S(t) – displacement of the shaking table. Proceeding from this, the vibrating influence on a beam is proportional to acceleration $\ddot{S}(t)$.

IMITATING MODEL OF OBJECT

The imitating model was under construction on the basis of the equation of bending fluctuations of a beam

$$m_o \frac{\partial^2 y(x,t)}{\partial t^2} + A \frac{\partial^4 y(x,t)}{\partial x^2} + \alpha \frac{\partial y(x,t)}{\partial t} = f(x,t) \qquad (1)$$

Under boundary conditions:

$y(x) = 0;$ $y'(x) = 0$ at $x = 0$;

$y''(x) = 0$; $y'''(x) = 0$ at $x = l$;

$y(x,t)$ – deflection of a beam;

$m_o = \rho F$ – weight of unit of length of a beam;

F – area of cross section;

l – length of a beam;

A=E I – bending tough;

E – module of elasticity of a material of a beam;

I – inertial moment of section;

α – factor of damping;

f(x, t) – external loading.

At cinematic excitation external loading is described by the following expression:

$$f(x,t) = -m_o \ddot{S}(t),$$

where S(t) – displacement of a table of the vibrator (jammed end of a beam).

Let's proceed from the equation (1) to the dimensionless form of record, having accepted $x = \varepsilon \cdot l$ γ=υ·l,

$$\frac{\partial^2 \upsilon}{\partial t^2} + \frac{EI}{m_0 l^4} \cdot \frac{\partial^4 \upsilon}{\partial \varepsilon^4} + \frac{\alpha}{m_0} \frac{\partial \upsilon}{\partial t} = -\frac{\ddot{S}}{l}. \tag{2}$$

On the basis of Galerckin method the transition from equations (2) was carried out to the system of linear differential equations.

The research of accuracy of modeling was carried out with 16 members. There was shown that it is possible to use only three members in Galerckin approximation.

So, the decision of the equation (2) is represented by the following decomposition:

$$(\varepsilon, t) = \sum_{k=1}^{3} y_k(\xi) \cdot f_k(t) ,$$

where

$y_k(\xi) = k_2(\lambda k) \cdot k_3(\lambda k \xi) - k_1(\lambda k) \cdot k_4(\lambda k \xi);$

$\lambda_1 = 1.875$ $\lambda_2 = 4.694,$ $\lambda_3 = 7.854,$ $\lambda_4 = 10.9986$;

k_1 ... k_4 – Krylov functions;

$$\begin{cases} \ddot{f}_1 \int_0^1 y_1^2(\xi)\partial\xi + \dot{f}_1 \cdot \dfrac{\alpha}{m_0} \int_0^1 y_1^2(\xi)\partial\xi + \dfrac{EI}{m_0 l^3} \cdot \lambda_1^4 \cdot \int_0^1 y_1^2(\xi)\partial\xi = -\dfrac{\ddot{S}}{l} \int_0^1 y_1(\xi)\partial\xi \\ \ddot{f}_2 \int_0^1 y_2^2(\xi)\partial\xi + \dot{f}_2 \cdot \dfrac{\alpha}{m_0} \int_0^1 y_2^2(\xi)\partial\xi + \dfrac{EI}{m_0 l^3} \cdot \lambda_2^4 \cdot \int_0^1 y_2^2(\xi)\partial\xi = -\dfrac{\ddot{S}}{l} \int_0^1 y_2(\xi)\partial\xi \\ \ddot{f}_3 \int_0^1 y_3^2(\xi)\partial\xi + \dot{f}_3 \cdot \dfrac{\alpha}{m_0} \int_0^1 y_3^2(\xi)\partial\xi + \dfrac{EI}{m_0 l^3} \cdot \lambda_3^4 \cdot \int_0^1 y_3^2(\xi)\partial\xi = -\dfrac{\ddot{S}}{l} \int_0^1 y_3(\xi)\partial\xi \end{cases} \quad (3)$$

The system (3) was numerically integrated by a Rhounge-Kutta method at random tension $\ddot{S}(t)$ formed, as it was specified, with generator of a random signal and filters.

In result the dependences of average square tension meaning were received

$$<\sigma^2>=\frac{1}{T}\int_0^T \sigma^2 dt \text{, where } \sigma=\frac{hE}{2}\frac{\partial^2 y(x,t)}{\partial x^2}=\frac{hE}{2l}\sum_{k=1}^{3} f_k(t) y_k'' .$$

In function of boundary frequency of vibrating excitation f_{2bd} (fig. 2) at constant root-mean-square meaning of vibrospeed

$$<\dot{S}^2>=\frac{1}{T}\int_0^T S^2 dt = const$$

Dependence of average square meaning of vibrospeed $<\dot{S}^2>$ also was received from boundary frequency of entrance influence f_{2bd} at given constant meaning of average square meaning of a tension $<\sigma^2>=$ const (fig. 3).

The received dependences (fig. 2 and 3) have extreme at boundary frequency 120 Hz$<f_{2bd}<$160 Hz. It is possible to accept f_{extr} about 140 Hz. The resonant frequencies for model (1) have meanings: f_1=28 Hz, f_2=175 Hz, f_3=485 Hz.

The carried out research allows to consider, that at constant average square of a tension (vibrospeed) of a beam the dependence of an average square of vibrospeed (tension) from boundary frequency of vibroaction has extremes, laying in the field of meanings between resonant frequencies of a beam, but is significantly closer to n-th own frequency.

SIMULATING RESULTS

The result, received by modeling, will well be coordinated with the fatigue experiment [4], in which vibroaction was formed with 1/3- octave filters with average frequency $f_{av}=f_1$ and width of a spectrum Δf=10, 30, 100 and 300 Hz.

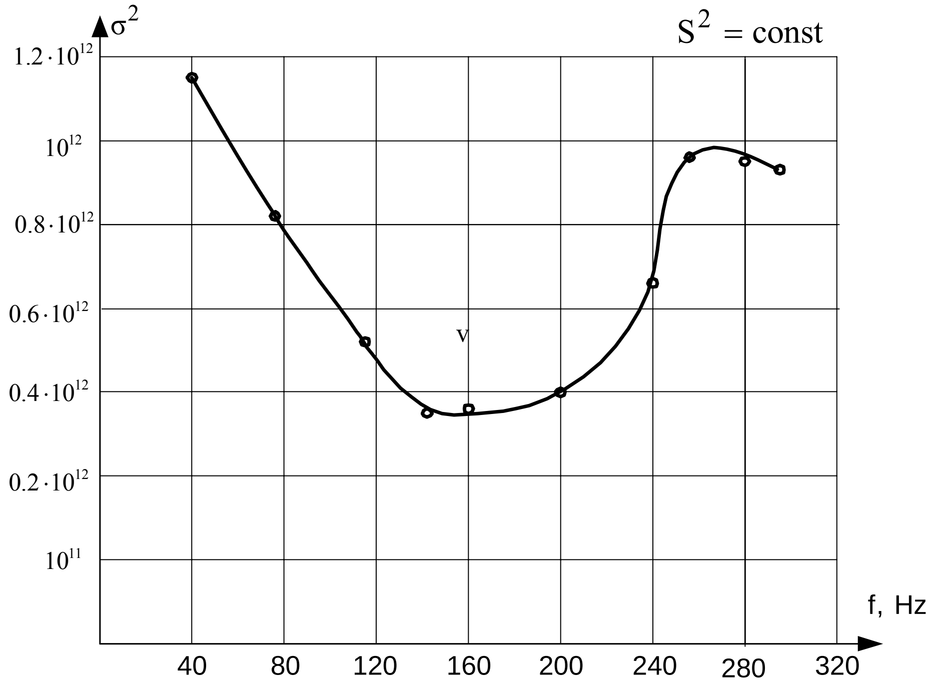

Figure 2. Extreme of root-mean-square meaning of a tension in function of boundary frequency of vibrating action

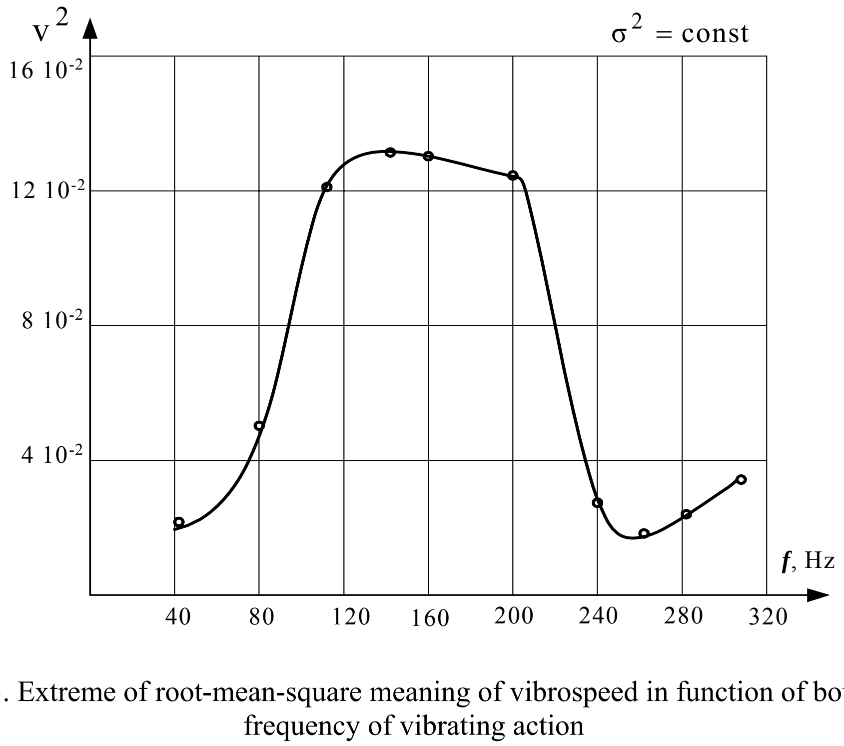

Figure 3. Extreme of root-mean-square meaning of vibrospeed in function of boundary frequency of vibrating action

Extreme meanings $\dot{S}^2$ and σ^2 are received for a mode with Δf=100 Hz, boundary frequency f_{bd} which with account flatness of characteristic of the filter is close to the meaning f_{2bd}, appropriate calculated extremes. On the mode with Δf=100 Hz the time before destruction of a beam was the least, i.e. the specified mode has appeared the hardest one.

At the first stages of revealing extremes in the described dependences it not so important the accuracy of concurrence of experimental and calculated results, how the fact of confirmation both in experiment, and in account of the existence of effective width of a spectrum, with which correspond extreme parameters of loading.

Modeling of influence of a broadband random spectrum on the console fixed beam by the described method was carried out in a range to sixteen first own frequencies of a beam. At capture by a loading spectrum of every one following own frequency of a beam vibrospeed and tension in dangerous section accepted the extreme meanings. Appropriate width of a spectrum was named " as an own strip of a spectrum ".

The received result can be treated as existence in mechanical systems "of own strips of a spectrum" (pass bands), in sense sensitivity to vibroaction, similar to own frequencies, but shown at broadband random vibroaction.

The received result is similar submitted in [3], where the theory of dynamic systems was used.

SUMMARY

The received result can be treated as existence at mechanical systems "of own strips of a spectrum" (pass bands), in sense of sensitivity to vibroaction, similar to own frequencies, but shown at broadband random action. Thus, received by modeling and in experiment, width of a loading spectrum may be accepted standard for a concrete beam with the aim of solving the reproducibility problem of test results and simultaneously may be used for the accelerated tests, as a hardest mode.

REFERENCES

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