# THE USE OF THE PROPAGATION EQUATION FOR THE CASE OF PLANE LONGITUDINAL WAVES THROUGH BARS WITH VARIABLE SECTION, TO STUDY THE IMPEDANCE TRANSFORMERS UTILIZED FOR ULTRASONIC WELDING. 

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#### Abstract

The ultrasonic vibrations, in the range (20-40) kHz , having high energy ( $1-2 \mathrm{~kW}$ ) are utilized for non-conventional welding. In this manner metal combination like aluminium - aluminium can be welded. This paper proceeds from the general form of the propagation equation of plane longitudinal waves through bars with different sections and thickness. The mathematical expressions are found for the variations in form of the mechanical tensions and for the amplitude of vibration as a function of the shape of the bars, and the material from which they are made. The curves which can be traced help to design the impedance transformers, the component which is placed between the ultrasound vibration generators (formed by piezoelectric elements) and the welding head. The derived equations are verified for the case of a welding machine of type TELSONIC-MPS-2. The paper proposes the realization of a program in order to calculate and then experimentally verify the case of an acoustic chain intended to be substituted for the defect acoustic chain, which exists on a welding machine of type TELSONIC-MPS-2. The paper presents the principle for the calculus, the calculus mode and experimental results obtained, for comparison with a few models selected for this purpose.


## 1. INTRODUCTION

In order to substitutes the defect acoustic chain, which exists on machine welding at KMP PRINT TECHNIK, it is necessary to find a calculus method able to assure the same efficiency and performances for new acoustic chain versus for original acoustic chain, which is made in Swiss. Also it must have possibilities to adaptation on work regime of welding machine TELSONIC-MPS-2. To realize a program, which may be applicable in this case, it starts from propagation equation of waves in solid environment, presented in paper [1]. It was realized a calculus program where it was studied the mechanical tensions and vibration amplitudes in the case of sundry shapes for acoustic chain as well as it was analysed the influence of geometrical and material parameters on ensemble acoustic chain which works on welding machine [2].

It was realized the optimum acoustic chain resulted by calculus and which it well matching to work in requisite regime. The acoustic chain it was experimental realised and verified. The rightness of calculus and realisation it was certifies by a good functionary on welding machine.

## 2. OPERATING PRINCIPLE

The operating principle consists by taking over, by intermediate of an acoustic chain, for mechanical vibrations product from a piezoelectric transducer. The ultrasonic vibration [3] is produced by piezoelectric pastille when it is applied on it of a sinus tension having great amplitude ( 2000 Vvv ).

The acoustic chain must transmit these vibrations and must realise: the vibration amplification, the impedance adaptation between piezoelectric transducer and acoustic charge and the realisation of strong mechanical catch for good operating of solder head. In solder time operation, the welding head must makes the following phases: to dispose the welding head on welding place, to press on welding place, to apply the necessary ultrasonic vibrations for realise the welding, to cool of solder and to elevate the welding head from welding place.

The ultrasonic vibrations propagate through acoustic chain by stationary waves [4]. The acoustic chain realises the necessary vibration amplitude on contact place between welding head and material for solder. For to realise the contract, it was built up the piezoelectric transducer - with helps of SL-4040W-W piezoelectric rings and the acoustic chain which corresponds, by point of view of geometrical dimensions and performances with defect acoustic chain which exists on welding machine TELSONIC-MPS-2.

The propagation of plane longitudinal waves through bar [1], with variable section, it makes by relation:

$$
\begin{equation*}
\frac{\partial}{\partial x}\left[A(x) \cdot \frac{\partial \zeta}{\partial x}\right]=\frac{A(x)}{c^{2}} \cdot \frac{\partial^{2} \zeta}{\partial t^{2}} \tag{1}
\end{equation*}
$$

with condition: $\left(\frac{\partial \zeta}{\partial x}\right)_{x=0}=\left(\frac{\partial \zeta}{\partial x}\right)_{x=l}$
That is to say that the amplitude vibrations are maximum amplitudes at ends of bar (for $x=0$ and for $x=1$ ). In above relation it is noted:

- $\zeta(\mathrm{x}, \mathrm{t}) \quad$ - represents the amplitude vibration in Ox direction;
- $\mathrm{A}(\mathrm{x}) \quad$ - represents the transversal section area at x distance;
- $c=\frac{E}{\rho} \quad$ - represents the propagation velocity of signal through bar;
- E - represents elasticity module of bar material;
- $\rho \quad$ - represents the density material of bar.
- 1 - represents the long of bar

The calculus for (1) equation it was presented in [7]. It looks for a solution by form:
$\zeta(\mathrm{x}, \mathrm{t})=\mathrm{X}(\mathrm{x}) \cdot \cos \omega_{1} \mathrm{t}$.
It substitutes in (1) and it obtains:

$$
\begin{equation*}
\frac{\partial}{\partial x}\left[A(x) \cdot \frac{\partial X(x)}{\partial x}\right]+\mu_{1}^{2} \cdot A(x) \cdot X(x)=0, \text { where: } \mu_{1}=\frac{\omega_{1}}{c} \tag{2}
\end{equation*}
$$

with maximum vibration condition at ends of bar, therefore we have: $\left.\frac{\partial X(x)}{\partial x}\right|_{x=0}=\left.\frac{\partial X(x)}{\partial x}\right|_{x=l}=0$

Equation (2) admits an infinity solutions by form $\left.X_{n}(x)\right|_{n=1,2, \ldots \infty}$. They are named proper functions and correspond with positive series numbers $\left.\mu_{\mathrm{n}}\right|_{\mathrm{n}=1,2, \ldots \infty}$ which are named proper values. The proper functions $\mathrm{X}_{\mathrm{n}}(\mathrm{x})$ are by form $\mathrm{C}^{1}$ class and so they may are developed in Fourier series converged to $X_{n}(x)$ series. For resolves this, it determines the proper values $\mu_{\mathrm{n}}$ and proper functions $\mathrm{X}_{\mathrm{n}}(\mathrm{x})$. The solution it is search by Galerkin method and using the complete system functions $(1, \cos \pi x / 1, \ldots \ldots . \cos n \pi x / l \ldots \ldots$ ) and it may satisfies the limit conditions $\left.\frac{\partial X(x)}{\partial x}\right|_{x=0}=\left.\frac{\partial X(x)}{\partial x}\right|_{x=l}=0$. For $\mathrm{X}(\mathrm{x})$ it is searching the solution by form:

$$
\begin{equation*}
X(x)=\sum_{n=0}^{s} a_{n} \cdot \cos \frac{n \pi}{l} x \tag{3}
\end{equation*}
$$

where: $\quad s$ - determines the precision calculus for finds of $X(x)$;
$a_{n}$ - are indeterminate constants solutions of a homogeneous system.
We have the solutions, for $\mathrm{a}_{\mathrm{n}}$, by resolving the system of equations:

$$
\begin{equation*}
\sum_{n=0}^{s} a_{n} \cdot\left(S_{m n}-\mu^{2} \cdot C_{m n}\right)=0 ; \mathrm{m}=0,1,2, \ldots \mathrm{~s} \tag{4}
\end{equation*}
$$

where we have, for $\mathrm{S}_{\mathrm{mn}}$ and $\mathrm{C}_{\mathrm{mn}}$, the values given by the following relations:

$$
\left\{\begin{array}{l}
S_{m n}=m n \cdot\left(\frac{\pi}{l}\right)^{2} \cdot \int_{0}^{l} A(x) \cdot \sin \left(\frac{m \pi}{l} x\right) \cdot \sin \left(\frac{n \pi}{l} x\right) \cdot d x  \tag{5}\\
C_{m n}=\int A(x) \cdot \cos \left(\frac{m \pi}{l} x\right) \cdot \cos \left(\frac{n \pi}{l} x\right) \cdot d x
\end{array}\right.
$$

The homogeneous system (4) has no common solution, if and only if we have the conditions:

$$
\begin{equation*}
\left|\mathrm{S}_{\mathrm{mn}}-\mu^{2} \cdot \mathrm{C}_{\mathrm{mn}}\right|=0 \mathrm{cu} \quad \mathrm{~m}, \mathrm{n}=0,1,2, \ldots \mathrm{~s} \tag{6}
\end{equation*}
$$

The (6) equation has, for $\mu$, a $s+1$ positive number solutions, having property: $\mu_{1}<\mu_{2}<\mu_{3}<\ldots<\mu_{\mathrm{s}+1}$.

It introduces $\mu_{\mathrm{I}}(\mathrm{i}=1,2, \ldots \mathrm{~s}+1)$ in (4) system and resolves it for obtains the solutions:

$$
\begin{equation*}
\frac{a_{1}^{i}}{a_{o}^{i}}, \frac{a_{2}^{i}}{a_{o}^{i}}, \ldots . \frac{a_{n}^{i}}{a_{o}^{i}} \quad \text { where } \mathrm{i}=1,2, \ldots \mathrm{~s}+1 \quad \text { and } \quad \mathrm{n}=1,2, \ldots \mathrm{~s} \tag{7}
\end{equation*}
$$

These values are introduced in (3) equation and it obtains:

$$
\begin{equation*}
X(x)=a_{o}^{i} \cdot \cos 0+\sum_{n=1}^{s} a_{n}^{i} \cdot \cos \frac{n \pi}{l} x \quad \mathrm{i}=1,2, \ldots \mathrm{~s}+1 \tag{8}
\end{equation*}
$$

It takes $a_{o}^{i}=1$ and it obtains:

$$
\begin{equation*}
X^{i}(x)=1+\sum_{n=1}^{s} a_{n}^{i} \cdot \cos \frac{n \pi}{l} x \quad \mathrm{i}=1,2, \ldots \mathrm{~s}+1 \tag{9}
\end{equation*}
$$

It is use the following procedure:

- it takes $\mu=\mu_{1}$;
- it calculates the values: $\frac{a_{1}^{1}}{a_{o}^{1}}, \frac{a_{2}^{1}}{a_{o}^{1}}, \ldots . \frac{a_{n}^{1}}{a_{o}^{1}} \quad$ where: $\quad \mathrm{i}=1$ and $\mathrm{n}=1,2, \ldots \mathrm{~s}$
- it introduces these values in (9) equation and it obtains: $X^{1}(x)=1+\sum_{n=1}^{s} a_{n}^{1} \cdot \cos \frac{n \pi}{l} x$


## 3. EXPERIMENTAL RESULTS

To calculate the shape and functional parameters of acoustic chain, it is using the relations given in [7]. It is choose [5] that a calculus precision s, for finds of X(x)- having an optimum value for $\mathrm{s}=2$ value. So, we have:

$$
\begin{equation*}
X(x)=\sum_{n=0}^{2} a_{n} \cdot \cos \frac{n \pi}{l} x=a_{0}+a_{1} \cdot \cos \frac{\pi}{l} x+a_{1} \cdot \cos \frac{2 \pi}{l} x \tag{10}
\end{equation*}
$$

The homogeneous system (4) becomes:

$$
\begin{equation*}
\sum_{n=0}^{2} a_{n} \cdot\left(S_{m n}-\mu^{2} \cdot C_{m n}\right)=0 ; \mathrm{m}=0,1,2 \tag{11}
\end{equation*}
$$

This homogeneous system has no common solution if and only if the determinant has a zero value:

$$
\begin{equation*}
\left|\mathrm{S}_{\mathrm{mn}}-\mu^{2} \cdot \mathrm{C}_{\mathrm{mn}}\right|=0 \mathrm{cu} \quad \mathrm{~m}, \mathrm{n}=0,1,2 \tag{12}
\end{equation*}
$$

From this equation it is found the proper values $\left.\mu_{\mathrm{n}}\right|_{\mathrm{n}=1,2, \ldots \mathrm{~s}+1}$ that are $\mu_{1}<\mu_{2}<\mu_{3}$
For each proper value $\mu_{1}, \mu_{2}, \mu_{3}$ it is obtain 3 sets values for $a_{0}, a_{1}, a_{2}$ :

$$
\begin{equation*}
\mu_{1} \rightarrow a_{0}^{1}, a_{1}^{1}, a_{2}^{1} . \text { or.. } \frac{a_{1}^{1}}{a_{0}^{1}}, \frac{a_{2}^{1}}{a_{0}^{1}} ; \mu_{2} \rightarrow a_{0}^{2}, a_{1}^{2}, a_{2}^{2} . \text {.or.. } \frac{a_{1}^{2}}{a_{0}^{2}}, \frac{a_{2}^{2}}{a_{0}^{2}} ; \mu_{3} \rightarrow a_{0}^{3}, a_{1}^{3}, a_{2}^{3} . \text {.or.. } \frac{a_{1}^{3}}{a_{0}^{3}}, \frac{a_{2}^{3}}{a_{0}^{3}} \tag{13}
\end{equation*}
$$

It takes $a_{0}^{i}=1$ and it obtains:

$$
\begin{equation*}
\mu_{1} \rightarrow 1, a_{1}^{1}, a_{2}^{1} . \quad ; \quad \mu_{2} \rightarrow 1, a_{1}^{2}, a_{2}^{2} . \quad ; \quad \mu_{3} \rightarrow 1, a_{1}^{3}, a_{2}^{3} . \tag{14}
\end{equation*}
$$

For finds the solution it takes $\mu=\mu_{1}$ and it resolves the system:

$$
\begin{equation*}
\sum_{n=0}^{2} a_{n} \cdot\left(S_{m n}-\mu^{2} \cdot C_{m n}\right)=0 ; \mathrm{m}=0,1,2 \tag{15}
\end{equation*}
$$

In conditions when the system has no banal solution, the value of $\mu$ results from equation $\left|S_{m n}-\mu^{2} \cdot C_{m n}\right|=0$. From this it results the values for $a_{1}^{1}$ and $a_{2}^{1}$

The solution it is determined by matrix mode: $\left[a^{1}\right]=\frac{[B]}{[D]}$, where:

$$
\left[a^{1}\right]=\left[\begin{array}{l}
a_{1}^{1}  \tag{16}\\
a_{2}^{1}
\end{array}\right] ; \quad[B]=\left[\begin{array}{l}
-S_{10}+\mu_{1}^{2} \cdot C_{10} \\
-S_{20}+\mu_{1}^{2} \cdot C_{20}
\end{array}\right] ; \quad[D]=\left[\begin{array}{l}
S_{11}-\mu_{1}^{2} \cdot C_{11} \ldots \ldots . . . S_{12}-\mu_{1}^{2} \cdot C_{12} \\
S_{21}-\mu_{1}^{2} \cdot C_{21} \ldots \ldots . S_{22}-\mu_{1}^{2} \cdot C_{22}
\end{array}\right]
$$

So it is obtained the new matrix with elements:

$$
\left\{\begin{array} { l } 
{ D _ { k , l } = S _ { k + 1 , l + 1 } - \mu _ { 1 } ^ { 2 } \cdot C _ { k + 1 , l + 1 } }  \tag{17}\\
{ B _ { k } = - S _ { k + 1 , 0 } + \mu _ { 1 } ^ { 2 } \cdot C _ { k + 1 , 0 } }
\end{array} \quad \text { where } \quad \left\{\begin{array}{l}
l=0,1 \\
k=0,1
\end{array}\right.\right.
$$

in general: $1, \mathrm{k}=0,1,2, \ldots . \mathrm{s}-1$.
Having the values for $a_{1}^{1}$ and $a_{2}^{1}$, it is possible to calculate $\mathrm{X}^{1}(\mathrm{x})$ with $\mathrm{i}=1$ :

$$
\begin{equation*}
X^{1}(x)=1+a_{1}^{1} \cdot \cos \frac{\pi}{l} x+a_{2}^{1} \cdot \cos \frac{2 \pi}{l} x \tag{18}
\end{equation*}
$$

Using the relations presented it is possible to calculate:

1. The vibration amplitude in $O x$ direction with relation: $\zeta^{1}(x, t)=X^{1}(x) \cdot \cos \omega_{1} t$, where vibration amplitude at moment $\mathrm{t}=0$ will be $\zeta^{1}(\mathrm{x}, 0)=\mathrm{X}^{1}(\mathrm{x})$;
2. The amplification through bar which it is given by ratio between vibration amplitude at the end of bar $(x=1)$ and vibration amplitude at begin of bar $(x=0)$. So we can write:

$$
G=\frac{\zeta^{1}(l, 0)}{\zeta^{1}(0,0)}
$$

3. The areas ratio, in the case of a bar having different sections, it is given by relation: $\frac{A(0)}{A(l)}$, where $\mathrm{A}(\mathrm{x})$ represents the equation which defines the transversal section area at x distance for the bar having different sections along Ox axis.
4. If it is note $\mathrm{X}_{0}=\mathrm{q}$, the distance when the vibration amplitude $\zeta(\mathrm{x}, \mathrm{t})$ it is null, will have: $\zeta^{1}(q, 0)=0 \Rightarrow X_{0}$
5. It can trace the curve of mechanical tensions along bar $-\mathrm{T}_{\mathrm{m}}$ - starting from relation $\mathrm{T}_{\mathrm{m}}(\mathrm{x})=\rho \cdot \mathrm{c} \cdot \mathrm{v}_{\mathrm{m}}(\mathrm{x})$
where: $\quad \rho \quad$ - represents the density of bar material;
c - represents the propagation velocity of signal along bar;
$\mathrm{v}_{\mathrm{m}}(\mathrm{x})$ - represents the propagation velocity of vibration particles through bar;
The propagation velocity of vibration particles through bar - $\mathrm{v}_{\mathrm{m}}(\mathrm{x})$ - is direct proportional with derivative of vibration amplitude $\zeta^{1}(\mathrm{x}, 0)$. So, it can write:

$$
\mathrm{T}_{\mathrm{m}}(\mathrm{x}) \sim \mathrm{v}_{\mathrm{m}}(\mathrm{x}) \sim \frac{d}{d x}\left[\zeta^{1}(x, 0)\right]
$$

For a higher precision of calculus, when it is find $\mathrm{X}(\mathrm{x})$, it can take, for s , a higher values than $s=2$. For $s$ having a higher value, it obtains a higher precision, but the calculus volume rises too much. It has observed [7] that to choice a higher values for s, more than 7,9 , it isn't justifier in report with the rising of calculus volume and necessary time to it calculates.

With help of this program, that uses the relations and methodology that it exposes in this paper, it can trace the curves that define the acoustic chain by point of vibration amplitude and mechanical tensions along it length as seed in fig. 1 . In fig. 2 it is presented the case when it is absence the diameter rise anterior of interface connector. It can be observed that this diameter rise has the following advantages:

- assures a easy catch in zone when vibration has small amplitude;
- takes away and diminishes the maximum of mechanical tension into termination catch of welding head with to minimise of mechanical solicitations at linkage zone from those two cylinders which forms the acoustic chain.

From presented program, it was calculates that resonance frequency of acoustic chain is $36,5 \mathrm{kHz}$, frequency that is very close from necessary frequency for a good work of original welding machine. The whole calculus and laboratory measurements effectuated was verified by a good working of welding machine and by quality of solder effectuated with that new acoustic chain. The acoustic chain, which it is put instead of original acoustic chain, it is presented in fig. 3. It is formed by:

- a reflector, having $\varnothing=41 \mathrm{~mm}$ diameter, realised from steel;
- a pair of piezoelectric rings, by SL-4040W-W type;
- a director, realised from aluminium alloy, having $\varnothing=41 \mathrm{~mm}$ diameter;
- a transmitted chain, realised from aluminium alloy, having $\varnothing=49 \mathrm{~mm}$ diameter;
- a transmitted chain, having a fixture system from welding machine, realised from aluminium alloy and having $\varnothing=55 \mathrm{~mm}$ diameter. This part of acoustic chain has an important contribution for to cool the solder head;
- a solder head, having a variable section for amplification of vibration, realised from titanium.


Fig. 1 The evolution of vibration amplitude and mechanical tensions along of acoustic chain in case of a rise diameter before interface connector salt.


Fig. 2 The evolution of vibration amplitude and mechanical tension along of acoustic chain in case of a constant diameter before interface connector salt.


Fig. 3 The acoustic chain realised which equips the welding machine.


Fig. 4 The welding machine equipped with the new acoustic chain.


Fig. 5 The acoustic chain realised and mounted on welding machine.

## 4. CONCLUSIONS

It was verified the propagation theory for longitudinal plane waves through bars having variable section presented in [7].

It was applied this theory in the case of an acoustic chain used on welding machine TELSONIC - MPS - 2. by:

- finding an optimum acoustic chain by point of view of shape, of component materials and of energy transfer;
- finding the mathematical relations which define this acoustic chain, relations introduced in calculus program;
- writing and putting into practice this calculus program;
- analysis of obtained results and influence study of different parameters on final result;
- realisation of acoustic chain, with help of this analysis;
- measuring and verifying of acoustic chain in our laboratory and on welding machine with occasion of put it in function with the new acoustic chain realised.


## REFERENCES

[1] DINCĂ F., ZAHARIA E., ROŞU I., Vibratiile longitudinale ale barelor cu sectiune variabila, Studii si Cercetari de Mecanica Aplicata, nr.2, tom 49, 1999, pag. (141-156);
[2] ODOBESCU G. L. , RUGINĂ I., A control method of mechanical vibration in the high intensity ultrasonic systems, Communication at INTER - NOISE, ( $25 \div 27$ ) august 1997, Budapest, Section: "Vibration and shock: generation, transmission, isolation and reduction ", position 525.
[3] ODOBESCU G. L., RUGINĂ I., The efficiency analysis of high ultrasonic systems supplied from electronic generators working in commutation, The Annual Symposium of the Institute of Solid Mechanics - SISOM $97-(20 \div 21)$ 11.1997. page. $(243 \div 250)$, Bucharest 1997 .
[4] ODOBESCU G. L., Method for determination of mechanical tensions which appear in bars with step variable sections, excited by ultrasonic mechanical vibrations, Inter-noise 99, The 1999 Congress and Exposition on Noise Control Engineering, Fort Lauderdale, Florida USA, 1999 December $6-8$, page.(835-840), INCE Subject Classification 43,Paper number : 126, Session number: 2354
[5] ODOBESCU G. L., RUGINĂ I.., LALA. C., Metode de creştere a eficieței sistemelor acustice de mare putere, Sesiunea de comunicari stiintifice - Comisia de Acustica-Academia Romana, Bucuresti 12-13 octombrie 2000, pag. (31-36).
[6] ODOBESCU G. L., The propagation study of ultrasound high intensity waves through resonance bodies transformation / adaptation with acoustic charge, The $17^{\text {th }}$ International Congress on Acoustic, (2-7) September 2001, Rome, Italy, abstract ID 1522, Oral session on 7A.02.07 " Non-linear acoustics", 7.09.2001, or 10.00, page 351- Book of Abstracts; page 89 Scientific program, page 71 abstract.
[7] ODOBESCU G. L., The calculation of amplitude vibrations and mechanical tensions for velocity transformations used for adaptation between piezoelectric transducers and acoustic charge, Eleventh International Congress on Sound and Vibration, 6-8 July 2004, Sankt Petersburg, Russia, p (2591-2598).

