

# THE MASS-LOADING EFFECT OF ACCELEROMETERS ON MAC CRITERION

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# Abstract

Traditionally, the mass-loading effect of accelerometers is avoided by using small accelerometers and, as a result, the effects of these mechanical devices on the measured FRFs are often considered to be negligible. However, there is no known proof to this belief. There are many cases in which the mass of accelerometers are not negligible compared to the mass of structures.

The MAC graph is considered as one of the most important criterions for comparison of mode shapes of different models of a structure. This article considers the effect of mass of accelerometers on MAC criterion.

# **1. INTRODUCTION**

One of the most important applications of modal testing is the validation of the mathematical model for the dynamic analysis of a structure by comparing experimentally determined modal parameters with those of analytical model. Once the analytical model is validated, it can be used with confidence for further analysis such as response prediction, structural coupling, stress analysis, life time prediction, etc [1]. However, Measurement of the dynamic behaviour of a structure in terms of Frequency Response Functions (FRFs) is always vulnerable to measurement errors. The mass-loading effect of transducers is one of the sources of error in modal testing. The accelerometer mounted on the structure changes the dynamics of the structure and introduces errors into the measured FRFs. One problem with this is the production of unrealistic results, which cause the measured resonant frequency to be less than the real value [1]. A further problem is the inconsistency of the data base when using a series of measurements with roving transducers in order to acquire data for modal analysis. Data inconsistency causes problems in the curve-fitting process, particularly when applying a modern global parameter estimation algorithm.

From a practical point of view, comparison between the natural frequencies of two mathematical models of a structure is a simple way of assessing the quality of these models. The MAC criterion is one suitable way for comparison of mode shapes of the models of a

structure. The mass-loading effect of accelerometers on MAC criterion is studied in this paper.

# 2. TECHNIQUES OF COMPARISON

#### 2.1 Direct comparison

The most common method for comparison of two different models is plotting the natural frequencies of one model against those of the other model. If the resulting curve lies on a straight line of slope 1, the models are perfectly correlated [2].

#### 2.2 The Modal Assurance Criterion (MAC)

The Modal Assurance Criterion (MAC), between analytical mode i and experimental mode j is defined as

$$MAC(X, A) = \frac{\left|\sum_{j=1}^{n} (\phi_{X})_{j} (\phi_{A})^{*}_{j}\right|^{2}}{\left(\sum_{j=1}^{n} (\phi_{X})_{j} (\phi_{X})^{*}_{j}\right) \left(\sum_{j=1}^{n} (\phi_{A})_{j} (\phi_{A})^{*}_{j}\right)}$$
(1)

A MAC value close to 1 suggests that the two modes are well correlated and a value close to 0 indicates uncorrelated modes [3].

### **3. CASE STUDY**

A beam was tested with two different configurations. At first, an accelerometer was used for measurement of response at 13 different points of beam. Next, 13 accelerometers of the same type were used for the measurement of responses of same points of beam.



Figure 1. Natural Frequency and MAC comparison for 13 accelerometers



Figure 2. Natural Frequency and MAC comparison for 1 accelerometer

Figure.1 shows the comparison of the mode shapes of experimental model and Finite Element model in the first test. Two mode shapes are well correlated, but the natural frequencies curve does not lie on a straight line of slope 1. Figure.2 shows the comparison of the mode shapes of experimental model and Finite Element model in the second test. It can be seen that the mode shapes correlation are not well, but the natural frequencies approximately stand in a line of slope 1.

Figure 3 shows the results of test in these two cases.



Figure 3. Measured FRFs of beam with 13 accelerometers (right) & with 1 accelerometer at 13 points (left)

As the results of second test show (Figure 3-left), the FRFs do not match at resonance point. Data inconsistency due to measurements with roving transducers causes poor correlation between the mode shapes. In each test we have a different structure due to mass-loading effect of accelerometer.

As the results of first test show (Figure 3-right), when 13 accelerometers were used at the same time, the data was consistent. However, as the mass of accelerometers was 234g

(13\*18g) which is about  $\frac{1}{7}$  of the mass of beam, the natural frequencies curve did not lie on a straight line of slope 1.

The number of measurement points affects the mode shapes extracted from modal testing due to spatial aliasing. Figure 4 shows the MAC graph of FE and Modal Testing models when only 5 measurement points were used. Figures 5 and 6 show the same MAC graph when 7 and 13 measurement points were used respectively.



Figure 4. MAC value with 5 measurement points



Figure 5. MAC value with 7 measurement points Figure 6. MAC value with 13 measurement points

It can be concluded that with 5 measurement points only 3 first modes can be compared, with 7 measurements 6 first modes can be compared. To compare 7 modes 13 measurement points are needed. This is due to spatial aliasing problem and the AUTOMAC [4] function should be calculated before comparison of the FE and Modal testing models. This also shows that the mass-loading effect of accelerometers can not be removed by decreasing the number of measurement points and instead aliasing problem arises [5].

# **4. CONCLUSIONS**

In light structures the mass-loading effect of accelerometers is an important problem. Using several accelerometers at the same time in order to conduct modal testing has a considerable effect on measurement due to excessive mass of accelerometers and bias the results. On the other hand, measurement using a roving accelerometer creates an inconsistent data which causes a poor correlation between the measured mode shapes and the real ones. Moreover, the number of measurement points is very important. A lack of enough measurement points causes the problem of spatial aliasing and uncorrelated data. To overcome this problem the AUTOMAC of mode shapes should be computed and checked before comparison of the models of structure. The mass-loading effect of accelerometers can not be removed by decreasing the number of measurement points this causes the spatial aliasing problem.

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