ICSV14
Cairns • Australia
9-12 July, 2007

14th International Congress on Sound & Vibration

PARTITION HOLE DESIGN FOR MAXIMIZING OR MINIMIZING THE FUNDAMENTAL EIGENFREQUENCY OF A DOUBLE CAVITY BY TOPOLOGY OPTIMIZATION

Jin Woo Lee[1] and Yoon Young Kim[1]

[1]National Creative Research Initiatives Center for Multiscale Design,
School of Mechanical and Aerospace Engineering, Seoul National University
San 56-1 Shilim-9-dong, Kwanak-gu, Seoul, 151-742, Republic of Korea
jw062nam@yahoo.co.kr

Abstract

A topology optimization method is developed to minimize or maximize the fundamental eigenfrequency of a double cavity. The acoustic model consists of two rectangular cavities and a holey partition. Because the number and locations of holes in the partition affect the eigenfrequencies of the double cavity significantly, the underlying acoustical characteristics of the double cavity can be controlled by adjusting them. In this work, the eigenfrequency control problem is formulated as an acoustical topology optimization problem where the fundamental acoustic eigenfrequency is minimized or maximized for an allowed hole volume. For the formulation, the partition is divided into sub-partitions, each of which has variable material properties. When a sub-partition has acoustical properties of air, it is regarded as a hole. Intermediate states between air and a rigid body are interpolated by a carefully-selected penalized function in order to produce a clear hole distribution at the converged iteration.

1. INTRODUCTION

An acoustical topology optimization method is applied to the partition hole design of a hole-partitioned double cavity. The double cavity is a simplified cavity model suggested in the previous paper [1], where the acoustical characteristics of a passenger vehicle compartment with a trunk were investigated. Recently, a topology optimization method has been employed to acoustical device design since it was suggested by Bendsøe and Kikuchi [2]. Jensen and Sigmund [3] formulated an acoustical topology optimization problem for systematic design of acoustical devices. Lee et al. [4] optimized the topology of thin-body for the radiation and scattering of sound using genetic algorithms. Wadbro and Berggren [5] presented an acoustic horn design method by topology optimization. Dühring [6] applied topology optimization to obtain optimal material distribution in the ceiling. However, an acoustical topology optimization problem for partition hole design has not been reported.

A double cavity consists of two cavities of the same cross-section and a holey partition between them. The total cross-sectional area and the position of holes strongly affect eigenfrequencies of longitudinal acoustic modes of the double cavity. The goal of this work is to find an optimal hole distribution for the minimum and maximum fundamental eigenfrequencies of the double cavity for a given constraint on the total hole volume. In this paper, the partition hole design problem is formulated as an acoustical topology optimization problem. To this end, the partition between two cavities is divided into sub-partitions, whose acoustical properties can vary during optimization process. Carefully-selected interpolation functions are used to ensure that each sub-partition becomes a hole or a rigid sub-partition at the final optimized results. To carry out eigenfrequency analysis, finite element model is employed and a gradient-based optimization algorithm is employed to update design variables.

2. ACOUSTIC TOPOLOGY FORMULATION

As shown in Figure 1, a double cavity consists of two rectangular cavities and a partition. Cavity 1 and Cavity 2 are filled with air, and the partition between the two cavities is divided into several sub-partitions, each of which is filled with an artificial material. Assuming that the width and the height of the cavity are smaller than its length, only longitudinal acoustic modes are considered. Acoustic pressure p of the acoustic system is governed by the Helmholtz equation:

$$\nabla \cdot \left(\frac{1}{\rho} \nabla p \right) + \frac{\omega^2}{K} \cdot p = 0 \,, \tag{1}$$

where ρ is the density of acoustic medium, and the angular frequency ω is defined by $\omega = 2\pi \cdot f$, where f is a frequency. The bulk modulus K is defined by $K = \rho c^2$, where c is the sound speed in acoustic medium.

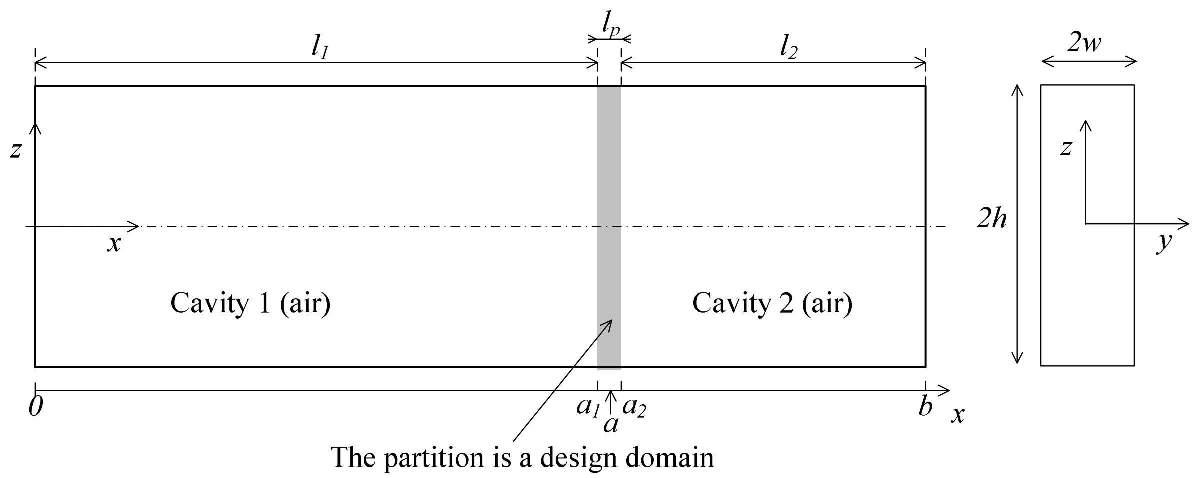

Figure 1. A double cavity with a holey partition

Using a finite element method, equation (1) is converted to

$$\left[\mathbf{K} - \omega^2 \mathbf{M} \right] \mathbf{P} = 0 \tag{2}$$

where $\mathbf{P}$ is the acoustic pressure vector. The global stiffness matrix $\mathbf{K}$ and the global mass matrix $\mathbf{M}$ are expressed by using an assembly operator $\mathop{\mathrm{A}}\limits_{n=1}^{N}$ as follows:

$$\mathbf{K} = \mathop{\mathrm{A}}_{n=1}^{N} \mathbf{k}_n \qquad \mathbf{M} = \mathop{\mathrm{A}}_{n=1}^{N} \mathbf{m}_n \tag{3}$$

From the condition to obtain a nontrivial solution from equation (2), the following characteristic equation of the double cavity is obtained:

$$\det\left[\mathbf{K} - \omega^2 \mathbf{M}\right] = 0 \tag{4}$$

The design domain is the partition which is divided into sub-partitions. The width of a sub-partition is equal to that of the partition. Each sub-partition is supposed to become a hole or a rigid sub-partition at the end of optimization iteration steps. One design variable χ_e is assigned to each sub-partition and it has values of between 0 and 1. The two limiting values correspond to air ($\chi_e = 0$) or a rigid body ($\chi_e = 1$). Therefore, it changes acoustic properties of the sub-partition and is updated by the MMA (Method of Moving Asymptotes [7]) during the optimization process. It requires the sensitivity of the r$^{\mathrm{th}}$ eigenfrequency with respect to a design variable χ_e, which is given as

$$\frac{\partial \omega_r^2}{\partial \chi_e} = \frac{1}{\mathbf{P}_\mathbf{r}^\mathbf{T} \mathbf{M} \mathbf{P}_\mathbf{r}} \left[\mathbf{P}_\mathbf{r}^\mathbf{T} \frac{\partial \mathbf{K}}{\partial \chi_e} \mathbf{P}_\mathbf{r} - \omega_r^2 \mathbf{P}_\mathbf{r}^\mathbf{T} \frac{\partial \mathbf{M}}{\partial \chi_e} \mathbf{P}_\mathbf{r} \right] \tag{5}$$

2.1 Design problem I: eigenfrequency minimization

To minimize the eigenfrequency of the fundamental acoustic mode of the double cavity, the following objective function and the constraint are employed.

$$\min_{0 \le \chi_e \le 1} L = f_1 \tag{6-a}$$

$$\text{Subject to } 1 - \sum_{e=1}^{N_e} \chi_e / N_e \ge V_o \tag{6-b}$$

where f_1 is the fundamental eigenfrequency and V_o is the ratio of the total hole volume to the partition volume. The symbol of N_e is the number of total sub-partitions. Interpolation functions of density ρ_e and bulk modulus K_e of the e$^{\mathrm{th}}$ sub-partition affect significantly final optimized results; without proper choices, it would be difficult to identify hole distributions precisely. For this minimization problem, we employed

$$1/\rho_e(\chi_e) = 1/\rho_{\mathrm{air}} + \chi_e^p \left(1/\rho_{\mathrm{rigid}} - 1/\rho_{\mathrm{air}}\right) \tag{7-a}$$

$$1/\mathrm{K}_e(\chi_e) = 1/\mathrm{K}_{\mathrm{air}} + \chi_e^p \left(1/\mathrm{K}_{\mathrm{rigid}} - 1/\mathrm{K}_{\mathrm{air}}\right) \tag{7-b}$$

where the subscripts ‘air’ and ‘rigid’ stand for air and a rigid body, and p is the penalization parameter.

2.2 Design problem II: eigenfrequency maximization

To obtain the maximum eigenfrequency of the fundamental acoustic mode of the double cavity, the following acoustical topology optimization problem is set up:

$$\underset{0\le\chi_e\le 1}{\mathrm{Max}}\, L = \log f_1 \tag{8-a}$$

$$\text{Subject to } 1-\sum_{e=1}^{N_e}\chi_e / N_e \le V_o \tag{8-b}$$

In this case, the following interpolation functions are shown to be appropriate:

$$\rho_e(\chi_e) = \rho_{\mathrm{air}} + \chi_e^p(\rho_{\mathrm{rigid}} - \rho_{\mathrm{air}}) \tag{9-a}$$

$$\mathrm{K}_e(\chi_e) = \mathrm{K}_{\mathrm{air}} + \chi_e^p(\mathrm{K}_{\mathrm{rigid}} - \mathrm{K}_{\mathrm{air}}) \tag{9-b}$$

3. NUMERICAL RESULTS

Since a hole distribution in the z direction was considered, a two-dimensional finite element model in x-z plane was employed for the convenience of analysis. The numerical data used for all design problems were as follows:

$$l_1 = 1.50\ \mathrm{m},\ l_2 = 0.50\ \mathrm{m},\ l_p = 0.012\ \mathrm{m}$$

$$h = 0.32\ \mathrm{m},\ N_e = 20,\ V_o = 0.1$$

$$\rho_{\mathrm{air}} = 1.21\ \mathrm{kg/m^3},\ c_{\mathrm{air}} = 343\ \mathrm{m/s},\ \rho_{\mathrm{rigid}} = 10^5\cdot\rho_{\mathrm{air}},\ c_{\mathrm{rigid}} = 10^1\cdot c_{\mathrm{air}}$$

Considering the hole volume ratio $V_o = 0.1$, two sub-partitions were expected to become holes. The values of the penalty exponents were $p = 1$ for the minimization problem and $p = 3$ for the maximization problem. To minimize the risk of obtaining a local minimum or maximum, ten initial guesses were used. It was expected that this strategy could yield the global minimum or maximum eigenfrequency.

3.1 Design problem I: numerical results

The hole partition design problem formulated in equations (6) and (7) was solved to obtain optimal hole distribution for the minimum eigenfrequency of the first eigenmode. Figure 2 plots iteration histories of the first eigenfrequency for ten initial design variables. Among 10 cases, two cases yielded 57.3 Hz as a minimum eigenfrequency. Figure 3 shows several initial design variables and optimized design variables. Black represents a rigid body and white represents a hole in an optimized topology. Neither filtering nor post-processing was not used to obtain this optimal hole distribution. Case 4 is taken as the solution to this topology optimization problem. Two holes are located consecutively at the edge of the partition. Figure 4 shows acoustic mode of the double cavity at the minimum eigenfrequency.

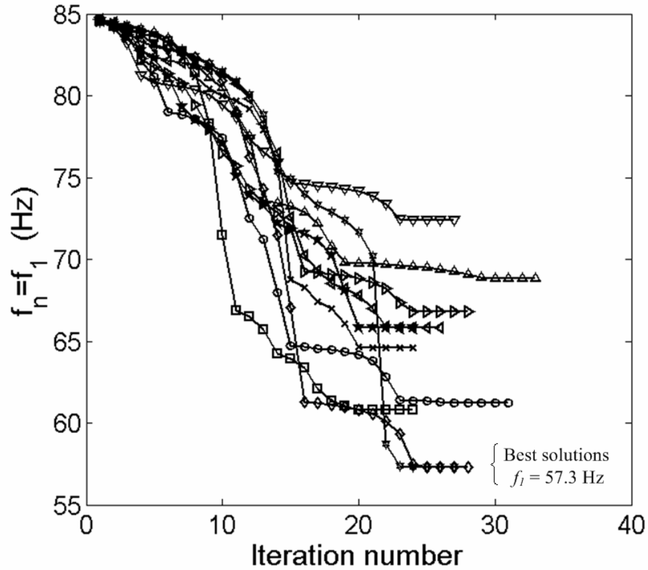

Figure 2. Iteration histories of the 1st eigenfrequency in the Design problem I.

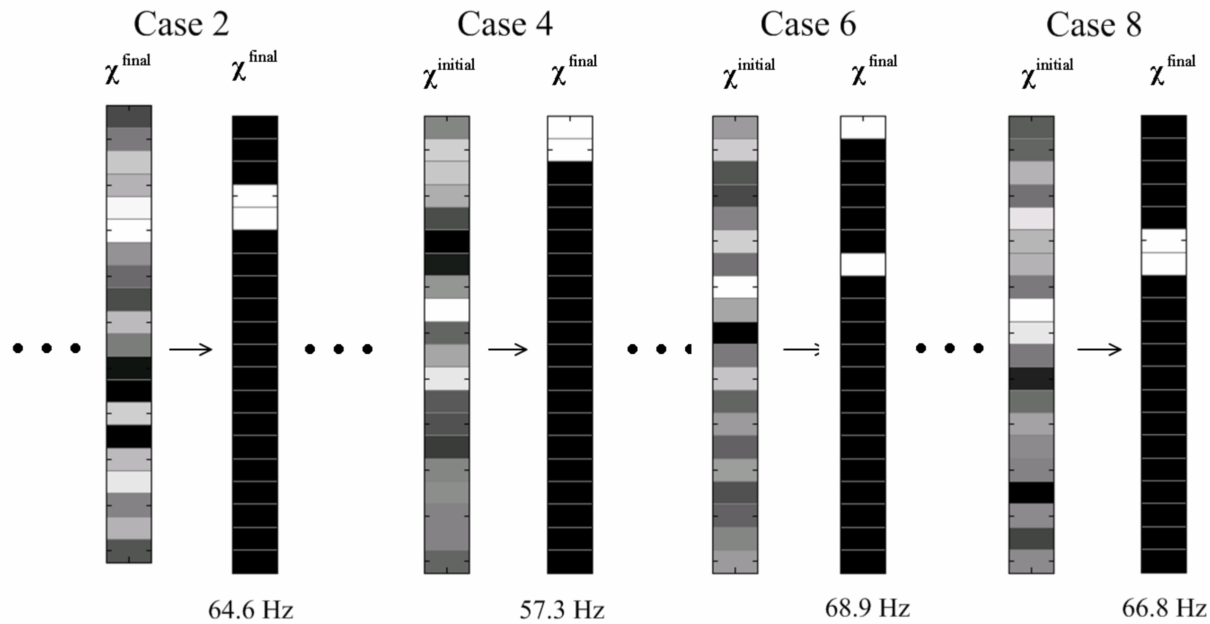

Figure 3. Changes in optimal partition layouts for initial design variables in the Design problem I.

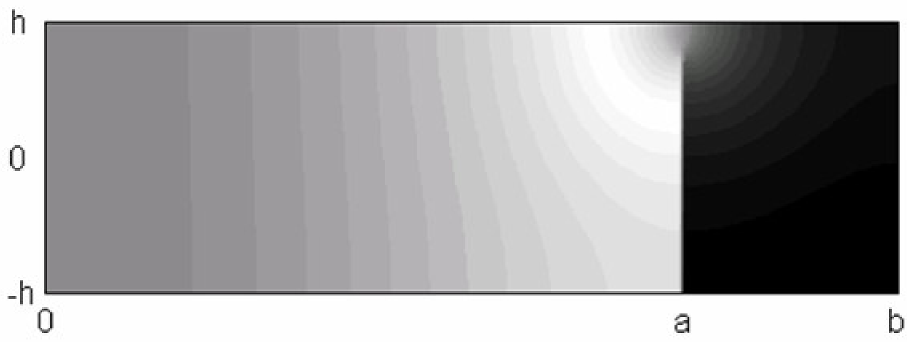

Figure 4. Acoustic mode at a minimum eigenfrequency of the 1st eigenmode (57.3Hz)

3.2 Design problem II: numerical results

The hole partition design problem set up in equations (8) and (9) was solved for the first eigenfrequency. As in Design problem I, a maximum value among ten cases is taken as the solution to this design problem ($f_1^{\max} = 76.2$ Hz). Figure 5 compares iteration histories of the first eigenfrequency. Figure 6 shows a hole distribution in the partition of the optimized result, where two holes are located symmetrically around the y-axis on the partition. Figure 7 displays the acoustic mode of the double cavity at the maximum fundamental eigenfrequency. As in Figure 4, it is proved that holes distribution strongly affects acoustic pressure distribution as well as eigenfrequency of the hole-partitioned double cavity. We calculated the 1st eigenfrequency of the double cavity for several symmetrically-located holes as shown in Figure 8. One interesting point is that the eigenfrequency for two holes located at the center of partition is equal to that for two holes located at both ends of the partition ($f_1 = 68.4$ Hz). That is to say, a maximum eigenfrequency could be expected when two separated holes are located some places between center and edge, respectively. This brief investigation shows the validity of the result obtained in this maximization problem.

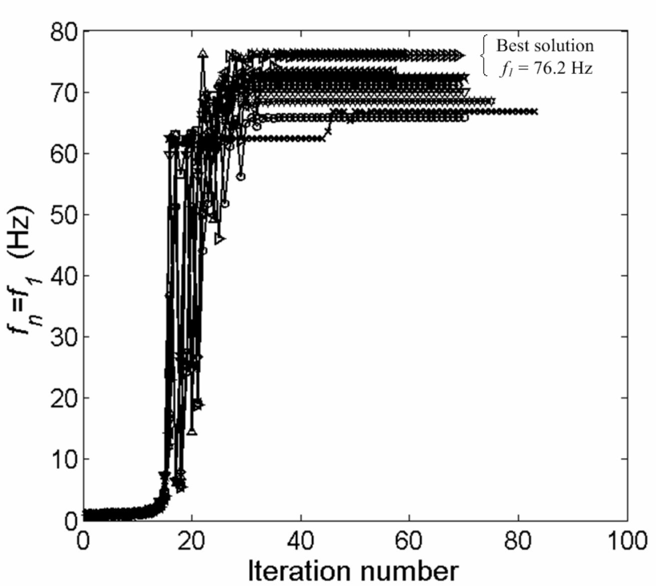

Figure 5. Iteration histories of the 1st eigenfrequency in Design problem II.

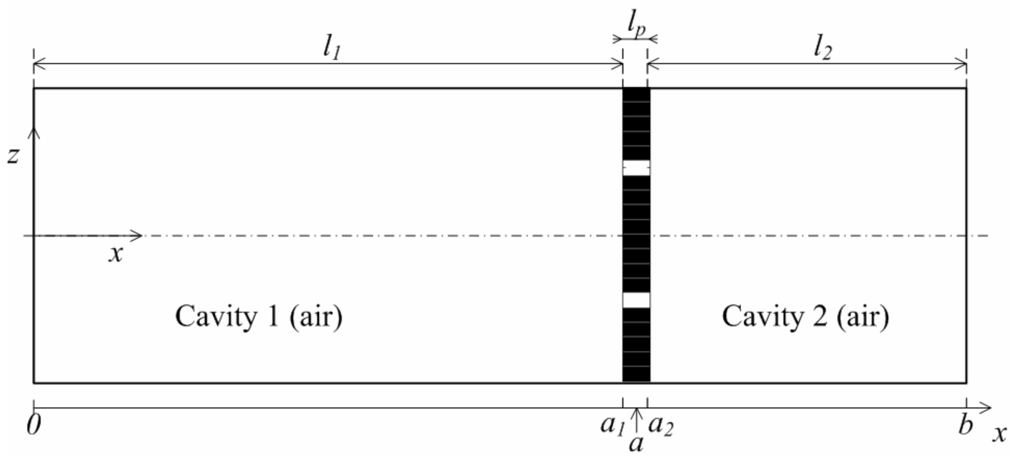

Figure 6. Hole distribution in the partition of the optimized result.

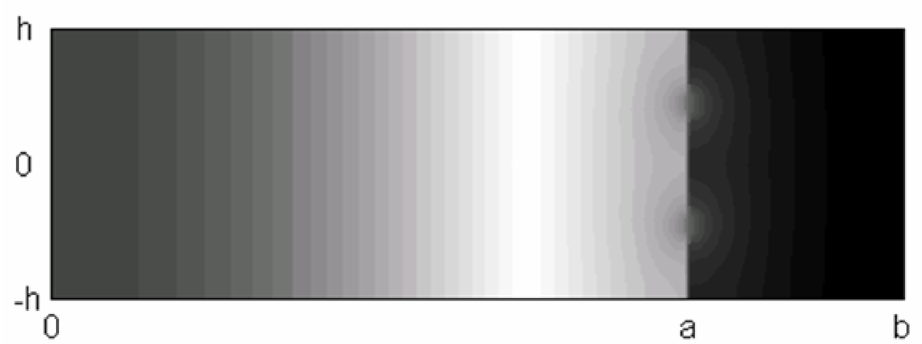

Figure 7. Acoustic mode at a maximum eigenfrequency of the 1st eigenmode (76.2 Hz).

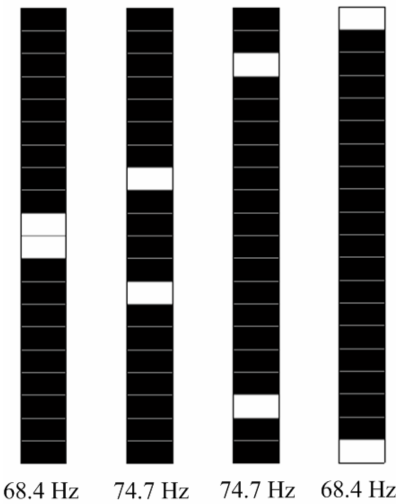

Figure 8. Comparison of eigenfrequencies for several symmetric hole distributions.

4. CONCLUSIONS

We formulated two acoustical topology optimization problems to effectively design a holey partition so that the hole-partitioned double cavity could have the desired acoustical characteristics. Different objective functions, constraints and interpolation functions were employed to obtain the minimum and maximum eigenfrequency of the fundamental eigenmode of the double cavity. We successfully obtained the reasonable hole distribution on the partition for minimum or maximum eigenfrequencies, where acoustic modes were compared focusing on the acoustic pressure distribution around the partition. The hole distribution in the partition affects not only an eigenfrequency of the double cavity but also its eigenmode strongly. The numerical results showed the possibility that our design strategy based on topology optimization could be applied to a two-dimensional holey partition design problem. Although we divided the partition into only 20 sub-partitions, the formulation developed in this work can be easily applied to the hole-partitioned double cavity having more than 20 sub-partitions.

ACKNOWLEDGMENT

This research was supported by the National Creative Research Initiatives Program (Korea Science and Technology Foundation grant No. 2006-033) contracted through the Institute of Advanced Machinery and Design at Seoul National University.

REFERENCES

[1] J.W. Lee, J.M. Lee and S.H. Kim, "Acoustical analysis of multiple cavities connected by necks in series with a consideration of evanescent waves," *Journal of Sound and Vibration* **273**, 515-542 (2004).

[2] M.P. Bendsøe and N. Kikuchi, "Generating optimal topologies in structural design using a homogenization method," *Computer Methods in Applied Mechanics and Engineering* **71**, 197-224 (1988).

[3] J. S. Jensen and O. Sigmund, "Systematic design of acoustic devices by topology optimization," *Proceedings of the Twelfth International Congress on Sound and Vibration* (ICSV12), 11-14 July 2005, Lisbon, Portugal

[4] J. Lee, S. Wang and A. Dikec, "Topology optimization for the radiation and scattering of sound from thin-body using genetic algorithms," Journal *of Sound and Vibration* **276**, 899-918 (2004).

[5] E. Wadbro and M. Berggren, "Topology optimization of an acoustic horn," *Computer Methods in Applied Mechanics and Engineering* **196**, 420-436 (2006).

[6] M.B. Dühring, *Topological design optimization of structures, Machines and Materials*, Springer **137**, 375-385 (2006).

[7] K. Svanberg, "The method of moving asymptotes – a new method for structural optimization," *International Journal for Numerical Methods in Engineering* **24**, 359-373 (1987).