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AN EFFICIENT NUMERICAL PROCEDURE FOR APPROXIMATING ACOUSTIC DIFFUSE FIELDS

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ABSTRACT

Aerospace structures can be subjected to severe acoustic excitations which exhibit random characteristics. The handling of such random excitations in a numerical model is the subject of this paper. Starting from the assimilation of the excitation to a stationary random process, it is shown how to characterize acoustical excitations (and, more specifically, diffuse fields) and how to evaluate efficiently the random structural response. This evaluation is performed with an hybrid FEM/BEM model which takes into account elasto-acoustic effects. Algorithmic issues related to the decomposition of the diffuse excitation into a set of uncorrelated pseudo-load cases are addressed. Convergence properties of the proposed scheme are illustrated through numerical applications.

1. INTRODUCTION

Space vehicles are subjected to a severe fluctuating external-pressure loading when their rocket-propulsion systems are operated in the atmosphere and this loading may be critical for some vehicle components and their supporting structures. On the other hand, acoustic loading tests are usually carried out in reverberant chambers where diffuse field conditions are achieved. Such acoustic excitations can be assimilated to random processes [1,2] and are usually described in terms of overall sound pressure level (and its related frequency spectrum) and appropriate spatial correlation functions.

The evaluation of the random response of exposed mechanical structures must refer to the characteristics of the load (e.g. frequency content) but also to the structural properties (stiffness, mass and damping). These properties are assumed to be strictly deterministic in the present study.

In many practical cases, studied mechanical structures (usually lightweight) can interact with the surrounding acoustic medium in a way such that the dynamic response is perturbed. The handling of related kinematical and mechanical coupling effects requires the selection of appropriate numerical models. The unbounded character of the fluid domain calls also for the use of a boundary integral formulation which is able to handle exactly the Sommerfeld radiation condition at infinite distance.

The coupled model used in the present development is based on the selection of a displacement-based FEM model for the structure while a BEM model is selected for the acoustic fluid. The BEM model relies on an advanced indirect boundary integral representation [4] particularly well suited for thin structures (thickness is assumed to be small versus the acoustic wavelength).

The paper is organized as follows. The first section is devoted to the characterization of random excitations as stationary random processes. A special attention is devoted to diffuse fields. The second section is related to the coupled discrete model used for setting up the FRF matrix. The third section shows how to proceed for getting the random response of the coupled system. Numerical examples related to plate structures are presented in the last section.

2. RANDOM ACOUSTIC EXCITATIONS

Acoustical excitations can be induced by discrete sources with random amplitudes. The particular case of a diffuse acoustic field (as produced in a reverberant room) deserves some attention. A diffuse field is defined as an acoustic field in which the time average of the mean square sound pressure is the same at any location and the flow of acoustic energy in all directions is equally probable. Such a diffuse field is usually obtained experimentally by activating acoustic sources in a reverberant chamber (Figure 1). The multiple reflections along the rigid boundary walls lead to the so-called 'diffuse' state.

At a modeling point of view, a diffuse field could be reproduced by activating a large number of discrete sources. Tuning these sources for achieving a 'diffuse' field could be impractical in many cases so that an alternative procedure based on an asymptotic approximation can be preferred. Such an approximation can be obtained from the analytical investigation of an infinite number of plane waves with random amplitudes and phases[5].

Whatever the key characteristic of the random excitation is, his random nature could be defined in the same way. Let us denote by x_1 such an excitation. The random character of $x_1(t)$ can be described by referring to a weakly stationary random process. The key characteristics of any weakly stationary process $x_1(t)$ are the mean (which is constant and usually zero in the present context), the auto-correlation function $R_{x_1}(\tau)$ and the power spectral density (PSD) function $S_{x_1}(\omega)$ as given by:

$$R_{x_1}(\tau) = E\left[x_1(t)x_1(t+\tau)\right] \qquad S_{x_1}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} R_{x_1}(\tau)e^{-i\omega\tau}d\tau \tag{1}$$

where E[...] denotes the mathematical expectation operator.

When a system is subjected to different random excitations, cross-correlation and cross-PSD functions $R_{x_i x_j}(\tau)$ and $S_{x_i x_j}(\omega)$ have to be introduced in order to characterize the interdependency between load components.

The formal way for getting a diffuse field (superimposing an infinite number of plane waves with different propagation directions) can also be followed in order to set up the cross-PSD function for the acoustic pressure or the pressure gradient at two different locations within the chamber (Figure 1). The whole procedure relies on the analytical expression of the steady-state acoustic pressure induced by a plane wave at two points labeled 1 and 2:

$$p_1 = p(\vec{r}_1, t) = P(\omega)\exp(-i\vec{k}\cdot\vec{r}_1)\exp(i\omega t) \quad p_2 = p(\vec{r}_2, t) = P(\omega)\exp(-i\vec{k}\cdot\vec{r}_2)\exp(i\omega t) \qquad (2)$$

where $\vec{r}_1$ and $\vec{r}_2$ give the location of the two considered points, $\vec{k}$ is the wave vector and ω is the circular frequency.

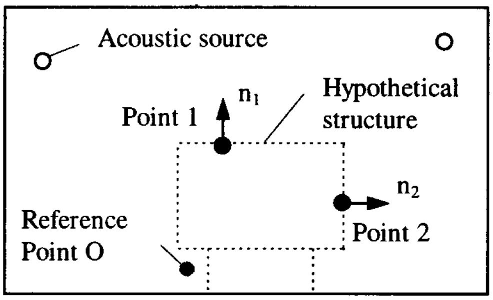

Figure 1: Reverberant chamber.

The cross-PSD function for the pressure normal gradients g_1 and g_2 at two arbitrary locations within the room can be obtained by summing up the effects of an infinite number of plane waves with random amplitudes and phases. The result turns out to be:

$$\tilde{S}_{g_1 g_2}(\omega) = \frac{\int \tilde{S}_{g_1 g_2}(\omega) d\vec{k}/|\vec{k}|}{\int d\vec{k}/|\vec{k}|} = S_{p_0}(\omega) f_g(\vec{r}_1, \vec{r}_2, \vec{n}_1, \vec{n}_2, \omega) \qquad (3)$$

with

$$f_g(\vec{r}_1, \vec{r}_2, \vec{n}_1, \vec{n}_2, \omega) = \frac{\int \exp(-i\vec{k}\cdot(\vec{r}_1 - \vec{r}_2))(\vec{k}\cdot\vec{n}_1)(\vec{k}\cdot\vec{n}_2) d\vec{k}/|\vec{k}|}{\int d\vec{k}/|\vec{k}|} \qquad (4)$$

As indicated by expression (3), the diffuse acoustic field is simply characterized by the data related to the reference point (S_{p_0}) and the spatial correlation function f_g which depends only on the frequency and the relative positions/orientations of considered points. It should be emphasized that the derivation of the correlation function related to pressure gradients is motivated by the selection of an elasto-acoustic model (section 3) where the acoustic excitation is defined in terms of incident pressure normal gradients along the exposed structure.

Function (4) is an extension of the well known correlation function for pressures ($\sin(kd)/kd$ where $d = |\vec{r}_1 - \vec{r}_2|$). The spatial correlation function (4) is available in a closed analytical form from which previously derived particular cases can be retrieved [5]. Additionally the correlation function has been verified experimentally [10] and some related results can be found in section 5 of the present paper.

3. COUPLED FEM-BEM MODEL

The evaluation of the response of a mechanical elastic structure interacting with a surrounding acoustic fluid can be obtained in many different ways. In the present study, a displacement-based finite element model is selected for the structure while an acoustic boundary element model is used for the acoustic fluid. This choice is quite natural for handling the low frequency response of a conventional structure.

The boundary integral representation which sustains the development of the BEM model presents several advantages (the radiation boundary condition at infinity is automatically satisfied while the mesh requirements are reduced since only the boundary (i.e. radiating) surface has to be discretized. Several boundary integral representations [6] are available in order to describe the acoustic field around a vibrating structure. The selected formulation is based on an indirect representation in terms of single and double layer potential densities along the mean boundary surface [7]. This leads to the following integral representation for the pressure at a point P outside from the boundary surface S:

$$p(P) = p_I(P) + \int_S \left\{ \mu(Q) \frac{\partial G(P,Q)}{\partial n_Q} - \sigma(Q) G(P,Q) \right\} dS(Q) \tag{5}$$

where p_I is the incident pressure, σ and μ are the single and double layer potentials, G is the Green's function and Q is a generic point along S.

This integral representation relies on the preliminary determination of layer potentials along the boundary surface S. This requires in turn to set up appropriate boundary conditions. In this context, a generalized approach [4] has been selected such that either pressure, normal velocity or normal admittance can be independently constrained along both sides of the boundary surface. Additionally transfer admittance boundary conditions and coupling effects with a mechanical structure can be accounted for.

The discrete coupled model relies on a structural FEM model for the structure while a variational solution scheme [8] supports the implementation of the above indirect boundary integral representation. The discrete coupled system appears as

$$\begin{bmatrix} Z^S & C & 0 \\ C^T & Z^F_{cc} & Z^F_{cu} \\ 0 & Z^F_{uc} & Z^F_{uu} \end{bmatrix} \begin{pmatrix} y^S \\ y^F_c \\ y^F_u \end{pmatrix} = \begin{pmatrix} x^S \\ x^F_c \\ x^F_u \end{pmatrix} \tag{6}$$

where Z^S is the structural impedance matrix, Z^F is the fluid impedance matrix, C is the geometrical coupling matrix, x^S is the structural load vector, x^F is the fluid load vector, y^S is the structural response vector (nodal displacements or modal participation factors) and y^F is fluid response vector (single and double layer nodal potentials). Subscripts c and u denote respectively coupled and uncoupled fluid degrees of freedom.

This formulation assumes implicitly that coupling effects occur only on one part of the boundary surface. The other part (which could be reduced to zero) involves purely acoustic boundary conditions. In a compact form, (6) could be rewritten as

$$Z(\omega) \cdot y = x \quad \text{or} \quad y = H(\omega) \cdot x \tag{7}$$

where Z is the global impedance matrix of the coupled system, H is the global admittance matrix of the coupled system (also called the FRF matrix), x is the load vector and y is the response vector.

4. EVALUATION OF THE COUPLED RESPONSE

It can be shown [1,2] that the relation between the response PSD matrix S_y and the excitation PSD matrix S_x is given by:

$$S_y(\omega) = H(-\omega)\cdot S_x(\omega)\cdot H^T(\omega) \tag{8}$$

The practical evaluation of the random response is easier if one assumes that the load vector can be expressed as a combination of 'l' deterministic load patterns (or load cases):

$$x(t) = L\cdot f(t) \tag{9}$$

where $L \in R^{nxl}$ and $f \in R^{l}$.

Components of f are assimilated to stationary random processes characterized by a correlation matrix R_f (and its Fourier transform S_f which is the related PSD matrix):

$$R_f(\tau) = E\left[f(t)\cdot f^T(t+\tau)\right] \tag{10}$$

In such circumstances, the PSD matrix S_y of the output can be obtained from:

$$S_y(\omega) = H(-\omega)\cdot L\cdot S_f(\omega)\cdot L^T\cdot H^T(\omega) \tag{11}$$

Defining by x_l and x_l' the matrices of responses related to all load patterns:

$$x_l = H(\omega)\cdot L,\ x_l' = H(-\omega)\cdot L \tag{12}$$

the power spectral density matrix S_y can be evaluated from

$$S_y(\omega) = x_l'\cdot S_f(\omega)\cdot x_l^T \tag{13}$$

The computational effort required is substantially reduced because the key operation is the evaluation (at each discrete frequency) of the responses associated to the different load patterns. This procedure is usually followed for handling mechanical excitations.

If the excitation is a diffuse acoustic field, the coupled model is 'driven' by the incident pressure normal gradient acting along the boundary surface since the acoustic BEM model relies on the normal derivative form of (5) expressed along the mean boundary surface. In the discrete context, this means that each coefficient of the cross-PSD matrix of the excitation is given by an expression like (3). A direct treatment based on (8) is not efficient and better performances could be obtained if matrix S_x can be replaced by a truncated decomposition based on the extraction of the dominant eigenmodes of S_x:

$$S_x(\omega) \approx L_m(\omega)\cdot T_m(\omega)\cdot L_m(\omega)^T \tag{14}$$

where $S_x \in C^{nxn}$, $L_m \in C^{nxm}$ and $T_m \in C^{mxm}$ with m<n.

Substitution of (14) into (8) leads to:

$$S_y(\omega) \approx H(-\omega)\cdot L_m(\omega)\cdot T_m(\omega)\cdot L_m(\omega)^T\cdot H^T(\omega) \tag{15}$$

The practical evaluation of (15) could rely on the process described by (12) and (13) due to the strong analogy between (11) and (15). This analogy leads to interprete the columns of L_m as 'pseudo-load' cases. It should be noted that these pseudo-load cases are uncorrelated if matrix T_m is diagonal. The effectiveness of the proposed procedure relies on the availability of low cost, truncated decompositions (SVD or SSD methods).

5. NUMERICAL EXAMPLES

5.1 Verification of the approximate decomposition procedure

A rectangular plate (a=0.5 m, b=0.25 m, thickness=0.00144 m) simply supported along its edges is excited by an acoustic diffuse field. The constitutive material has the following mechanical properties: Young modulus = 1×10^{11} N/m^2, Poisson ratio = 0.3, mass density = 1800 kg/m^3). The excitation is defined as a white noise (PSD of the acoustic pressure is 1 Pa^2/Hz). The structure is discretized into 320 QUAD4 elements and a modal representation is selected (21 modes in the frequency range 0-1500 Hz). A viscous damping ratio of 1% is assumed for all the modes. The response is evaluated in the frequency range 50-500 Hz with a frequency step of 5 Hz (1Hz around resonance frequencies).

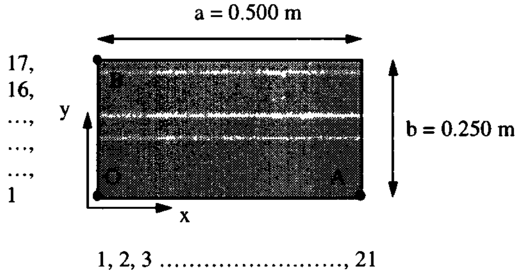

Figure 2: Locations of nodal positions along sides OA/OB for sampling the approximate correlation function.

The approximation of the spatial correlation function versus point O (at 500 Hz) along the two sides OA and OB (Figure 2) using a variable number (5, 10 and 15) of pseudo-load cases is shown in Figures 3(a) and 3(b) together with the exact form of the correlation function.

As it can be seen from inspection of these figures, a very good approximation can be obtained with only a few vectors. The present calculation has been done using 15 vectors.

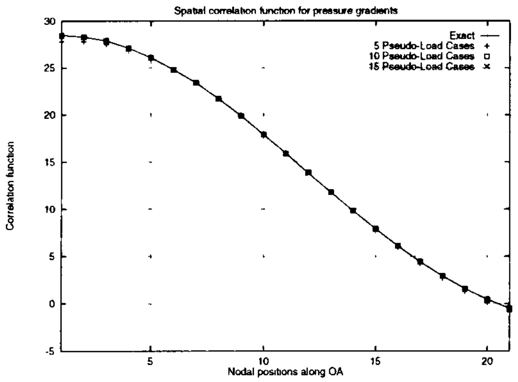

Figure 3(a): Comparison of exact and approximate correlation functions along side OA (at 500 Hz) using 5,10 and 15 pseudo-load cases.

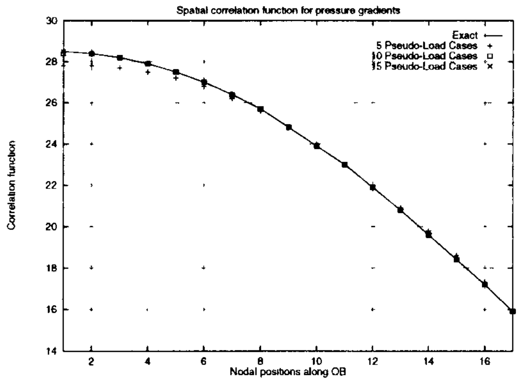

Figure 3(b): Comparison of exact and approximate correlation functions along side OB (at 500 Hz) using 5,10 and 15 pseudo-load cases.

Figure 4 shows the PSD of normal acceleration at location (x=0.1 m, y=0.0625 m) as computed by SYSNOISE [9] and compared to available reference results obtained by activating 63 plane wave sources around the plane in order to create 'artificially' the diffuse field. An excellent agreement can be observed between the two models.

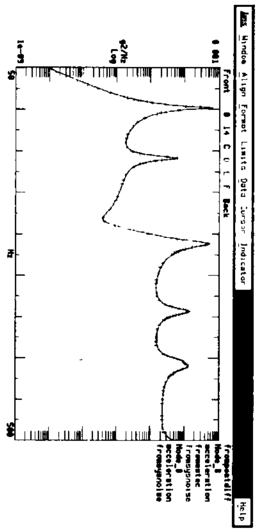

Figure 4: PSD of acceleration at (x=0.1 m, y=0.0625 m)
(line: plane wave model model, crosses: present approach)

5.2 Experimental validation

An experimental study [10] has been carried out in order to verify the correlation function for pressure gradients and the capability of the procedure to capture effectively the random response of a plate. This plate (a=0.6 m, b=0.4 m, thickness=0.001 m) is suspended vertically (free-free condition) and is excited by an acoustic diffuse field (Figure 5). The constitutive material has the following mechanical properties: Young modulus = 7×10^{11} N/m^2, Poisson ratio = 0.33, mass density = 2700 kg/m^3). The excitation is defined as a white noise. The structure is discretized into 1536 QUAD4 elements and a modal representation is selected (126 modes in the frequency range 0-1300 Hz). The response is evaluated in the frequency range 150-400 Hz with a frequency step of 5 Hz (1Hz around resonance frequencies).

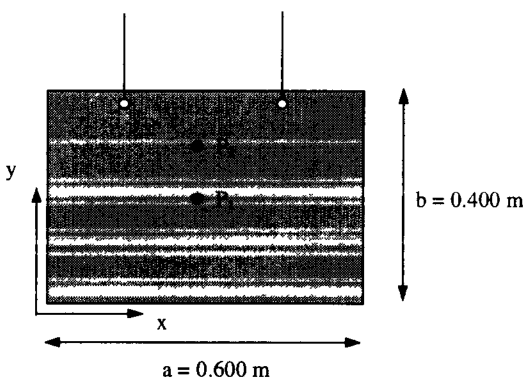

Figure 5: Elastic plate in a diffuse field (experimental case).

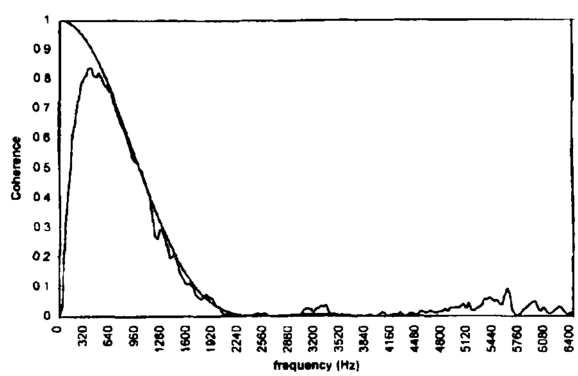

Figure 6: Comparison of measured and computed coherence function for two parallel velocity components 0.10 m apart (from [10]).

The measured coherence function related to velocity components along z for two points 0.1 m apart is compared to the analytical expression based on (4) at Figure 6.

The measured and predicted acceleration PSD at point P_2 (x=0.3 m, y=0.3 m) are compared in Figure 7. Maximum 30 pseudo-load cases are involved in the numerical evaluation. As it can be observed from these figures, a good agreement has been obtained between numerical predictions and measurements.

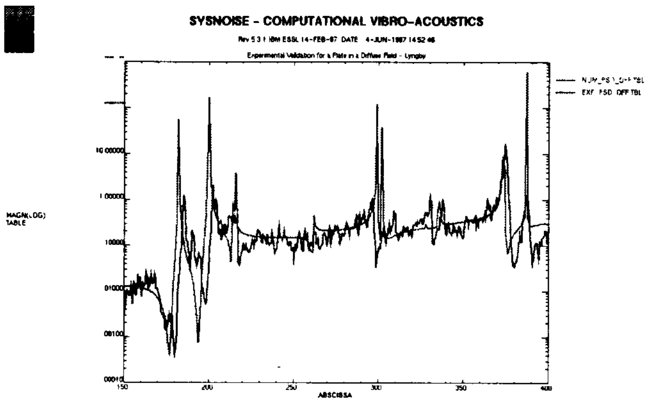

Figure 7: Comparison of predicted and computed PSD of acceleration at point P_2 (x=0.3 m, y=0.3 m) (from [10])

6. CONCLUSIONS

The numerical treatment of a coupled elastic structure subjected to random acoustic excitations has been described. The assimilation to stationary random processes provides the required framework for handling the random nature of the excitation. The diffuse acoustic field deserves some attention. An appropriate analytical model and an approximate decomposition procedure into a set of correlated or uncorrelated pseudo-load cases allows to extract efficiently reliable response results. Numerical examples related to a plate in a diffuse field illustrate the computational procedure.

7. REFERENCES

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