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*Invited Paper*

# FINITE ELEMENT ANALYSIS OF ACTIVE VIBRATION ISOLATION

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**Abstract**

Finite element analysis was used to predict the power transmission from an actively isolated vibrating rigid mass to a simply supported beam. Vibrational power transmission was used as the cost function to be minimised. The work demonstrated that neglect of power transmission due to moments in experimental work is the reason why negative power transmission in the vertical direction at some frequencies has been reported in the literature. Simulations show that under active control when power transmission in the vertical direction is used as a cost function to be minimised, the overall vibration isolation performance of the active isolator can be worse than without control.

## 1 INTRODUCTION

Active vibration isolation requires the selection of a suitable cost function to be minimised. Typical cost functions are point acceleration or force between two mating parts on the structure. However, selection of one parameter to minimised such as acceleration, will not necessarily lead to the minimisation of force (Howard & Hansen 1997). A cost function which takes account of both velocity and force is the vibrational power transmission which should reduce the overall transmitted vibrational energy under active control.

An experiment was conducted by one of the authors to minimise the power transmission of a vibrating mass actively isolated from a simply supported beam. An accelerometer and force transducer combination was used to measure the power transmission from an active vibration isolator into a simply supported beam. A heterodyning technique was used to combine velocity and force signals at the base of the isolator, into a signal which was proportional to the vibrational power transmission at the driving frequency (Howard & Hansen 1996). It was found that at some frequencies the power transmission under active control was worse than the passive case. It was reported that power transmission from moments was suspected to be causing this effect.

The effect of moment power transmission was used to advantage by Koh & White (1996) to reduce the power transmission from a vibrating machine to its support structure, by selecting appropriately dimensioned mounts such that the rotational moment and linear force would combine to reduce the vibrational power transmission.

This paper demonstrates through the use of Finite Element Analysis (FEA) that if moments are neglected in active isolation using power as a cost function, the overall power transmission can be greater than for the passive isolation case.

## 2 THE FINITE ELEMENT METHOD

A three dimensional Finite Element Model (FEM), using the software package ANSYS (©Ansys Inc.), was constructed of the experimental arrangement presented in Howard & Hansen (1997), as shown in Figure 1.

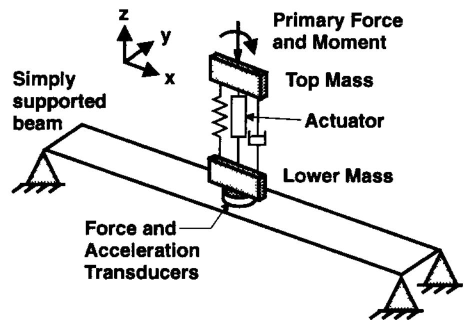


Figure 1: Schematic of the 3-D beam system.

A script file was written which contained ANSYS instructions and a FORTRAN program was used to determine the optimum control forces. The details of the steps involved are described in the following sections.

The method is similar to that used by Jenkins (1989) and Hollingsworth & Bernhard (1994) who used displacement as the cost function to be minimised. However the method presented here differs from the previous work in that the cost function used is the vibrational power transmission into the support structure and also, the effects of moments on the cost function are investigated.

The program follows these steps:

### 2.1 Definition of the problem

A FEM is constructed of the system, the node locations are defined for the application of the primary, control forces and error transducers. The ANSYS program is started and proceeds without user interaction. The program was written so that any structure could be used with any primary, control and error sensor locations.

### 2.2 System identification

The response of the system is determined by measuring the influence coefficients for the primary and control forces. The control forces are set to zero and in turn, each primary force is set to a unit load and the displacement and force responses are measured at each error sensor over the analysis frequency range. This process is repeated for the control forces. These transfer functions are saved to external files for an external FORTRAN program to determine the optimal control forces.

### 2.3 Determine optimal control forces

It can be shown that the displacement and force at the error sensor will be given by

$$\text{disp} = -\mathbf{Z}_{dp}\mathbf{F}_p + \mathbf{Z}_{dc}\mathbf{F}_c \quad (1)$$

$$\text{force} = -\mathbf{Z}_{fp}\mathbf{F}_p + \mathbf{Z}_{fc}\mathbf{F}_c \quad (2)$$

where $\mathbf{F}_p$ and $\mathbf{F}_c$ are the primary and control force column vectors respectively, $\mathbf{Z}_{ij}$ is a transfer function between displacement or force ($i$) and primary or control force ($j$). For example, $\mathbf{Z}_{fc}$ is the transfer function matrix between the force response measured at the error sensor and the driving control force. These definitions can be used to define the harmonic vibrational power transmission into the structure as

$$\text{Power} = \frac{\omega}{2}\,\text{Re}\left(\text{disp}^{\text{H}} \times \text{force}\right) \tag{3}$$

where the superscript $^{\text{H}}$ is the Hermitian transpose and $\omega$ is the angular frequency in rad/s. Substitution of Eqs. (1) and (2) into Eq. (3) and rearrangement will result in a quadratic expression in terms of the control force $\mathbf{F}_c$ given by Howard, Pan & Hansen (1997) as

$$\text{Power} = \frac{\omega}{2}\left(\mathbf{F}_c^{\text{H}}\boldsymbol{\alpha}\mathbf{F}_c + \mathbf{F}_c^{\text{H}}\boldsymbol{\beta} + \boldsymbol{\beta}^{\text{H}}\mathbf{F}_c^{\text{H}} + c^i\right) \tag{4}$$

where

$$\boldsymbol{\alpha} = \boldsymbol{\alpha}^{\text{T}} = \frac{1}{2}\begin{bmatrix} \mathbf{a}^i + (\mathbf{a}^i)^{\text{T}} & \mathbf{a}^r - (\mathbf{a}^r)^{\text{T}} \\ -\mathbf{a}^r + (\mathbf{a}^r)^{\text{T}} & \mathbf{a}^i + (\mathbf{a}^i)^{\text{T}} \end{bmatrix} \tag{5}$$

$$\boldsymbol{\beta} = \frac{1}{2}\begin{bmatrix} (\mathbf{b}_2^i)^{\text{T}} + \mathbf{b}_1^i \\ (\mathbf{b}_2^r)^{\text{T}} - \mathbf{b}_1^r \end{bmatrix} \tag{6}$$

and the real matrices $\mathbf{a}^r, \mathbf{a}^i, \mathbf{b}_1^r, \cdots$ represent, respectively, the real and imaginary parts of the complex matrices $\mathbf{a}, \mathbf{b}_1, \mathbf{b}_2$ and $c$ which are defined as

$$\mathbf{a} = \mathbf{Z}_{dc}^{\text{H}}\mathbf{Z}_{fc} \tag{7}$$

$$\mathbf{b}_1 = -\mathbf{Z}_{dc}^{\text{H}}\mathbf{Z}_{fp}\mathbf{F}_p \tag{8}$$

$$\mathbf{b}_2 = -\mathbf{F}_p^{\text{H}}\mathbf{Z}_{dp}^{\text{H}}\mathbf{Z}_{fc} \tag{9}$$

$$c = \mathbf{F}_p^{\text{H}}\mathbf{Z}_{dp}^{\text{H}}\mathbf{Z}_{fp}\mathbf{F}_p \tag{10}$$

The power transmission into the system for passive vibration isolation $\mathbf{F}_c = 0$ is given by $\omega c^i/2$. The minimum of Eq. (4) is given by

$$\text{Power}_{\text{min}} = -\frac{\omega}{2}\boldsymbol{\beta}^{\text{T}}\boldsymbol{\alpha}^{-1}\boldsymbol{\beta} \tag{11}$$

corresponding to an optimum control force vector given by

$$(\mathbf{F}_c)_{\text{opt}} = -\boldsymbol{\alpha}^{-1}\boldsymbol{\beta} \tag{12}$$

This optimum control force is calculated using a FORTRAN program which writes a file for each control force or moment. At the completion of the program, execution returns to ANSYS.

## 2.4 Calculation of the response for active control

The matrices of optimum control forces are loaded into ANSYS and the response is determined for a single frequency. The responses at the error sensors are recorded, along with additional measurement points. This is saved to another file for post-processing and analysis.

| | | | |
|---|---|---|---|
| Beam length | 1.500m | Beam width | 0.160m |
| Beam thickness | 0.010m | Isolator location | 0.760m |
| Young's modulus | 71 GPa | Moment of inertia | $1.6 \times 10^{-5}$ m$^4$ |
| Beam density | 2800 kg/m$^3$ | Beam damping | $7.48 \times 10^{-6}$ sN/m |
| Isolator stiffness $k_z$ | 45870 N/m | Isolator damping $c_z$ | 140 sN/m |
| Isolator stiffness $\theta_y$ | 216 N/rad | Isolator damping $c_{\theta_y}$ | 140 sN/rad |
| Top mass | 7.44 kg | Bottom mass | 7.88 kg |

Table 1: The parameters used in the modelling.

### 2.5 Analysis of results

The power transmission under active isolation is calculated using the response determined in the previous section and a MATLAB (©Mathworks) script which uses Eq. (3).

## 3 MATLAB MODEL

For validation purposes, the FEM of the simply supported beam and the active isolator was compared with the 2 dimensional theoretical model presented by Pan, Hansen & Pan (1993). The parameters which were used in the model are shown in Table 1.

## 4 RESULTS

The FEA of the beam system was compared with the theoretical model presented by Pan et al. (1993). A unit harmonic primary force was applied to the top mass ($F_z = 1$ N). The power sensors are placed between the active isolator and the simply supported beam. The control actuator acts against the lower mass and reacts against the top mass. Figures 2 and 3 compare the theoretical and FEA predictions of power transmission into the simply supported beam for passive and active vibration isolation.

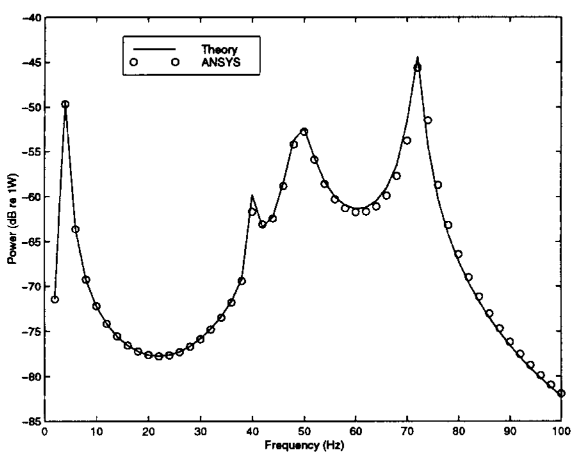


Figure 2: Comparison of theoretical and FEA predicted power transmission into a beam for passive isolation.

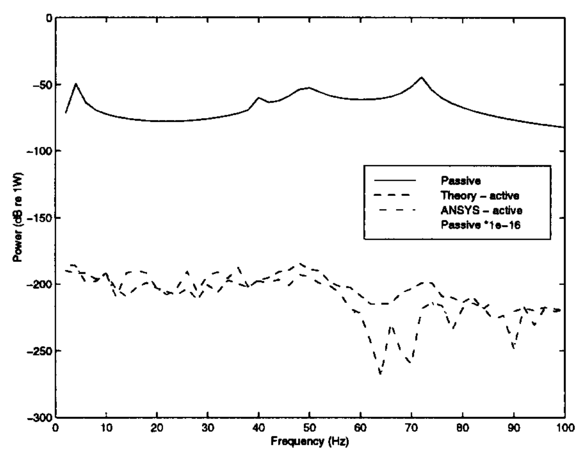


Figure 3: Comparison of theoretical and FEA predicted power transmission into a beam for optimal active isolation.

For the active case, the vibrational power transmission through the error sensor was used as the cost function to be minimised. Figure 3 shows that the active control results are close to the passive power transmission values minus 160 dB. This can be interpreted as the control force having completely cancelled the action of the single primary force, to within the numerical precision of the software. This result differs from the results presented by Pan et al. (1993) who predicted a finite power transmission for active control.

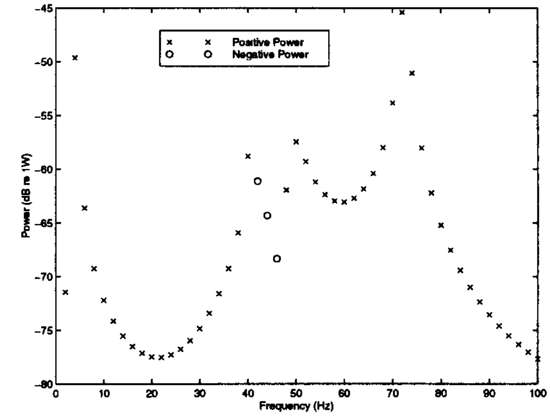


Figure 4: Power transmission for passive isolation when $F_z = 1$ N and $M_y = 0.005$ Nm.

Figure 4 shows the power transmission when the primary force is a unit harmonic load in the vertical direction $F_z = 1$ N, with a rotational moment of $M_y = 0.005$ Nm. A rotational moment can be generated by misalignments of the primary force with the centroid of the top mass. Figure 4 shows that negative power flow occurs in the frequency range of 40 to 50 Hz. In this case a 5mm misalignment of the primary shaker with the centroid of the top mass would generate the required rotational moment. The phenomenon also exists for 2mm of misalignment, which is likely to occur in experimental setups.

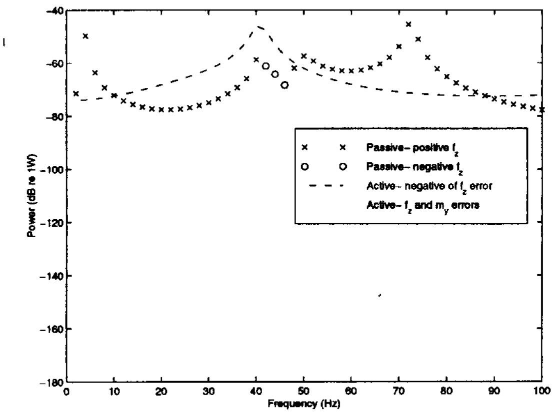


Figure 5: Comparison of the power transmission in the $u_z$ direction for different error criteria.

Negative power flow means that power is being transmitted into the vibration source. For this to occur, the linear power returning to the isolator due to the rotational moments ($M_y$) is greater than the power from the linear primary force ($F_z$).

This interesting phenomena has an effect on the optimal active isolation performance. Figures 5, 6 and 7 show the power transmission into the beam in the $u_z$ and $\theta_y$ directions and total power transmission respectively, for the cases of passive and active isolation for a single error sensor in the $u_z$ direction and error sensors in the $u_z$ and $\theta_y$ directions. In figure 5 it can be seen that the power transmission under active control for a single error sensor in the $u_z$ direction is negative at all frequencies. The controller will "see" an optimum response which is shown to correspond to negative power transmission in the $u_z$ direction. Clearly a negative power transmission value is lower than the positive power transmission value shown for the passive isolation case. However, at some frequencies, the negative

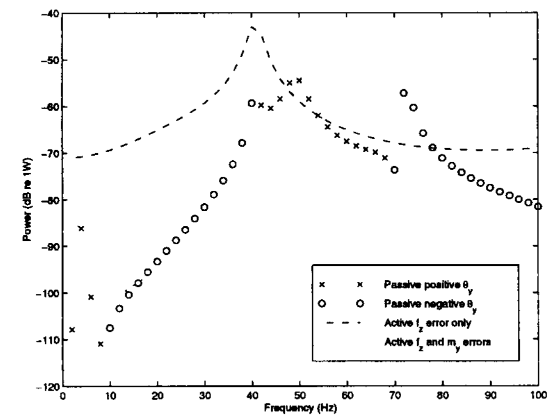


Figure 6: Comparison of the power transmission in the $\theta_y$ direction for different error criteria.

power transmission value will have an absolute value that is greater than the value of the passive isolation case. In this case, the active control has increased the total power transmission into the support structure compared to passive isolation, which can be seen in figure 7.

It can also be seen from figure 5 that when two error sensors are used to measure power in both the $u_z$ and $\theta_y$ direction, the power transmission in the $u_z$ direction is positive and much smaller (i.e. essentially zero) than for the passive case. As illustrated in figure 6, the power transmission in the $\theta_y$ direction for a single sensor measuring power in the $u_z$ direction is much worse than for the passive case. Even with 2 sensors, one measuring power in the $u_z$ direction and the other measuring in the $\theta_y$ direction, the power transmission in the $\theta_y$ direction is never negative as it is for the passive case.

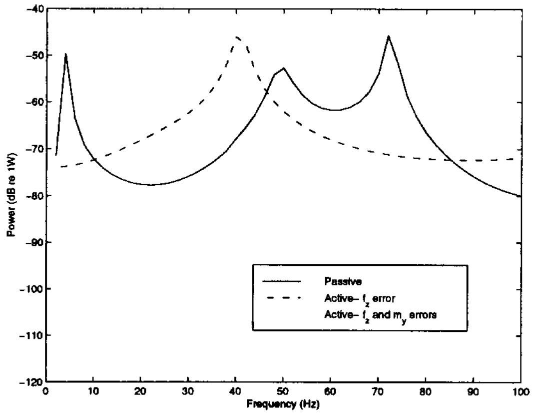


Figure 7: Total power transmission for passive and active isolation when $F_z = 1$ N and $M_y = 0.005$ Nm.

The total power transmission for the passive case and for two active control cases involving the minimisation of power in the $u_z$ direction and then the $u_z$ and $\theta_y$ directions respectively is shown in figure 7. It can be seen that just minimising power in the $u_z$ direction can lead to increases in the total power transmission over a substantial frequency range. However, minimizing power in both $u_z$ and $\theta_y$ results in a small positive power transmission at all frequencies which is a substantial reduction over the passive case.

When two error sensors are used, the power transmission in both the $u_z$ and $\theta_y$ directions are measured by the cost function, and although they might individually exhibit positive or negative values at certain frequencies, the sum will always be positive.

## 5 CONCLUSION

A finite element method has been used to predict the vibrational power transmission from a vibrating mass to a simply supported beam through an active isolator. The finite element method compared well with the passive performance theoretically predicted by Pan et al. (1993) and demonstrated that it is theoretically possible to completely cancel the power transmission if no rotational moments are present. When the primary excitation includes rotational moments and linear forces, the power transmitted into the beam as measured by a linear force and acceleration transducer combination can appear negative at certain frequencies. By neglecting the power transmission caused by rotational moments, the overall vibration isolation under active control can be worse than the passive isolation case, even though the power transmission in the vertical direction is minimised.

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