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UNSTEADY PRESSURE MEASUREMENT: CORRECTION OF THE RESONANCE OF THE PNEUMATIC LINE

S. GABARD, J. PEUBE and C. PICARD

Laboratoire d'Etudes Aérodynamiques, URA C.N.R.S. 191, 40 avenue du Recteur Pineau
86022 Poitiers Cedex, France

Abstract: The reliability of a real time unsteady pressure measurement system is of great interest in many domains of industry and research. Experimental constraints often implies the presence of a pneumatic line between the pressure tap and the transducer, which causes an amplitude and a phase distortion of the pressure signal to be measured. Different ways for correction of this resonance phenomenon are proposed in the literature : analogical (active filters), numerical (inverse transfer function), or mechanical (reduction of the tube section on a certain length (restrictor)). We studied this last mechanical correction method that is generally empirically used , without any a priori suitable choice of the position or the geometry of the restrictor.

This paper presents a modelization of the transfer function of the pneumatic line: the analytical study displays the respective influences of the different geometrical elements of the pneumatic line. Nevertheless, the complexity of the transfer function doesn't allow the a priori determination of the best geometry of the pneumatic line. Our study provides a methodology to optimize the geometry of the restrictor, using a non-dissipative model of the flow in the tube, associated with a quasi-steady flow assumption in the restrictor.

I. INTRODUCTION

In unsteady aeroacoustics, it is necessary to measure surface pressure fluctuations on models: the physical size of models may prohibit the mounting of pressure transducers internally, closed to the measurement point. The pressure tap is relied to the transducer by a pneumatic line, which resonance phenomenon requires a correction of the measurement system.

The use of an analogical compensation system made up of active filters allows, from successive approaches, to reproduce the transfer function of the pneumatic line, with a satisfying damping of the amplitude, but a phase difference which may be a non linear function of frequency. Otherwise, a numerical method using the sampling of the measured pressure and based on the calculation of the Inverse Transfer Function (I. T. F. method), provides a good correction of the amplitude and of the phase distortion, but presents the disadvantage of delayed computations.

Mechanical methods consisting in correcting the geometry of the pneumatic line have been developed. The more frequently used consists in reducing the line tube section on a certain length (restrictor) between the pressure tap and the transducer. The number of geometrical parameters is important (volume before the transducer, diaphragm flexibility, length and diameter of the tube, geometry of the restrictor (length, diameter) and position of this one along the tube). Theoretical results have been obtained by Bergh & Tijdeman [BT65] from the complete movement equations, for a series connection of N tubes and N volumes. These results have been applied by the authors, and more recently by Holmes & Lewis [HL87], to different systems, composed of a line tube with or without instrumentation volume, and with one or two restrictors inserted along the tube. By comparing the transfer functions obtained for different configurations, Holmes & Lewis tried to achieve systems of optimum geometry, that is geometry leading to a modulus of the transfer function equal to unity on the largest possible frequency band and a phase difference which is a linear function of frequency. However, the complexity of the analytical expression of the pneumatic line transfer function, prevents any a priori optimization of its geometry.

II. DETERMINATION OF AN OPTIMUM GEOMETRY USING A SIMPLIFIED THEORETICAL PROPAGATION MODEL

Our purpose is to study a mechanical correction of the pneumatic line of an unsteady pressures measurement system, using a restrictor. The complete theoretical model shows the influence of the geometrical parameters of the pneumatic line, by using different values of these ones . The transfer function being too much complex to allow an a priori choice of the geometry, suitable physical assumptions are proposed and lead to a simplified model, which provides an a priori approximate choice of the right geometry to use.

In the first two paragraphs, the theoretical complete model for a simple tube is resumed and the influence of the length and radius tube and of a volume is discussed. The complete and simplified model for a tube with a restrictor are presented in the third paragraph.

II.1. Dissipative propagation in a simple tube

The basic equations for the propagation of sound waves in air in cylindrical tubes are the Navier-Stokes equations in axial and radial directions, the equation of continuity, the equation of state for an ideal gas and the energy equation. Introducing simplifying assumptions, Tijdeman [Tij75] shows that these equations are governed by four parameters:

$s = R_t\sqrt{\rho_s\omega / \mu}$ the Stokes number, ratio of tube radius R_t and boundary-layer thickness δ

$k_r = \omega\, R_t/c$ the reduced frequency, ratio of tube radius R_t and $\lambda/2\pi$ (λ: wavelength)

$\sigma = \sqrt{\mu C_p / \lambda_0}$ the square root of the Prandtl number: $\sigma = \sqrt{P_r}$ (λ_0: thermal conductivity)

$\gamma = C_p/C_v$ the ratio of specific heats.

with $\omega = 2\,\pi f$, f being the frequency, c the velocity of sound and ρ_s the mean density of fluid.

The complex propagation constant Γ, which appears in the solution of the problem, consists of a real part Γ' and an imaginary part Γ'', representing respectively the attenuation and the phase shift over a unit distance in the axial direction. The expression **(1)**, established by Zwikker & Kosten [ZK49] and independently obtained by Iberall [Ibe50], called "low reduced frequency solution" has been shown by Tijdeman to be valid over the complete range of the values of s for $k_r \ll 1$ and $k_r/s \ll 1$, from Rayleigh's solution to Kirchhoff's one:

$$\Gamma = \sqrt{\frac{J_0(i^{3/2}s)}{J_2(i^{3/2}s)}}\sqrt{\frac{\gamma}{n}} \tag{1}$$

with:

$$n = \left[1 + \frac{\gamma - 1}{\gamma}\sqrt{\frac{J_0(i^{3/2}\sigma s)}{J_2(i^{3/2}\sigma s)}}\right]^{-1} \tag{2}$$

The case of our study is the air flow in a pneumatic line tube. For air at 20 °C in a tube with an internal radius $R_t = 0{,}75$ mm:

At 100 Hz: $k_r = 4{,}37.10^{-4}\,\pi$; $s = 4{,}885$
$\Gamma' = 0{,}2132$; $\Gamma'' = 1{,}2064$

At 500 Hz: $k_r = 0.0022\,\pi$; $s = 10{,}923$
$\Gamma' = 0{,}1061$; $\Gamma'' = 1{,}0957$

Above 500 Hz, $s = R_t/\delta$ becomes very much superior to unity (the boundary layer is thinner than the tube radius) and the attenuation of acoustical waves decreases. The small values of k_r obtained above are consistent with the domain of the "low reduced frequency" approximation. The acoustical pressure and velocity in dimensional form may be written (Tijdeman, [Tij75]):

$$p(\xi,t) = \frac{\rho_s c^2}{\gamma}\left[A\,e^{\Gamma\xi} + B\,e^{-\Gamma\xi}\right]e^{i\omega t} \tag{3}$$

$$u(\xi,\eta,t) = \frac{i\Gamma c}{\gamma}\left[1 - \frac{J_0(i^{3/2}\eta s)}{J_0(i^{3/2}s)}\right]\left[Ae^{\Gamma\xi} - Be^{-\Gamma\xi}\right]e^{i\omega t} \tag{4}$$

where $\eta = r/R_t$ and $\xi = \omega x/c$ are the radial and longitudinal reduced coordonates.

II.2. Influence of geometrical parameters of the pneumatic line

Assuming pressure fluctuations imposed at the entry of the tube to be harmonic and the air velocity to be equal to zero at the transducer (measurement surface assumed to be rigid), relations **(3)** and **(4)** lead to the transfer function between the pressure measured by the transducer (x = 0) and the pressure to be measured at the entry of the tube (x = L). According to a non-dissipative model, the resonance occurs when the tube length is equal to a quarter of wavelength; with dissipative model the resonance frequency is a little lower. The L length of the tube has to be as small as possible to get a higher resonance frequency. When the R_t value of the internal radius of the tube decreases, the frequency and the amplitude of the resonance also decrease. The need for the bandwidth to be as large as possible leads to choose a low enough value of the tube radius to decrease the resonance amplitude, without choosing a too weak value, which produces an excessive damping phenomenon after the resonance.

The pneumatic line of an unsteady pressure measurement system often includes an instrumentation volume between the tube and the measurement surface of the transducer. A first approach, based on a non-dissipative model of the flow in the volume, allows to conclude that the value of this volume has to be as small as possible. The same conclusion may be obtained by observing that the tube is equivalent to a one degree oscillator with stiffness near the close end, mass near the open end and viscous damping. Adding a volume decreases stiffness, so as the resonance frequency is lowered, which is unfavourable for pressure measurements ; the pressure amplitude that may be shown, at resonance frequency, proportional to the square of the stiffness, decreases also that is favourable but this decreasing is slight.

II.3. Models for a tube with a restrictor

To correct the resonance induced by the line tube, a mechanical method consists in introducing a narrowing (restrictor) of the tube between the pressure tap and the transducer. The most reliable restrictors and the easiest to modelize, are made up with a peripheral restriction of section, by inserting in the tube a smaller internal diameter tube. According to Bergh & Tijdeman's model, the length to diameter ratios of the tube and of the restrictor are assumed large, so that end effects can be neglected. The distance between the restrictor and the transducer's diaphragm, assumed tò be rigid, is noted L_I. The S tube section is locally reduced to the S_r one in the restrictor. The L_r length of the restrictor is assumed to be small in comparison with the L one of the tube:

$$L \approx L_I + L_2 \qquad (5)$$

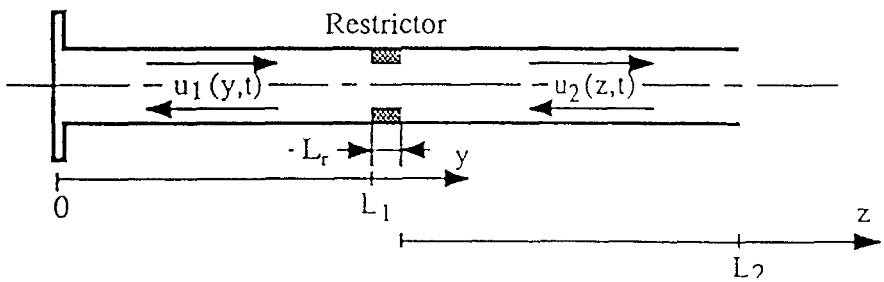

Figure 1. Tube with a restrictor.

II.3.1. *Dissipative model of the system*

The transfer function is obtained from the following assumptions and boundary conditions:
- the diaphragm of the transducer is rigid
- the restrictor length is small as compared as the length tube or the wavelength : by integrating the equation of continuity on the restrictor volume, it can be shown that the relative difference between the axial velocity at the entry and at the end of the restrictor is of order of L_r / λ. Compressibility effects in the restrictor can be neglected and the equality of these velocities may be used .
- the pressure drop induced by the restrictor is expressed as a function of a coefficient noted R (for small values of the Stokes number s, s << 1, we shall see that R is the Poiseuille resistance of a narrow tube for a steady-flow):

$$p_2 - p_I = R u^*_{I_m} = R\, u^*_m \qquad (6)$$

with $u^*_{I_m} = S\, u_{I_m}$ and $u^*_m = S_r\, u_m$, where u_{I_m} and u_m are respectively the mean velocity in the tube and in the restrictor and u^*_i is the volumic flow rate. The value of R , acoustic impedance, will be determined in the next paragraph .
By noting $Z_c = \rho_s c / S$, the characteristic impedance of the tube and $k = \omega/c$ the reduced frequency, the transfer function can then be expressed

$$\frac{p_1(o,t)}{p_2(\omega L_2/c,t)} = \frac{1}{\cosh \Gamma kL - i\theta \frac{R}{Z_c}(\sinh \Gamma kL - \sinh \Gamma k(L-2L_1))} \quad (7)$$

with

$$\theta = \int_0^1 \frac{\Gamma}{\gamma} \left[1 - \frac{J_0(i^{3/2}\eta s)}{J_0(i^{3/2} s)}\right] \eta \, d\eta \quad (8)$$

II.3.2. *Flow model in the restrictor*

With a small L_r restrictor length

$$\frac{dp}{dx} = \frac{p_2 - p_1}{L_r} \quad (9)$$

With the assumptions of the "low reduced frequency solution" and writing the relation **(9)** in a form similar to the relation **(6)** leads to a frequency dependent expression of R.

$$R = i\,\rho_s\,\omega \frac{L_r}{S_r} \frac{J_0(i^{3/2}s)}{J_2(i^{3/2}s)} \quad (10)$$

For small values of the Stokes number, $s \ll 1$, the limited expansions in series of s of the Bessel functions J_0 et J_2 lead to a frequency independent and real expression of the coefficient R of the restrictor, noted R_1, a being the internal radius of the restrictor :

$$R_1 = \frac{8\mu}{a^2} \frac{L_r}{S_r} \quad (11)$$

The ratio between R and R_1 is a complex function of frequency, which real part tends to unity and imaginary part to zero, when ω tends to zero. The difference between these two terms remains weak on a larger frequency bandwidth than the one fixed by $s = 1$.

II.3.3 *Simplified model of the system*

The transfer function **(7)** is complicated that prevents any a priori choice of a good geometry of the restrictor. We propose a simplified model, based on *a non-dissipative model of the flow in the tube* associated with *a quasi-steady flow assumption in the restrictor ($s \ll 1$)*, that provides the simplified expression:

$$\frac{p(0,t)}{p(L_2,t)} = \frac{1}{\cos kL + i\frac{R_1}{2Z_c}(\sin kL - \sin k(L-2L_1))} \quad (12)$$

- $L_1 = 0$: The R_1 term disappears in relation **(12)**: the restrictor is ineffective with this model, placed in an area where the flow velocity is nearly equal to zero. It comes to the case of a simple tube.

- $L_1 = L_2 \approx L/2$: The restrictor is centred. The modulus of the pressure ratio M verifies :

$$1/M^2 = 1 + \left(\frac{R_1^2}{4Z_c^2} - 1\right) \sin^2 kL \quad (13)$$

The first M^2 extremum is obtained for $kL = \pi / 2$:

$$M^2 = \frac{4Z_c^2}{R_1^2} \tag{14}$$

If $R_l < 2\ Z_C$ it is a maximum: the restrictor produces an under-damping. If $R_l > 2\ Z_C$ it is a minimum: the restrictor produces an over-damping. If $R_l = 2\ Z_C$: M^2 is a constant; the restrictor is optimum. The phase φ, that verifies (15), is then a linear function of frequency.

$$tg\ \varphi = \frac{R_1}{2Z_c}\ tg\frac{\omega L}{c} \tag{15}$$

- $L_1 = L$: The restrictor is placed at the tube entry; it produces an under-damping if $R_l < Z_C$, an over-damping if $R_l > Z_C$, and is optimum for $R_l = Z_C$, the phase φ being then a linear function of frequency.

According to this simplified model, a centred restrictor which acoustic resistance is twice the characteristic impedance of the tube (or a restrictor placed at the tube entry which resistance is equal to the characteristic impedance) is optimum: the pressure amplitude ratio remains constant with frequency and equal to unity, and the phase is a linear function of frequency. The internal radius of the restrictor must be small enough and the frequency low enough to verify the quasi-steady flow assumption $s << 1$. For restrictors used in the experimental study (see Section III.), this last assumption is strictly respected only for low frequencies ($N \leq 100$ Hz); however, the difference between the theoretical responses for a tube with an optimum restrictor, using the R_l acoustic resistance or the R exact acoustic impedance, remains weak on a larger frequency band.

Nevertheless, theoretical responses calculated with the dissipative model of the flow in the tube, often show a residual resonance peak: a further adjustment, obtained by moving the restrictor towards the tube entry or by using a R resistance a little higher, may then be used.

III. EXPERIMENTAL STUDY

This section presents several experimental results of the correction obtained with restrictors. The geometrical parameters were chosen on the basis of the properties of the simplified model presented in Section II.3.3.

III.1. Great length tube with optimum centred restrictor

The Figure 2. presents the experimental and the dissipative theoretical (with the frequency dependent R impedance) responses for a tube (length $L = 350$ mm, radius $R_t = 0{,}75$ mm) with a centred optimum restrictor (length $L_r = 8{,}56$ mm, radius $a = 0{,}17$ mm). The agreement between theory and experiment is satisfying, and the result shows the existence of a small residual peak: the damping of the first resonance of the simple tube is obtained with the optimum centred restrictor, the amplitude ratio remaining close to unity within the limits of ± 10 % on the bandwidth 0-200 Hz. A better accuracy could be obtained by moving the restrictor towards the tube entry, but the damping after the resonance would then be faster.

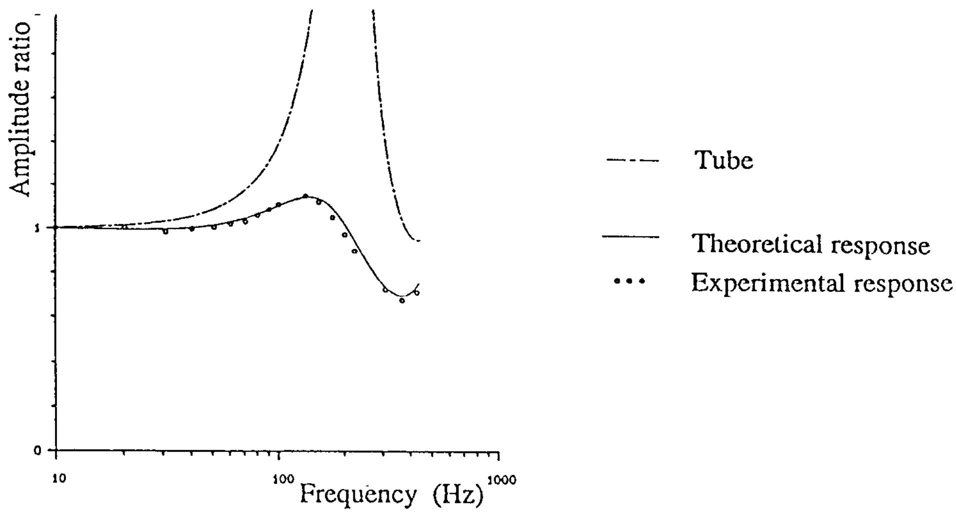

Figure 2. Experimental and theoretical characteristics for a tube (L = 350 mm, R_t = 0,75 mm) with an optimum centred restrictor (L_r = 8,56 mm, a = 0,17 mm).

III.2. Short length tubes with unknown volume and optimum restrictor

The Figure 3. presents the experimental amplitude characteristics for two short tubes (L = 51 mm and 56 mm, R_t = 0,75 mm) connected to a small instrumentation volume (whose value is unknown), and the diaphragm flexibility is non negligible. For these two tubes, the useful bandwidth before correction is limited to the frequency band 0 - 250 Hz.

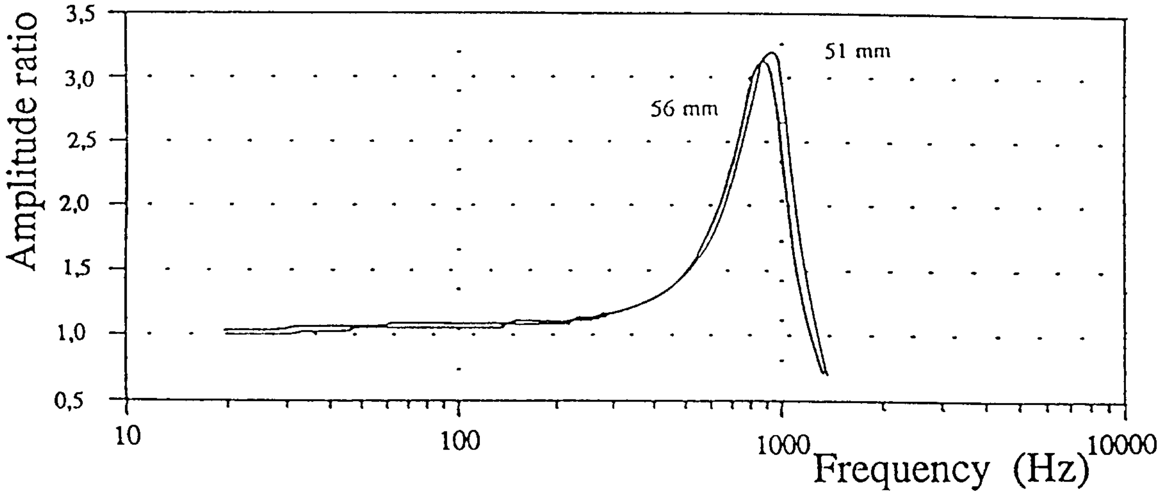

Figure 3. Experimental characteristics for two tubes (L = 51 mm and L = 56 mm , R_t = 0,75 mm) with an unknown volume.

The correction method consists in inserting a restrictor whose geometry has been calculated to be optimum in a centred position, according to the simplified model (L_r = 5,2 mm, a = 0,15 mm). The existence of the unknown volume and the diaphragm flexibility lead to an over-damping, when the restrictor is centred: after a further experimental adjustment, the restrictor is placed in the position $\alpha = L_1 / L = 0{,}4$. The Figure 4. presents the experimental amplitude characteristics obtained after correction. The useful bandwidth after correction for the L = 51 mm tube is 0 - 600 Hz, the amplitude ratio between the pressures measured and to be measured remaining close to unity within the limits of ± 10 %. For the L = 56 mm tube, the useful bandwidth after correction is 0 - 500 Hz, the amplitude ratio remaining close to unity within the limits of ± 5 % .

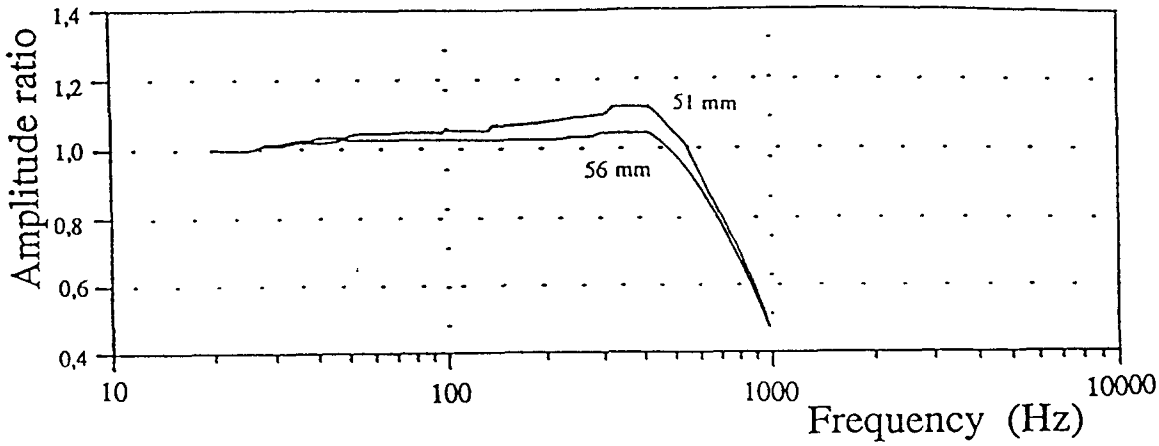

Figure 4. Experimental characteristics for two tubes (L = 51 mm and L = 56 mm , R_t = 0,75 mm) with an optimum restrictor (L_r = 5,2 mm, a = 0,15 mm, $\alpha = L_l / L = 0,4$).

IV. CONCLUSION

From the study of the transfer function of the pneumatic line between the pressure tap and the transducer, we tried to foresee the geometry of this liaison, so as it would perturb as less as possible the measurement of unsteady pressures, on a large enough frequency band.

Minimising the tube length and the instrumentation volume value provides a higher first resonance frequency. Using a small enough internal radius of the tube decreases the resonance amplitude; nevertheless, a too weak value would produce an over-damping.

A pneumatic line including a *restrictor* is then used to damp the resonance amplitudes. We proposed a simple model, with a non-dissipative model of the flow in the tube and a *quasi-steady flow assumption* in the restrictor ($s << 1$): for a centred restrictor whose acoustic resistance is twice the characteristic impedance of the tube (or a restrictor placed at the tube entry, its resistance being equal to the characteristic impedance), there is no frequency dependence of the modulus of the pressure transfer function, and the phase is a linear function of frequency.

The amplitude of the first resonance calculated from the complete theory is a little higher than the approached one: moving the restrictor towards the tube entry provides a further adjustment of this amplitude closer to the unity. An example of a pneumatic line with an instrumentation volume and a non-negligible diaphragm flexibility is given : the first resonance amplitude and its frequency are then lowered : moving the restrictor towards the transducer provides a further adjustment.

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