Global sensitivity analysis of acoustic transmission models

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ABSTRACT

Optimization of noise reduction inside cavities is a particularly important topic in the aeronautic and space industries, as the very high noise levels outside the vehicle can cause passenger or payload comfort issues, introducing the need for sound packages. With the increasing use of composite materials in aerospace structures,acoustic transmission models may become quite complex and depend on numerous material or geometric parameters, in a broadband frequency range. It is therefore interesting to identify the most influential of these parameters in order to reduce the computing cost associated with any optimization task on these models. Sensitivity analysis methods can be used for this purpose. As the parameters may vary in broad design ranges, interactions between them may arise, making the use of global methods such as analysis of variance relevant. An application of the Fourier amplitude sensitivity test (FAST) to acoustic transmission models through multilayered composite plates are presented in this paper. Outcomes are consistent with classical results such as mass law, and new interpretations are presented in the case of plane wave and diffuse field excitation.

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1. INTRODUCTION

Noise reduction is a very important issue in the industry. Typical applications include interior noise inside plane fuselage or car bodies, or payload comfort for space launchers. Transmission loss through complex structures has been studied extensively throughout the last 40 years. The considered structures range from plates [1] to multilayered composite cylindrical shell [2]. More recently, numerical models based on the Wave finite element method [3] or taking into account periodicity of the structure [4] have been proposed.

As complexity of the structure increases, so does the number of parameters needed to accurately model its behavior. However, given industrial constraints, the design range of all these parameters may be so that some of them have major influence on the model output while the effect of others remains small. If these parameters can be identified and fixed to a reference value, the optimization problem need only be conducted on a reduced design space, thus saving computing time.

Several methods have been proposed for optimizing noise transmission through composite structures. Wang et al. [5] propose an application of genetic algorithm to find an optimal design of sandwich panel considering both structural and acoustic constraints. However, in order to be efficiently designed, these methods need a good insight in the mechanical behavior of the model with respect to its parameters.

Global sensitivity analysis (GSA) methods have been studied since the 70's for different purposes. Cukier et al. [6] developed the FAST method to study complex chemical reactions. Iooss et al. [7] used sensitivity analysis on radiologic risk assessment models. Ouisse et al. [8] applied the FAST method to porous material models, regarding acoustic impedance and absorption.

Contrarily to other models to which GSA has been applied, the ranking of parameters by order of influence on the output given by sensitivity analysis is not absolute in acoustic models, but depends on frequency. As the frequency ranges considered for noise transmission may be quite wide, the system will present very different behaviors in different regions of the excitation spectrum. The behavior in low frequencies is essentially governed only by the mass of the system, while damping and stiffness-like parameters will play a more important role for higher frequencies. However, the transition between these phases can be rather complex.

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The objective of this paper is to find interpretations of the results of sensitivity analysis applied to acoustic transmission models through single and double-walled partitions.

The paper is structured as follows. Section 2 presents an overview of the global sensitivity analysis method used in this work. Section 3 presents the model used to compute sound transmission loss through simple and double plates. Section 4 presents the results of the FAST analysis of a single plate model under plane wave and diffuse field excitation. The same analysis is presented in section 5 for the more complex case of double plates.

Finally conclusions are summarized in section 6.

2. GLOBAL SENSITIVITY ANALYSIS

2.1 Analysis of variance

In the analysis of variance technique, a parameter's influence on the model output is quantified by the impact it has on the variance in the given design range. In the following development, a generic mathematical model is considered. A model is a real valued function f defined over K^n, where $K = [0,1]$. With appropriate scaling and translations, any model defined over continuous ranges of parameters can be represented that way.

For a given model f linking input parameters $\mathbf{x} = (x_1, ...x_n)$ to a scalar output $y = f(\mathbf{x})$, there exists a unique partition of f so that

$$y = f(x_1, x_2, ...x_n) = f_0 + \sum_{i=1}^{n} f_i(x_i) + \sum_{i<j} f_{ij}(x_i, x_j) + ... + f_{1...n}(x_1, ..., x_n) \tag{1}$$

provided that each function f_I involved in the decomposition has zero mean over its range of variation

$$\int_{K|_I} f_I(x_I)\mathrm{d}x_I = 0. \tag{2}$$

The decomposition given by equation 1 is called the Hoeffding decomposition or high order model representation (HDMR).

For a given set of indices $I = \{i_1, \ldots, i_n\}$, the partial variance is therefore the variance of f_I

$$D_I = \int_{K|_I} f_I(x_I)^2 \mathrm{d}x_I \tag{3}$$

the sensitivity index relative to the set I is expressed as the ratio of the variance of the function f_I to the total variance of the model:

$$\mathrm{SI}(I) = \frac{D_I}{D}. \tag{4}$$

The computation of all the 2^n sensitivity indices is needed to represent completely the model, however this becomes quickly a very costly task in terms of computational time, as they have to be evaluated by numerical integration. However, most information about a parameter's influence can be found in the first-order sensitivity index and the total sensitivity index, which can be computed more efficiently with the FAST method.

For a given parameter $i \in [1, n]$, the main effect (ME) is then the sensitivity index relative to the 1-dimensional function f_i. Another interesting sensitivity measure for a given parameter i is the total sensitivity index, defined as the sum of the indices of all sets of parameters I to which i belong.

$$\mathrm{TSI}(i) = \sum_{\substack{I \subset [1,n] \\ i \in I}} \mathrm{SI}(I) \tag{5}$$

2.2 Interpretation

The first-order index represents the share of the output variance that is explained by the considered parameter alone. Most important parameters therefore have high ME, but a low ME does not mean the parameter has no influence, as it can be involved in interactions.

The total index is a measure of the share of the variance that is removed from the total variance when the considered parameter is fixed to its reference value. Therefore parameters with low TSI can be considered as non-influential.

2.3 Main effect computation

The idea of the FAST method is to avoid the evaluation of the n-dimensional integrals needed for the computation of the f_i functions, and replace them by a single 1-dimensional integral along a *space-filling* curve in the design space. This curve is defined so as to be periodic with different periods relative to each parameter. Saltelli [9] propose the sampling function defined by:

$$x_i = \frac{1}{2} + \frac{1}{\pi}\arcsin\left(\sin\left(\omega_i s + \varphi_i\right)\right) \tag{6}$$

The frequencies ω_i are integers chosen so as to minimize interference between parameters. The frequencies are said to be free of interference up to order M if all linear combinations

$$\sum_{i=1}^{n} \alpha_i \omega_i \neq 0 \tag{7}$$

where $\alpha_i \in \mathbb{Z}$ and $\sum_{i=1}^{n} |\alpha_i| < M$.

As all frequencies are integers, the resulting function is 2π-periodic with respect to variable s. The sampling is then done using $N > 2\omega_n + 1$ samples in the $[0, 2\pi]$ interval. Calling $y_k = f(x_k)$ the model output on each sample, the discrete Fourier transform $\hat{y}_k$ can be computed.

The total variance of the function in the design space is computed with Parseval's theorem as

$$D = \int_K f^2(x) - f_0^2 \mathrm{d}x \approx \sum_{k=1}^{N} y_k^2 = \sum_{k=1}^{N} \hat{y}_k^2 \tag{8}$$

The contribution of parameter i will then be:

$$D_i = \sum_{k=1}^{M} \hat{y}_{k\omega_i} \tag{9}$$

2.4 Total sensitivity index computation

A method proposed in [9] is to assign one frequency ω_i to parameter i and another $\omega_{\sim i}$ to all other parameters. The same sampling curve as defined in equation 6 is used with these two frequencies only. The total sensitivity index of parameter i is then

$$TSI(i) = 1 - \frac{D_{\sim i}}{D} \tag{10}$$

where $D_{\sim i}$ is the partial variance relative to all parameters but i.

3. ACOUSTIC TRANSMISSION THROUGH COMPOSITE PARTITIONS

The acoustic transmission through infinite partitions has been studied over several decades. The main interest of these models s the fact that analytical solutions can be derived. Models for orthotropic plates can be found for example in [10] for single isotropic plates, of [11] for more complex structures such as orthotropic plates or double-walled partitions.

3.1 The simplified transfer matrix method

The transfer matrix method was introduced with the work of Brouard et al. [12] to study the noise transmission through an arbitrary layering of plates, fluid layers and porous layers. It has been extended and standardized in the work of Allard and Atalla [13]. However the implementation proposed in the latter book uses a complex assembling algorithm to take into account all the propagating waves in elastic layers and porous layers. A simplified model valid for thin plates and fluids is proposed in the work of Hu [14], which will be recalled here.

We will only focus here on two types of layers, namely acoustic fluid and elastic plates. The idea of the simplified transfer matrix method is to represent the transmission of an acoustic pressure wave through a infinite plane structure composed of plates and fluid layers, impinged by an oblique plane wave with pulsation ω and incidence angle θ with respect to the normal of the plate, and backed by a semi-infinite cavity full of fluid. It is therefore a 1D system, the only position variable being in the thickness direction.

The pressure wave can be described uniquely by its pressure p and velocity along the normal direction v, which are time-harmonic functions, and depend on the location. As the structure is excited by a plane wave, the transverse component of the wavevector is forced across the system, and is equal to

$$k_x = \frac{\omega}{c_0}\sin\theta \tag{11}$$

A fluid layer of thickness h is characterized by its density ρ_0 and sound speed c_0. The wave equation in the fluid leads to the following relation between p, v at both ends of the layer:

$$\begin{pmatrix} p(0) \\ v(0) \end{pmatrix} = \mathbf{T_f} \begin{pmatrix} p(h) \\ v(h) \end{pmatrix}, \tag{12}$$

where the transfer matrix of the fluid layer is

$$\mathbf{T_f} = \begin{pmatrix} \cos(k_z h) & \dfrac{i\omega\rho_0}{k_z}\sin(k_z h) \\ \dfrac{ik_z}{\omega\rho_0}\sin(k_z h) & \cos(k_z h) \end{pmatrix}. \tag{13}$$

The symbol i is the imaginary unit $i^2 = -1$.

A thin plate is defined by its surface mass m_s and bending stiffness D, which in the isotropic case is constant and equal to

$$D = \frac{Eh^3}{12(1-\nu^2)}, \tag{14}$$

where E is the Young modulus, h the plate thickness, and ν the Poisson ratio. The wave transmission through an infinite plate is characterized by its impedance, defined by [15]

$$Z_s = i\omega m_s(1 - \frac{Dk_x^4}{\omega^2 m_s}), \tag{15}$$

and the transfer matrix of a single plate is

$$\mathbf{T_p} = \begin{pmatrix} 1 & Z_s \\ 0 & 1 \end{pmatrix} \tag{16}$$

The transfer matrix of a layering of fluids and plates will then be the product of the individual transfer matrices of each layer. For a single plate, we have

$$\mathbf{T} = \mathbf{T_p}, \tag{17}$$

and for a double plate

$$\mathbf{T} = \mathbf{T_p^{(2)}} \mathbf{T_f} \mathbf{T_p^{(1)}}. \tag{18}$$

with different matrices for the plates.

The transmission loss can then be computed by processing the elements of the global transfer matrix. The acoustic transparency is the ratio of the transmitted power over the incident power, which is also the ratio of the mean squared pressure relative to the positive-going waves on both sides of the system. The pressure field can be decomposed into the sum of a positive and negative-going waves on both sides of the structure:

$$P_i = P_i^+ + P_i^- \tag{19}$$

and the velocity field is related to this by the acoustic admittance of the surrounding fluid:

$$V_i = Y(P_i^+ - P_i^-), \tag{20}$$

where subscript $_i$ represents the left (L) or right (R) side of the system, and $Y = \dfrac{\cos\theta}{\rho_0 c_0}$ is the admittance of the fluid, assuming it is the same on both sides. After some manipulation of the equations, we come up to the scattering matrix **D** defined with the following equation:

$$\begin{pmatrix} P_L^- \\ P_R^+ \end{pmatrix} = \mathbf{D} \begin{pmatrix} P_L^+ \\ P_R^- \end{pmatrix} \tag{21}$$

The transmission loss is then defined as

$$\mathrm{STL} = -10\log_{10}\left(|D_{21}|^2\right), \tag{22}$$

where the coefficient D_{21} of matrix D can be computed as follows

$$\frac{1}{D_{21}} = \frac{1}{2}\left(T_{11} + T_{12}Y + \frac{T_{21}}{Y} + T_{22}\right) \tag{23}$$

The acoustic transparency relative to the plane wave with incidence θ and pulsation ω is

$$\tau(\omega,\theta) = |D_{21}|^2. \tag{24}$$

For single and double plates, the latter development is equivalent to the analytical models found (for example) in [11].

3.2 Diffuse field excitation

A diffuse field excitation is a random excitation from which the incident sound comes from all directions with equal probability. The diffuse field transparency is then defined as the average oblique plane wave transparency with an appropriate weighting. We therefore have

$$\tau_d(\omega) = \frac{\int_0^{\frac{\pi}{2}} \tau(\omega,\theta)\sin\theta\cos\theta \mathrm{d}\theta}{\int_0^{\frac{\pi}{2}} \sin\theta\cos\theta \mathrm{d}\theta}, \tag{25}$$

hence

$$\tau_d(\omega) = 2\int_0^{\frac{\pi}{2}} \tau(\omega,\theta)\sin\theta\cos\theta \mathrm{d}\theta. \tag{26}$$

From a numerical point of view, it is advisable to use slightly different values for the integration , and a high number of incidence values is needed (about 1000 for a good precision up to 30 kHz).

4. FAST ANALYSIS OF SINGLE PARTITIONS

4.1 Plane wave

The FAST analysis was conducted on the above model for an infinite plate 1mm thick, with parameters uniformly distributed in the ranges defined in table 1.

Results are presented in figure 1. As the considered model output is averaged over frequency bands, the sensitivity indices are plotted as histograms.

The continuous line represents the STL for the median values of the parameters (corresponding in this case to steel), divided by the maximum value over the whole range. This allows a visualization of the global trends of the model output for a special case. The three important zones are visible, below around and above coincidence.

The other interesting quantity shown on the graph is the normalized standard deviation, computed as the ratio of the global standard deviation to the mean value for each frequency band.

The density is shown to have a very high ME in low frequencies, while for higher frequencies, the Young modulus is predominant. This is consistent with the classical result that high frequencies are governed by stiffness effects and the low frequencies by mass effects.

As the design range has been chosen so that parameters lie within $\pm 10\%$ of typical values for steel, all coincidence frequencies occur rather close to each other.

A high level of interactions between parameters can be observed in the middle of the considered frequency range, where all parameters have rather low ME, but E and ρ exhibit high TSI. This high level of interaction can be explained by the fact that the coincidence frequency is characterized by an important drop in the TL, and that the value of this frequency

$$\omega_{coin} = \frac{c_0^2}{\sin^2\theta}\sqrt{\frac{\rho_s h}{D}}, \tag{27}$$

is a function of both density and stiffness.

4.1.1 Influence of design range

The influence of the design space on the result has been studied by setting the damping range to $[2.5 \cdot 10^{-3}, 0.5]$. The results are given in figure 2. An increased influence of damping in the coincidence region is observed, as expected from the theory There is practically no difference in the rest of the considered frequency range. However, varying more importantly the Young modulus and the density lead to a wider coincidence band, without changing the lower and higher frequency trends.

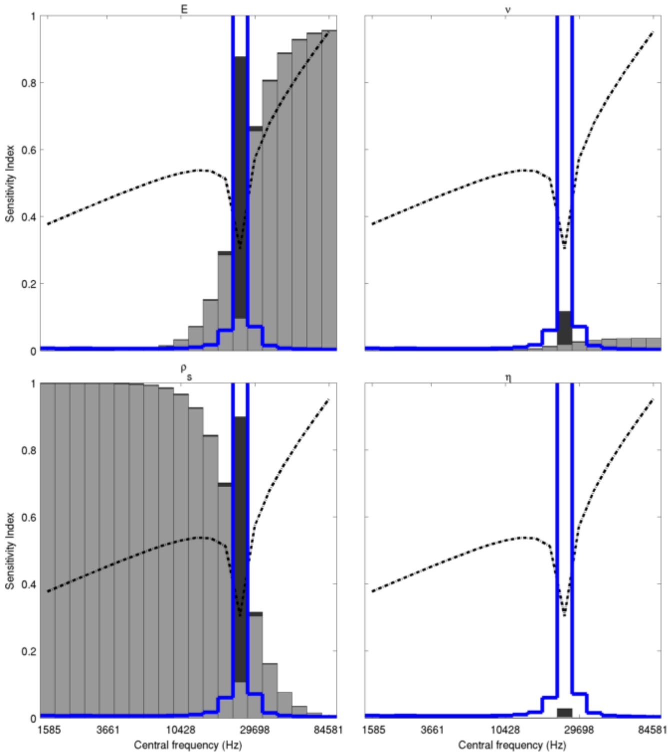

Figure 1 – FAST sensitivity analysis of infinite isotropic plate under 45^o incident plane wave. Dark grey: TSI; Light grey: ME; dashed line: STL trend for the median plate; solid line: normalized standard deviation.

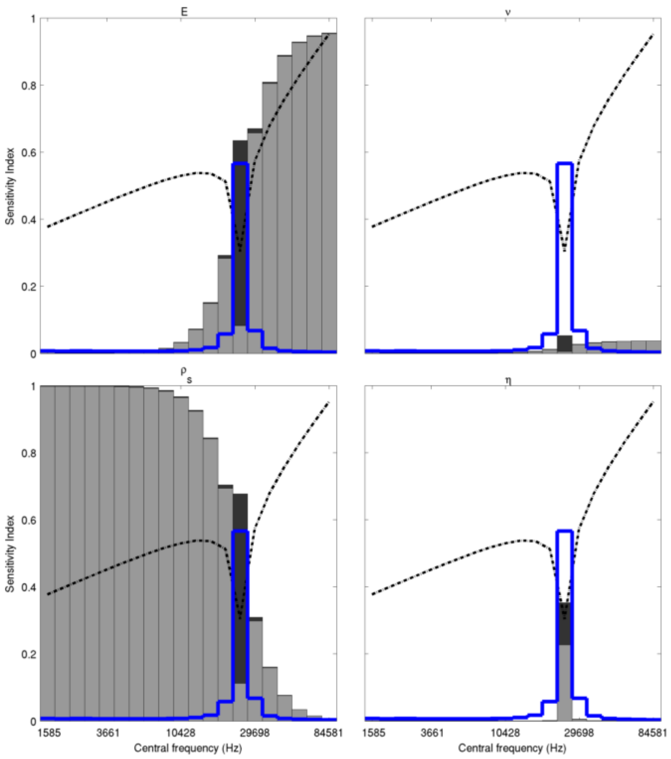

Figure 2 – FAST sensitivity analysis of infinite isotropic plate under 45^o incident plane wave with high damping variability. Dark grey: TSI; Light gray: ME; dashed line: STL trend for the median plate; solid line: NSD

Variable	Min. value	Max. value
E_x (GPa)	180	220
ν	0.27	0.33
ρ_s (kg.m^{-3})	7020	8580
η ($\cdot 10^{-3}$)	2.5	7.5

Table 1 – Variation ranges of parameters for isotropic models

4.2 Diffuse field

The diffuse field integration introduces a different behavior in the higher frequency range. Below f_{crit}, the mass law is still valid, but above it, the most influential parameter is by far damping, while stiffness parameters have an influence only on the location of the critical frequency

$$f_{crit} = f_{coin}(\theta = \pi/2). \tag{28}$$

This can be explained by the fact that above f_{crit}, coincidence occurs for some incidence at every frequency, so that the main effect governing the TL value is effectively the dip at coincidence, and no longer the coincidence frequency itself, so that damping becomes preponderant, even if it varies in the rather narrow range specified in table 1.

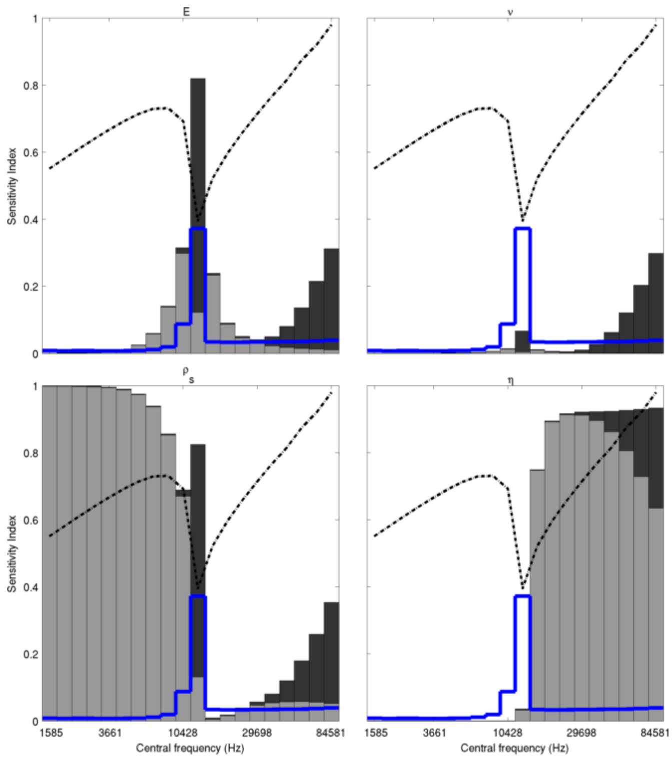

Figure 3 – FAST sensitivity analysis of infinite isotropic plate under diffuse field excitation. Dark grey: TSI; Light gray: ME; dashed line: STL trend for the median plate; solid line: NSD

5. FAST ANALYSIS OF DOUBLE PLATES

The acoustic transmission was studied with the model described in section 3 and especially equation 18. Two cases were studied, one where the two plates have similar characteristics, corresponding to steel, and one where one plate is made of metal and the second one of a typical polymer. Only the diffuse field case was studied, as it is the most representative of a real case.

The typical STL curve of a double plate system under plane wave excitation exhibits several drops, corresponding for a part to the coincidence frequency of each plate, and for the rest to resonances of the air cavity. Under diffuse field, these resonances, being proportional to $\frac{n}{\sin\theta}$, where n is an integer, are no longer visible

The transmission loss was averaged over third-octave bands. The results are presented on figure 4.

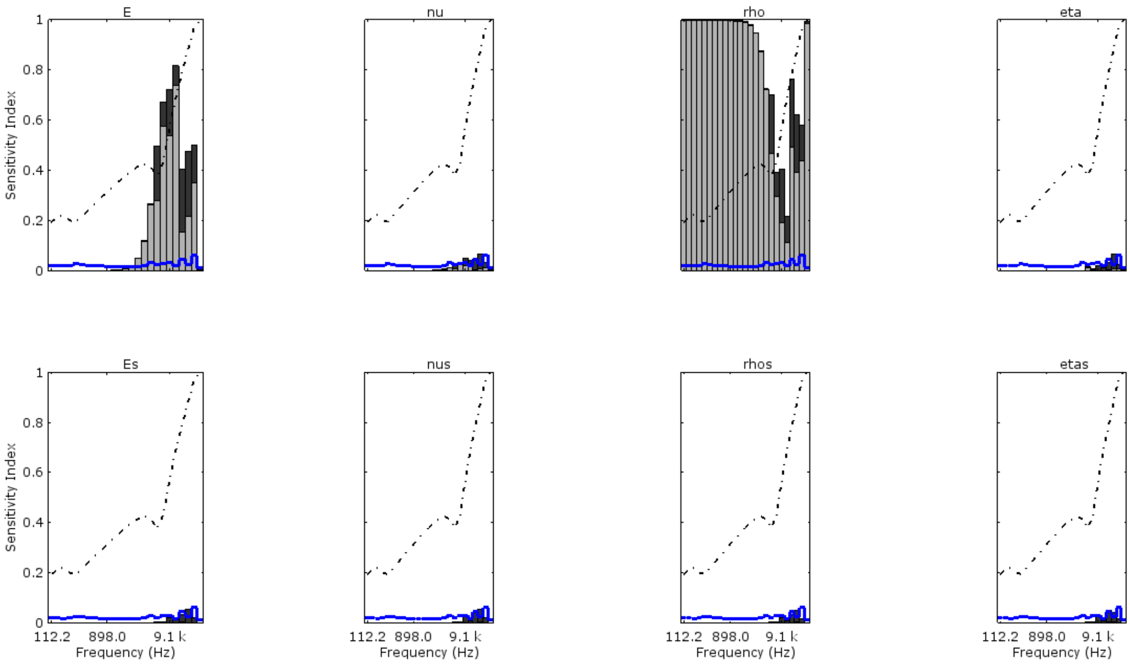

Figure 4 – FAST analysis of double steel under diffuse field excitation. Dark grey: TSI; Light gray: ME; dashed line: STL trend for the median plate; solid line: NSD

On that figure, the parameters on the first line relate to the first plate of thickness 2mm, and the second line to he second plate, with a thickness of 1mm. Both plates have their parameters varying in the same range as defined in table 1. It can be seen that only the parameters relative to the first plate have a noticeable influence on the output, and that the variability of the TL over the design range, characterized by the normalized standard deviation (NSD), remains very low over the considered frequency range. The mass law is still visible in low frequency, as well as the influence of stiffness around the two critical frequency (the peaks on the "E" curve, separated by one octave, as the critical frequencies are inversely proportional to thickness)

6. CONCLUSIONS

The Fourier amplitude sensitivity test (FAST) has been applied to several cases of acoustic transmission through partitions. Isotropic and orthotropic single plates are studied under plane wave excitation and diffuse field, which allow to validate the approach by finding known results. The mass law in low frequency, damping control around coincidence and stiffness control in higher frequencies are found in the plane wave case. The diffuse field case exhibits a different phenomenon, in that damping is dominant in higher frequencies, due to the superposition of plane waves with different coincidence frequencies. Double-layer plates are also studied. In this case the phenomena described previously are also observed, but the air gap between the two plates induces resonances in the mid-frequency range, which lead to the preponderance of the air gap thickness if this is considered a relevant parameter.

The FAST method is therefore an interesting method to determine the most influential variables in an acoustic transmission model, provided the uncertainties of the considered variables are relatively well known. That said, most of the models studied here present a small number of influential parameters over the whole frequency range, making it possible to focus only on these in an optimization task.

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