A study on the influence of model uncertainties on the performance of a feedback control based ASAC system

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ABSTRACT

An important problem in systems such as aerospace interiors, automobile passenger compartments and other cavities is the control of low frequency interior noise. Active control of this noise may offer a potentially better alternative to passive control due to constraints of weight and space. Active structural-acoustic control (ASAC) is considered as an attractive strategy especially for controlling structure-borne noise. LQG based feedback control is one of the popular strategies that has been used in the context of ASAC. The LQG based strategies utilize an observer that estimates the states of the plant for feedback and therefore is a key element in the whole control loop. Since it uses a copy of the model of the plant for estimation, the accuracy of the model is crucial for its performance. This paper reports a study on the influence of the model uncertainties on the LQG based ASAC system. The influence of the uncertainties on the accuracy of the estimated states as well as on the closed loop system performance is studied. Uncertainties in material properties, geometry and boundary conditions are considered. Study on a rectangular box cavity with a flexible plate is reported.

Keywords: ASAC, Kalman Filter, Uncertainties I-INCE Classification of Subjects Number(s): 38.2, 46.4

1. INTRODUCTION

Active noise control techniques involve utilization of secondary sources of sound or vibration to obtained noise levels less than that due to primary sources alone. This technique may be a preferred technique over passive control in the low frequency range. Interior noise in aerospace and automotive cavities are some of the problems where this has been applied and has potential for further development and application. Active structural acoustic control (ASAC) is a type of active control where the objective is to actively control the noise by controlling or modifying the vibration of the structure of the cavity (1). Due to this, the coupling of the structure and the acoustic domain has a significant effect on the noise produced when some disturbance acts on the structure (2). While some of the structural modes may be strongly coupled with the acoustic modes, leading to significant contribution to the noise produced, some other modes may have much less coupling and hence may not be contributing much to the noise. It has been shown that the structural modes are not the independent contributors to the acoustic field while the same may be formulated in terms of 'radiation modes' of the structure (3, 4).

The effectiveness of active noise control depends on the characteristics of the primary noise source. If the primary noise contains dominant harmonic frequencies, then active noise control can be quite effective with the feedforward control approach. However, if the disturbance is broadband or impulsive then there is difficulty in obtaining a suitable reference signal and satisfying the requirements of causality. For such cases, feedback control could be an effective strategy (5). Output feedback and optimal control are the two mainly used feedback control strategies. LQG is an optimal control strategy that utilizes an observer for state estimation using fewer numbers of measurements. Reference (6) considers feedback control of interior noise using structural sensing. An LQG based feedback control with Kalman-Bucy filter as the state estimator is used. Reference (7) used Kalman filter approach to develop a virtual sensing algorithm that computes optimal estimates of the error signals at the virtual locations and implemented that algorithm on an acoustic duct arrangement.

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It is observed that the LQG based strategies are based on an observer that estimates the states of the plant for feedback and therefore is a key element in the whole control loop. Since it utilizes a copy of the model of the plant for estimation, the accuracy of the model is crucial for its performance. This paper reports a study on the influence of the model uncertainties on the LQG based ASAC system. The influence of the uncertainties on the accuracy of the estimated states as well as on the closed loop system performance is studied. Uncertainties in material properties, geometry and boundary conditions are considered. Study on a rectangular box cavity with a flexible plate is reported.

2. ASAC SYSTEM BASED ON FEEDBACK CONTROL

A schematic of the ASAC system proposed is shown in figure 1. The ASAC system is based on structural sensors and actuators in the form of piezoelectric patches.

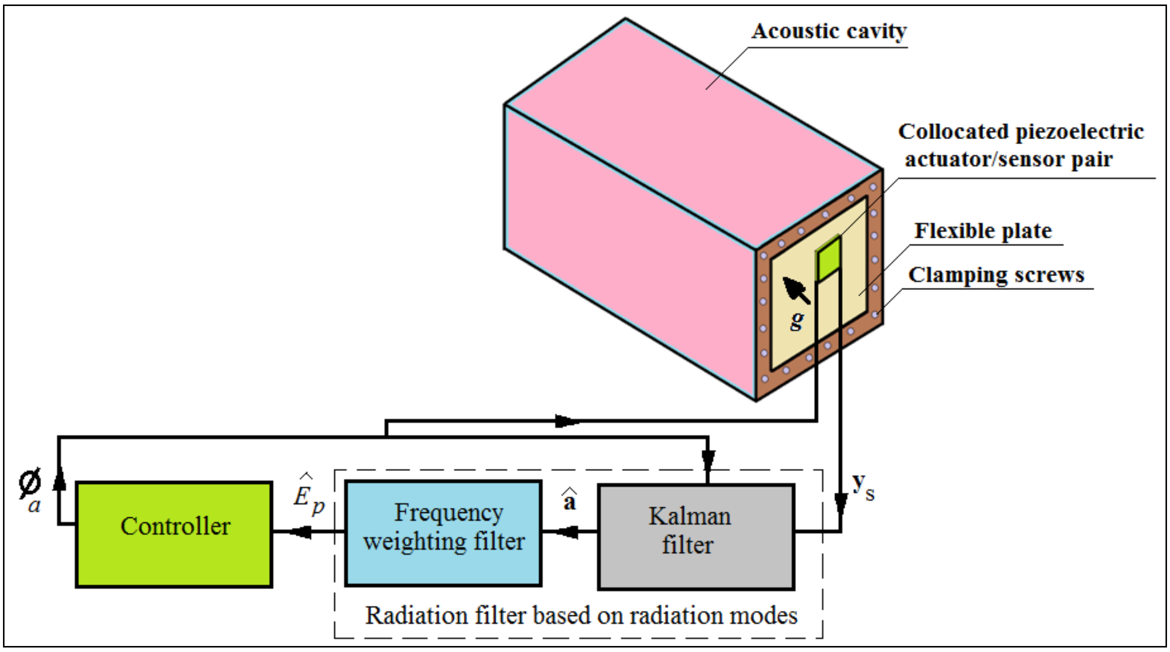

Figure 1 – Schematic representation of feedback control of interior sound

A radiation filter is developed for estimation of acoustic potential energy based on sensing of structural vibrations. The radiation filter estimates the modal amplitudes of the radiation modes along with the efficiency of each mode. Since, the radiation modes contribute independently to the acoustic potential energy it becomes possible to estimate acoustic potential energy from the knowledge of characteristics of the radiation modes. The filter in the current study is based on using only the first radiation mode. The radiation filter is composed of a Kalman filter and a frequency-weighting filter. A Kalman filter is designed in the modal space of the structural domain of the cavity. The Kalman filter estimates the modal displacement and modal velocities of the required structural modes of the cavity. Since the Kalman filter is designed in the modal domain of the cavity, the number of structural states that need to be tracked is far less than the size of the numerical model that may be used to model the plant. A frequency-weighting filter is developed that models the frequency dependence of the efficiencies of the radiation modes. The radiation filter therefore is designed for virtual sensing of the acoustic potential energy based on a small number of structural measurements. The filter is also expected to be robust against measurement noise.

The controller is a linear quadratic regulator (LQR) and is designed to minimize the weighted sum of the acoustic potential energy inside the cavity and the control effort. From figure 1 it is seen that a piezoelectric sensor senses the vibration response at its location and gives output voltage $\mathbf{y}_s$ that is fed to the Kalman filter. The Kalman filter estimates the modal amplitudes ($\hat{\mathbf{a}}$) of the radiation modes while frequency weighting filter estimates efficiency of the radiation modes. Then the estimate of the acoustic potential energy $\hat{E}_p$ is fed to the controller that produces a control voltage ϕ_a that is then fed to the actuators.

The above system is developed for a numerical case of a 3D rectangular box acoustic cavity of size 0.261m×0.300m×0.686 m as shown in figure 2. The density and speed of sound for the medium of sound is taken as 1.21 Kg/m^3 and 340 m/s respectively. The cavity walls are rigid from five sides and the sixth side is made up of a flexible steel plate of thickness 0.001 m clamped at its four edges. The density of the plate is 7800 Kg/m^3; the Young's modulus is 200 GPa and Poisson's ratio 0.30. A proportional viscous damping is simulated in the structural and the acoustic domain of the cavity. On the plate, a P-876 A12 Dura Act piezoelectric patch is bonded, whose in-plane dimensions are 0.0522 m×0.050 m and is 5×10^{-4} m thick. For P-876 A12 Dura ACT piezoelectric patch, the Young's modulus, density, Poisson's ratio , piezoelectric strain coefficients e_{31} and e_{32} , and dielectric constant ε_{33} are 23.3 GPa, 7800 Kg/m^3, 0.34, -8.9678 C/m^2 and 6.6075×10^{-9} F/m respectively.

The radiation filter is developed using a finite element based numerical model of the piezo-structural-acoustic system assuming weak structural-acoustic coupling. The flexible-plate is discretized using a mesh of 10×12 four-nodded Kirchhoff's thin plate bending elements that have three degrees of freedom (an out-of-plane displacement and the two rotations) at each of their nodes. The acoustic cavity is discretized using a mesh of 10×12×14 eight-nodded solid acoustic elements with acoustic pressure as the degree of freedom at each of its nodes (figure 2). The piezoelectric patches that are supposed to be glued on the flexible plate are modeled with classical lamination theory using piezo-electric constitutive relations and are discretized by 2×2 four-nodded rectangular bending element with 12 mechanical DOFs and 2 electric DOFs (voltage). One of the electrodes for each patch is grounded. Figure 3 shows the finite element mesh of the flexible plate of the cavity structure with piezos.

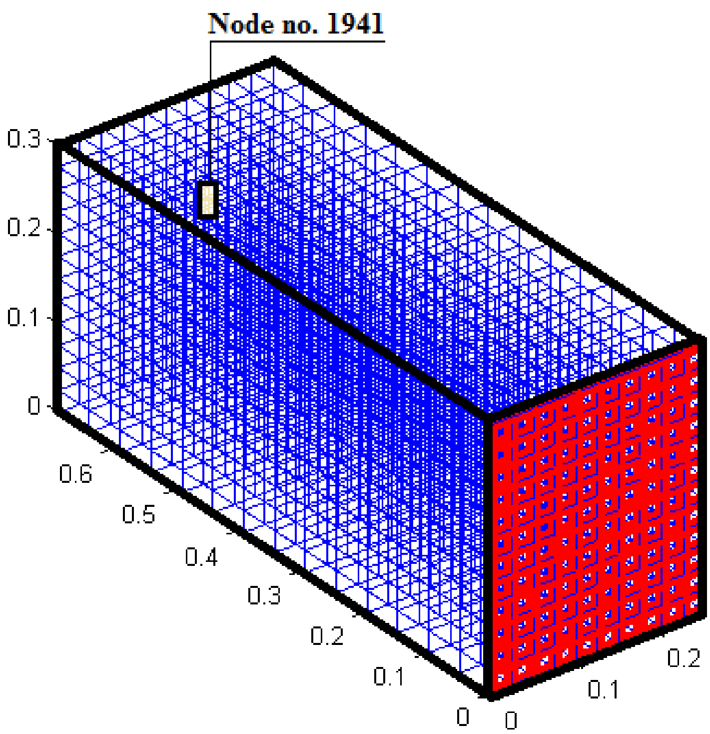

Figure 2 – Finite element mesh of the acoustic domain of the cavity

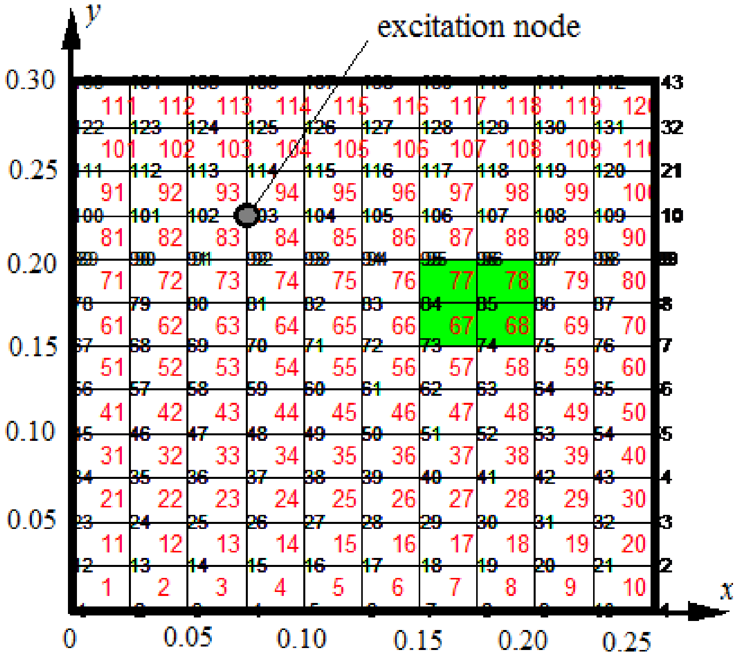

Figure 3 – Finite element mesh of the structure with piezoelectric patches

3. STUDY OF THE EFFECT OF MODEL UNCERTAINTIES ON ASAC SYSTEM PERFORMANCE

The controller used in the system described in the last section is a linear quadratic regulator (LQR). It is designed using the standard approach by making appropriate choices for weighting matrices in the cost function. The Kalman filter is designed based on the estimates of process and measurement noise covariances. In this way, a closed loop system based on LQG control strategy is obtained. This section studies the open and closed performance of the system when the model of the plant used in the Kalman filter design has uncertainties. The Kalman filter used in the strategy described in the previous section needs an FE model of the cavity structure. This model may have uncertainties associated with material and geometric properties as well as boundary conditions.

3.1 Effect of geometric Uncertainties

The model of the plant is built using certain value for the geometric parameters like thickness in case of a plate or a shell like structure. However, this may not be consistent with the actual structure that may have statistical variations of the thickness over its geometry. This leads to uncertainty in the model used for Kalman filter design. To simulate this situation while the model used in the Kalman filter is build with a unique value of thickness, the model used to simulate the experimental cavity structure is build with a thickness of different finite elements distributed normally with mean equal to the nominal thickness value of 1 mm and standard deviation of 10%. Table 1 shows a comparison of the natural frequencies of the actual plant and the model used in the Kalman filter. It is seen that the maximum error between the frequencies is 1.9%. Figure 4a-e shows the performance of the Kalman filter in tracking the measured piezoelectric sensor voltage and modal velocities of some structural modes. In these and all subsequent figures, the actual output of the Kalman filter is compared with the true value of the output. Figure 5 shows the estimation of modal amplitude of first radiation mode. It is seen from these figures that the tracking performance of the Kalman filter is severely hampered due to presence of the geometric uncertainties. Figure 6a shows a comparison of acoustic pressure at a node inside the cavity with and without control. With control the increase of acoustic pressure with time indicates that the closed loop system is unstable due to presence of geometric uncertainties considered above. Figure 6b also shows a pole-zero map of the closed loop system with uncertainties. Some closed loop poles are seen in the right half complex plane indicating an unstable system.

Table 1 – Comparison of natural frequencies (Hz)

S.No.	Plant Model	Kalman filter Model	% error
1	112.6	111.1	1.3
2	217.0	213.2	1.7
3	252.5	251.2	0.5
4	339.5	334.6	1.4
5	375.4	369.4	1.6
6	465.5	461.6	0.8
7	486.6	477.3	1.9
8	549.7	545.0	0.8

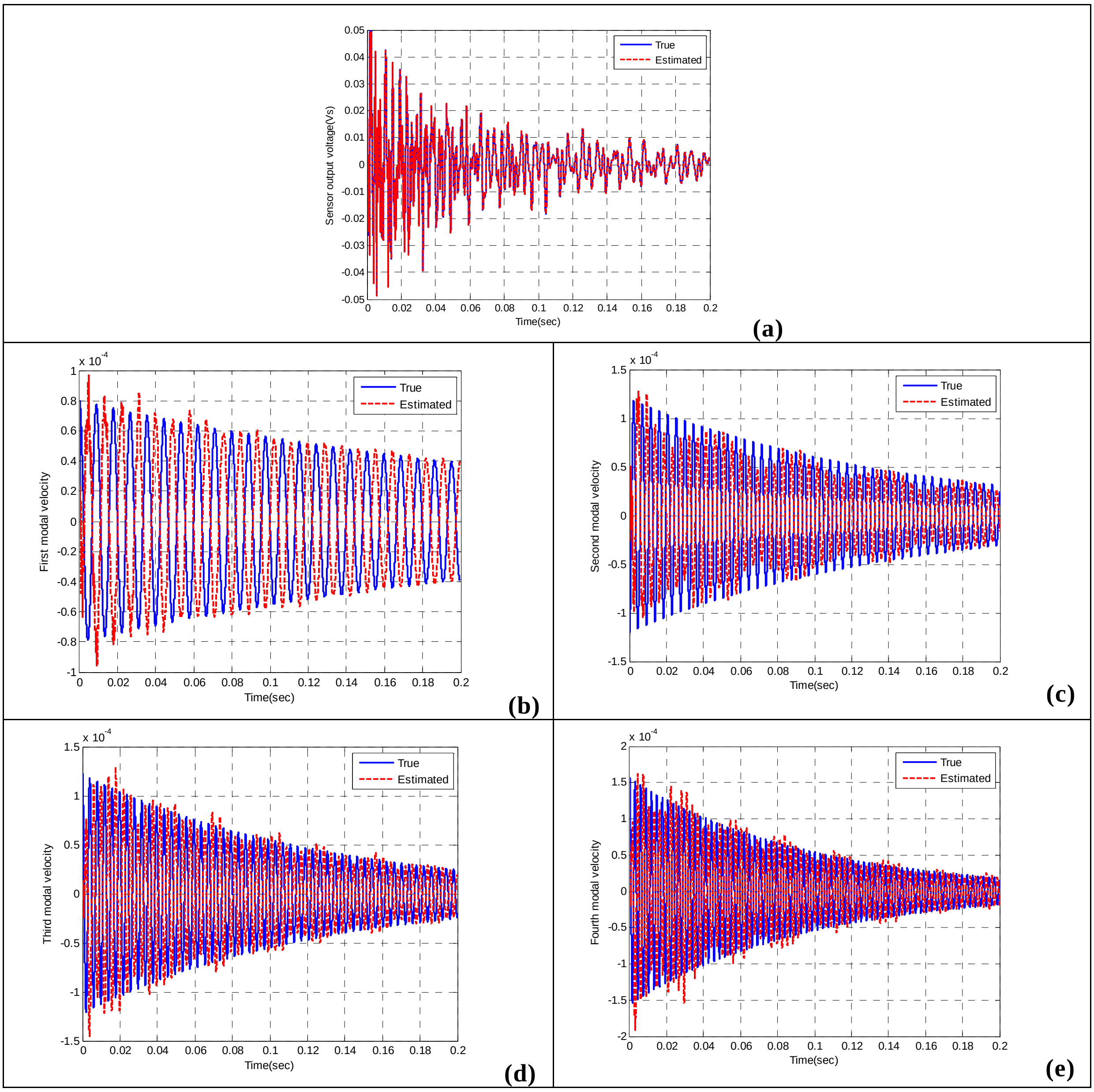

Figure 4 – Comparison of true and estimated values of a) piezoelectric sensor voltage and b)-e) modal velocity of structural modes

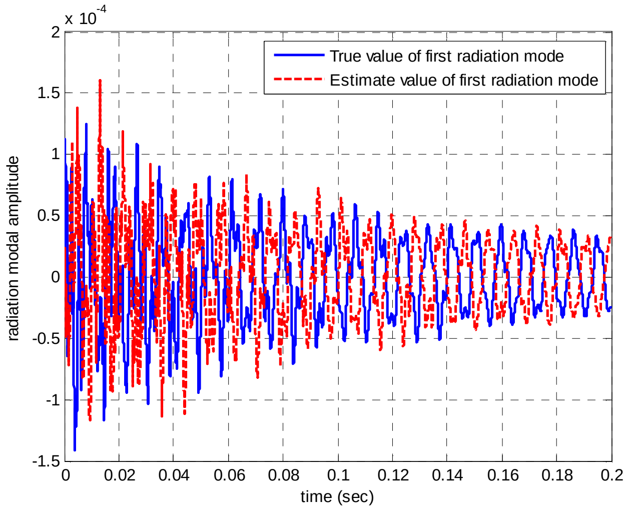

Figure 5 – Comparison of true and estimated value of modal amplitude of first radiation mode

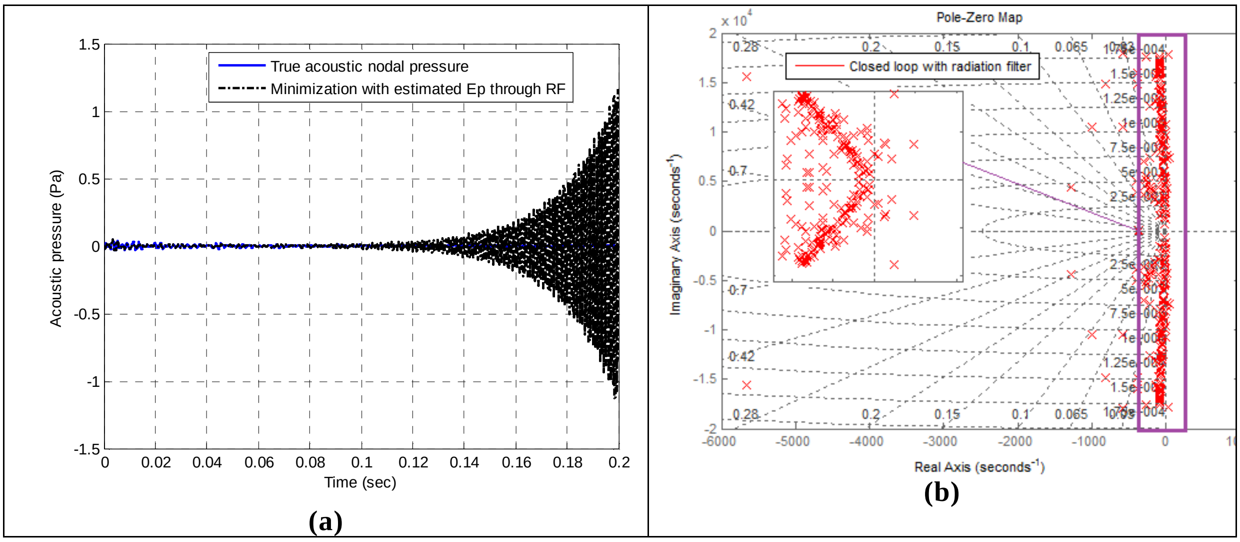

Figure 6 – a) Comparison of open loop and closed loop acoustic pressure b) pole-zero map

3.2 Uncertainty in Boundary-conditions

Boundary condition is another dominant source of uncertainty in structural models. Generally, some idealized boundary conditions are used in models, which quite often do not reflect the boundary conditions of the actual structure. For example, edges of the surfaces of the cavity might be modeled with fixed or simply supported boundary conditions while the physical structure may not be completely fixed or simply supported. This source of uncertainty is simulated in this section by considering a constant stiffness at the four edges of the flexible plate for each of the three degrees of freedom in the model used in the Kalman filter. On the contrary, the model used to simulate the experimental cavity structure is build with stiffnesses at the four edges with a normal statistical distribution. Figure 7a-d shows translational and rotational stiffness in the plant and Kalman filter model.

Table 2 shows a comparison of the natural frequencies of the plant and the Kalman filter model. It is seen that the maximum error between the frequencies is 0.13%. Figure 8 show tracking of measured sensor voltage and modal amplitude of the first radiation mode under open loop condition. It is seen that these quantities are tracked reasonably well. Figure 9a shows acoustic pressure response with and without control indicating that the closed loop system is stable under the presence of boundary condition uncertainty considered in the study. All the closed loop poles are also seen located in the left half plane as seen from the pole-zero map (figure 9b) indicating stability.

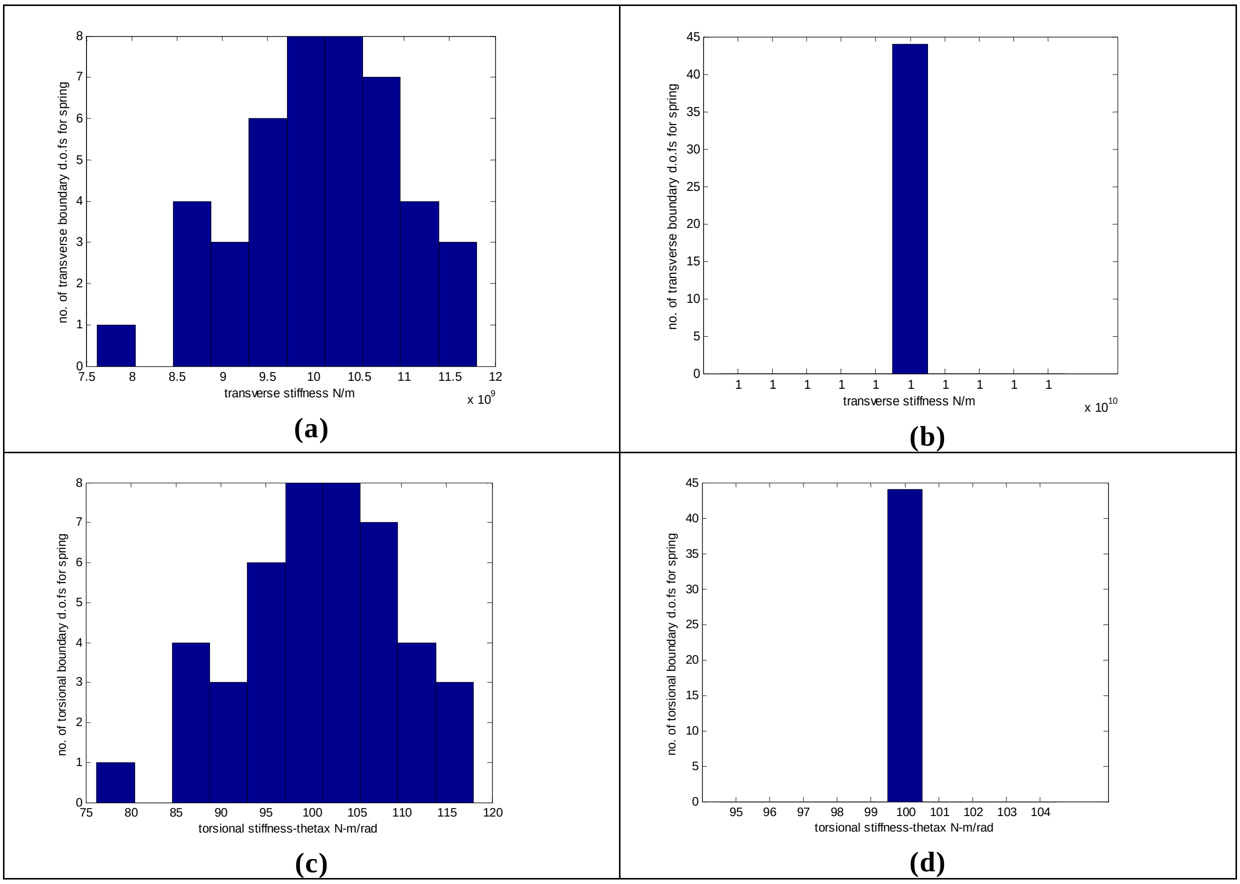

Figure 7 – Stiffness distribution at the boundary of the plate a) Transverse stiffness in plant model b) Transverse stiffness in Kalman filter model c) torsional stiffness in Plant model d) torsional stiffness in Kalman filter model

Table 2 – Comparison of natural frequencies (Hz)

S.No.	Plant Model	Kalman filter Model	% error
1	104.9	104.8	0.02
2	202.0	201.8	0.11
3	237.1	237.2	0.04
4	318.0	317.9	0.03
5	351.3	350.8	0.13
6	435.9	436.1	0.04
7	455.8	455.5	0.06
8	518.9	518.9	0.01

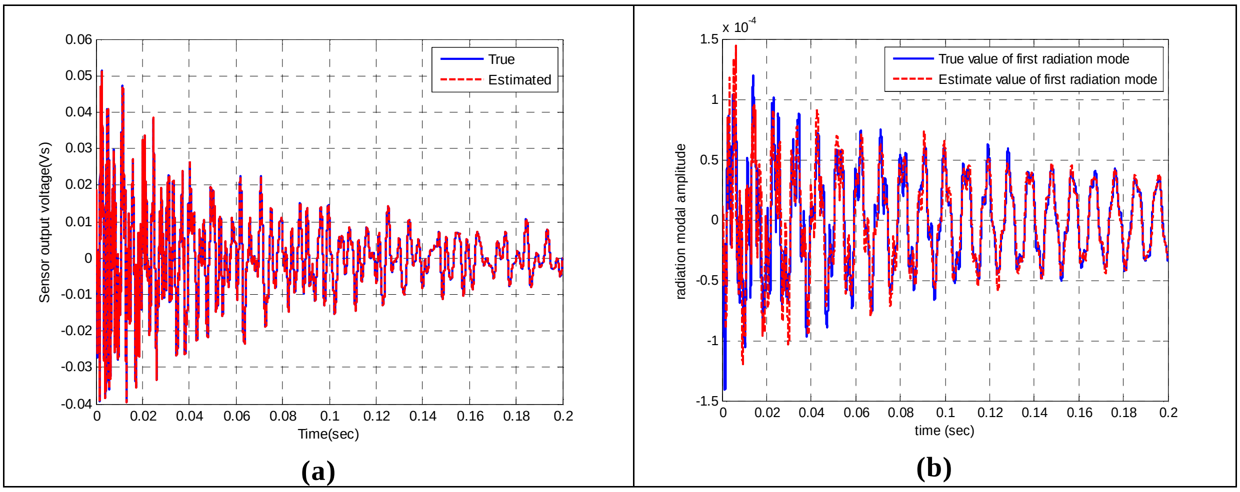

Figure 8 – Comparison of true and estimated a) piezoelectric sensor voltage b) modal amplitude of first radiation mode

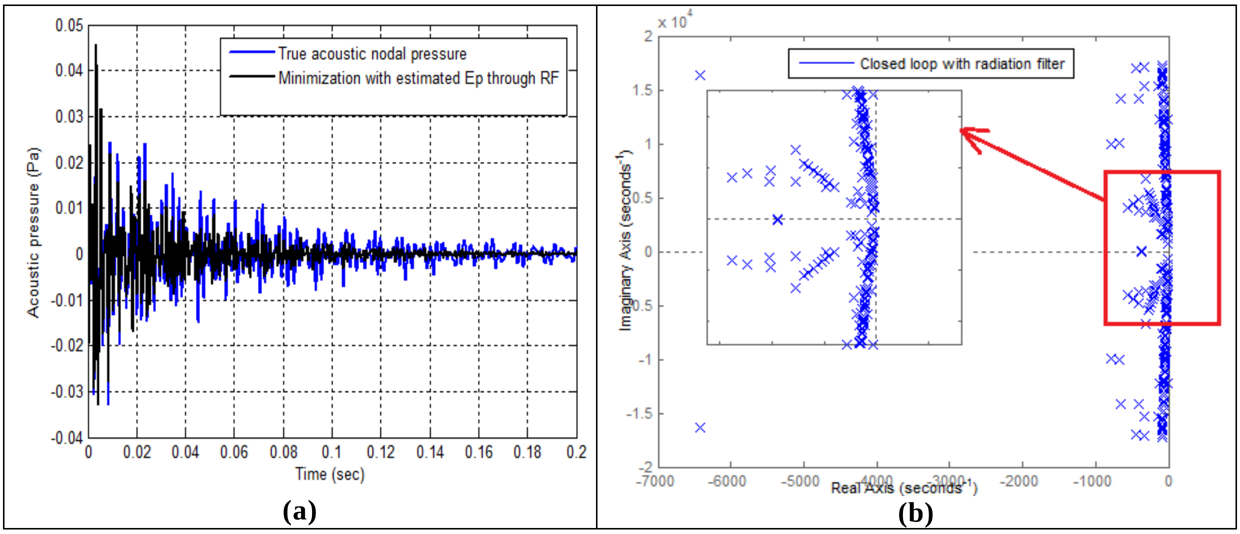

Figure 9 – a) Comparison of open loop and closed loop acoustic pressure b) pole-zero map

3.3 Uncertainty in material property

In this section, the uncertainty in material property is considered. Uncertainty in modulus of elasticity of the material is introduced by considering 1% and then 5% difference in the values used in the Kalman filter model and the simulated experimental model. The material property uncertainty can be avoided if an accurate knowledge of the material properties is available. In contrast, the uncertainties considered in the previous two sub-sections are more inherent in practice. Figure 10 shows tracking of modal amplitude of the first radiation mode under open loop condition for these two cases. It is seen that with 1% uncertainty estimation is reasonably well while with 5% the discrepancy is higher. Figure 11a-b) show acoustic pressure inside the cavity with and without control for these two cases. It is seen that in both the cases finally the response becomes unstable, though for the 1% case, initially the acoustic response is reduced with control.

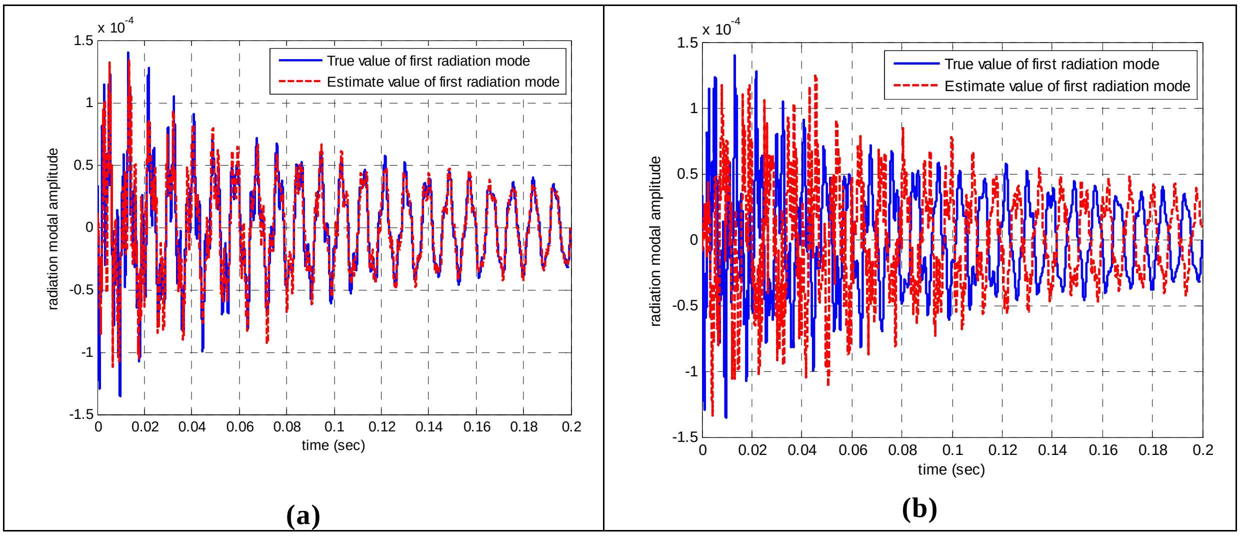

Figure 10 – Comparison of true and estimated modal amplitude of first radiation mode (a) 1% uncertainty (b) 5% uncertainty

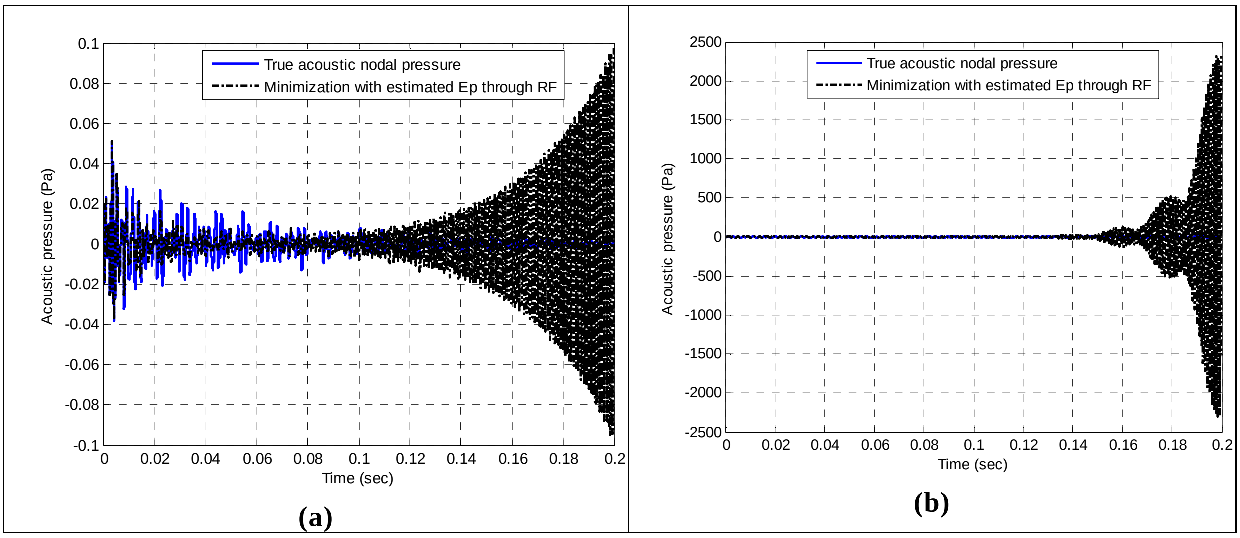

Figure 11 – Comparison of open loop and closed loop acoustic pressure (a) 1% (b) 5%

4. DEALING WITH MODEL UNCERTAINTIES

The design of control systems that are robust against model uncertainties has been addressed a lot in the literature (8). Many techniques have been proposed to deal with model uncertainties. In the Fictitious noise approach (9), the model uncertainties are represented by fictitious noise sources by estimating equivalent covariances of the process and measurement noise. In equivalent Kalman Filter Approach markov parameters of the auto-regressive model is calculated by solving a least square problem (9). H-infinity control is another method, which is robust with respect to a predefined level of structural uncertainty (10).

All the above approaches essentially aim at making the control system more robust and insensitive to the model uncertainties. For structural systems, it is possible to some extent address the model uncertainties directly by trying to update the structural model using experimental dynamic test data. If updating can be carried out accurately then this may eliminate the model uncertainties to a great deal and hence help avoid instabilities that may occur due to model uncertainties as observed in the results presented in the previous section.

Some of the approaches mentioned in this section would be taken to deal with the model uncertainties and study their effectiveness for ASAC feedback control system design in future work.

5. CONCLUSIONS

This paper presented a study of effect of model uncertainties on the performance of Kalman filter used in an ASAC system based on feedback control. Geometric, boundary conditions related and material property uncertainties are studied. It is observed that while smaller levels of uncertainties degrade the performance of the Kalman filter to track states of the cavity structure and radiation mode modal amplitude, higher levels of model uncertainties may make system even unstable. Future efforts would aim at extending some of the available approaches for dealing with model uncertainty to ASAC system based on feedback control described in this paper.

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