Moving boundary similarity method and its application on ship structural borne noise prediction

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ABSTRACT

Based on the principle of structural dynamics, the unitary of exciting force and moving boundary condition is proved by vibration analyzing of an equipment-base system. By constructing “virtual force” to keep the consistency of moving boundaries of structure with dynamic parameters locally unknown, moving boundary similarity method (MBSM) is proposed to get structural dynamic response of given moving boundary conditions with structural dynamic parameters locally unknown. The effectiveness of MBSM is also verified. On that basis, MBSM is applied to the study of ship’s underwater vibration and noise radiation. The underwater noise radiation characteristic of a ship is analyzed. Result shows that the underwater vibration and sound radiation of a ship is highly related with location and frequency of excitations. On one hand, ship structural borne noise radiation is mainly concentrated in the vicinity of excitation of the middle and aft region of ship, followed by aft and bow. On the other hand, the underwater noise radiation is more uniformly distributed along the ship length in low frequency band, while the inhomogeneity and directivity gradually increases as excitation frequency increases.

Keywords: Structural Dynamic Analysis, Moving Boundary Similarity Method, Underwater Radiated Noise

1. INTRODUCTION

Structural dynamic response predictions under given forces have been widely discussed in structural dynamics, while cases with moving boundary conditions (with given velocity /acceleration, for an example) are rarely mentioned, not even for the cases with dynamic parameters locally unknown. However, there are a large number of engineering practices for this kind of demand. Problems of structure dynamics prediction of ships by measuring the vibration of pedestals or base of power equipment with parameters unknown often occur in ship structure dynamics research field. Riser’s structure dynamics and strength are expected by surveying the vibration at junctions of platform and risers in marine engineering field.

Based on modal superposition and wave propagation theory, structural dynamics analysis methods such as theoretical method, Finite Difference Method (FDM), Finite Element Method (FEM), Boundary Element Method (BEM) and Statistical Energy Analysis Method (SEA) are developed to predict dynamic response of structures under given excitations, which meet the general requirements of theoretical research and engineering applications \textsuperscript{[1-8]}. In the dynamic analysis of structures under given moving boundary conditions, researchers carry out structural dynamic analysis with analytical method or numerical simulations by transforming given moving boundary conditions into exciting force. For an instance, when predicting vibration caused by excitations from an equipment, by measuring vibration acceleration and impedance at each end of isolators, a transform method from vibration acceleration into exciting force of the equipment is established, which makes the dynamic

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analysis of ship structure under given moving boundary conditions from the equipment possible \[9-10\]. However, it is noteworthy that, when transforming the vibration of the equipment into the exciting forces, both the vibration acceleration and impedance at each end of isolator should be measured \[11\]-\[15\]. If the impedance of the isolator is unknown, the transformation becomes impossible. Therefore, it is still hard to get the dynamic response of a structure under given moving boundary conditions with dynamic parameters locally unknown.

Actually, according to structural dynamics, boundary conditions of a structure can be expressed either in the form of displacement or force, which is accordant in essence. Taking advantage of the unity of displacement boundary conditions and mechanical boundary conditions, the displacement boundary conditions can be transformed into mechanical boundary conditions, which can effectively reduce the cost and errors caused by transformation.

Based on the theory of structural dynamics, the unity of displacement boundary conditions and mechanical boundary conditions is proved. By constructing a virtual force to keep the consistence of boundary conditions; the Moving Boundary Similarity Method (MBSM) is proposed to get the structural dynamic response by given moving boundary conditions with parameters locally unknown. Effectiveness and validations are also presented. On that basis, MBSM is applied to the analysis of underwater vibration and noise radiation of a ship. Characteristic of underwater noise radiation of the ship is also discussed.

2. MBSM IN STRUCTURAL DYNAMIC ANALYSIS

2.1 The Method to Calculate the Modal Loss Factor of Composite Plate Structure

In the field of structural dynamics analysis, a given moving boundary conditions can be expressed in the form of mechanical boundary conditions. And also a given mechanical boundary conditions can be presented as moving boundary conditions as well, which indicates that the moving boundary conditions and mechanical boundary conditions are equivalent. This principle not only suits for simple structures, but also for complex ones. To illustrate the correctness of the principle, a two DOF vibration model of an equipment-base system is established.

With regard to the equipment-base system shown in Fig.1(a), if the vibration acceleration of equipment \( m_e \) is \( a_e = x_e(t) \omega^2 e^{i(\omega t + \phi)} \), the motion of the system can be presented as:

\[
\begin{align*}
    x_1 &= x_0 e^{i\omega t} \\
    m_b \ddot{x}_2 + c_b \dot{x}_2 + k_b x_2 &= c_e \dot{x}_1 + k_e x_1
\end{align*}
\]

Solve Eq. (1) and yields

\[
\begin{align*}
    x_1 &= x_0 e^{i\omega t} \\
    x_2 &= (k_e + c_e \omega) e^{i\omega t} / (k_b + c_b \omega + m_e \omega^2)
\end{align*}
\]

If equipment \( m_e \) is excited by external force \( F = F_0 e^{i\omega t} \), the motion of system in figure 1(b) can then be expressed as:

\[
\begin{align*}
    m_e \ddot{x}_1 + c_e (\dot{x}_1 - \dot{x}_2) + k_e (x_1 - x_2) &= F_0 e^{i\omega t} \\
    m_b \ddot{x}_2 + (c_b + c_e) \dot{x}_2 - c_e \dot{x}_1 + (k_e + k_b) x_2 - k_e x_1 &= 0
\end{align*}
\]

The vibration of the equipment-base system in figure 1(b) is:
\[
\begin{align*}
D &= \begin{bmatrix}
(k_e - m_e \omega^2 + j(c_e \omega)) & -(k_e + j(c_e \omega)) \\
-(k_e + j(c_e \omega)) & (k_e - m_e \omega^2 + j(c_e + c_d) \omega)
\end{bmatrix}, \\
D_1 &= \begin{bmatrix}
F_0 & -(k_e + j(c_e \omega)) \\
0 & (k_e - m_e \omega^2 + j(c_e + c_d) \omega)
\end{bmatrix}, \\
D_2 &= \begin{bmatrix}
(k_e - m_e \omega^2 + j(c_e \omega)) & F_0 \\
-(k_e + j(c_e \omega)) & 0
\end{bmatrix},
\end{align*}
\]

Equals Eq.(2) and Eq.(4) and yields
\[
F_0 = D_1 x_1 / D_0
\]

Eqs. (5) shows that, if exerting force \( F = F_0 e^{j \omega t} = D_2 x_2 / D_1 \) on equipment \( m_e \) of the model shown in Figure 1(b), the vibration of the system in Fig. 1(a) and Fig. 1(b) will be identical, which proves that the moving boundary conditions can be presented in the form of the corresponding mechanical boundary conditions.

Similarly, if the displacement \( x_0 \) is unknown, equals Eq. (2) and Eq. (4), and yields:
\[
x_0 = F_0 D_1 / D
\]

That is, by keeping the displacement of equipment \( m_e \) in Fig. 1(a) equals to \( x_0 = D_1 x_2 / D_0 \), the vibration of the system in Fig. 1(b) will be identical, which indicates that mechanical boundary conditions can be also presented in the form of the corresponding moving boundary conditions.

Hence the moving boundary conditions and the mechanical boundary conditions are accordant.

2.2 Theory of MBSM

If dynamic parameters of a system are given, structural dynamic analysis can be performed by transforming the moving boundary conditions into the corresponding exciting forces with the method proposed in chapter 2.1. However, it’s still hard to determine the exciting force if dynamic parameters of the system are locally unknown.

To solve the problem caused by uncertainty of dynamic parameters of the system, a similar system should be established. Then, by ensuring consistency of the similar system and the original one, the virtual force, which is required for dynamic analysis of the system, is created. Therefore, the actual vibration response of the original system can be obtained from the analysis of the similar system.

\[
F' = m_0 \omega^2 e^{j(\omega t + \phi)}
\]

Equations of motion of the new system can be expressed as:

\[
a_t = x_0 \omega^2 e^{j(\omega t + \phi)}
\]
\[
(m_2 + m_3)x_1 + c_i(x_i - \dot{x}_2) + k_i(x_i - x_2) = -(m_0 + m_1)x_0\omega^2 e^{i\omega t} \\
(m_2 + m_3)x_1 + (c_i + c_2)x_2 - c_i\dot{x}_2 + (k_i + k_2)x_2 - k_i x_1 = 0
\]  

(9)

The response of the new system shown in Figure 2(b) is

\[
\begin{align*}
    x_1 &= E_1 e^{i\omega t} / E \\
    x_2 &= E_2 e^{i\omega t} / E
\end{align*}
\]  

(10)

Where \( E = \begin{bmatrix} k_i - (m_0 + m_1)\omega^2 + j(c_i + c_0)\omega & -(k_i + jc_i) \\
\frac{k_i - (m_0 + m_1)\omega^2 + j(c_i + c_0)\omega}{k_i + k_2 - m_0\omega^2 + j(c_i + c_2)\omega} & k_i + k_2 - m_0\omega^2 + j(c_i + c_2)\omega \end{bmatrix} \)

\( E_i = \begin{bmatrix} 0 \\
\frac{k_i - (m_0 + m_1)\omega^2 + j(c_i + c_0)\omega}{k_i + k_2 - m_0\omega^2 + j(c_i + c_2)\omega} & k_i + k_2 - m_0\omega^2 + j(c_i + c_2)\omega \\
0 & k_i + k_2 - m_0\omega^2 + j(c_i + c_2)\omega \end{bmatrix} \)

Since \( m_0 \gg m_r \) and \( m_0\omega^2 \gg |k_i + jc_i\omega| \), thus

\[
x_1 = -\frac{[k_i + k_2 - m_0\omega^2 + j(c_i + c_2)\omega]m_0 + m_1\omega^2}{[k_i + k_2 - m_0\omega^2 + j(c_i + c_2)\omega][k_i - (m_0 + m_1)\omega^2 + j(c_i + c_2)\omega] - (k_i + jc_i)\omega^2} x_0 e^{i\omega t} \approx x_0 e^{i\omega t} \quad (11)
\]

\[
x_2 = -\frac{(k_i + jc_i)\omega(m_0 + m_1)\omega^2}{[k_i + k_2 - m_0\omega^2 + j(c_i + c_2)\omega][k_i - (m_0 + m_1)\omega^2 + j(c_i + c_2)\omega] - (k_i + jc_i)\omega^2} x_0 e^{i\omega t} \approx D_x e^{i\omega t} / D \quad (12)
\]

Comparing Eq.(2) and Eq.(11)~(12), it can be seen that the dynamic response of the affinity system and the original one are the same, which indicates that MBS is feasible in structural dynamics analysis.

### 2.3 Basic Steps of MBSM

From the discussion mentioned above, it can be seen that whether parameters of a system are known or not, if the moving boundary conditions of the system is given, the dynamic response of the system can be derived by transforming the moving boundary conditions into the corresponding mechanical boundary conditions. When parameters of a system are unknown, the moving boundary conditions can be transformed into the corresponding mechanical boundary conditions by the following steps

1. Construct a rigid body with great mass \( m_0 \) on the moving boundary of the system, and ensure that \( m_0 \gg m_r \) and \( m_0\omega^2 \gg |k_i + jc_i\omega| \).
2. Calculate moving boundary similarity force \( F' = (m_0 + m_r)\omega \approx m_0\omega \) according to the mass \( m_0 \) and vibration acceleration \( \omega \) of rigid body.
3. Apply moving boundary similarity force \( F' \approx m_0\omega \) on the rigid body \( m_0 \).

What is worth noting here is that, MBSM is only valid on the premises that \( m_0 \gg m_r \) and \( m_0\omega^2 \gg |k_i + jc_i\omega| \). And the greater the mass of the rigid body is, the higher the calculation accuracy will be.

### 3. VALIDATIONS OF MBSM

The correctness of MBSM has been proved theoretically. The validity of this method will be verified in practical applications.

![Figure 3 – Structure of ribbed plate](image)
Example 1 is a ribbed plate (shown in Fig.3). The size of plate is $1000\,\text{mm} \times 2000\,\text{mm} \times 8\,\text{mm}$. The size of stiffeners is $50\,\text{mm} \times 10\,\text{mm}$, and the spacing is $250\,\text{mm}, 200\,\text{mm}$ along the longitudinal and transverse direction, respectively. Supposing the Young’s modulus of the stiffened plate $E = 2 \times 10^{11}\,\text{Pa}$, the Poisson’s ratio $\mu = 0.3$ and the structural damping $\eta = 0.01$, the stiffened plate is simply supported at four corner points E, F, G and H. Vibration of the stiffened plate excited by given vibration acceleration (which is induced by a unit force exerting at point A) at the center point A, shown in Figure 4, is desired.

According to the requirements of MBSM, this paper examines the dynamic responses of ribbed plate when $m_0 = 1\,\text{kg}, 100\,\text{kg}, 10\,\text{ton}$ and $1000\,\text{ton}$. For better comparison, vibration response of points A, B, C and D are shown in Fig.5 and Fig.6.

Fig.5 and Fig.6 has shown that if the mass of $m_0$ is small compared with $m_e$, the vibration calculated by MBSM will be quite different from that of the original model. However, when the mass of $m_0$ is greater than $10^4\,\text{kg}$, the vibration response calculated by MBSM tends to be consistent with the original model, and the greater $m_0$ is, the smaller different will be, which indicates that MBSM is valid on the premises: $m_x > m_e$ and $m_x\omega^2 \gg |k + j\omega|$

Example 2 is a cabin of a ship as shown in Fig.7. The cabin is a half circular structure, with the radius
$R=1500\text{mm}$, 2 bulkheads at both ends. The thickness of bulkheads and circular hull is $t=10\text{mm}$. The size of ribs, which is uniformly arranged in the inner side of the cabin with the spacing $L$ of $600\text{mm}$, is $t=10\text{mm}$, $h=200\text{mm}$. Given the cabin is excited by the vibration at points A and B (shown in Figure 8), the center of symmetry of the cabin, and supposing that the Young's modulus of ship structure $E = 2\times10^{11}\text{Pa}$, the Poisson's ratio $\mu=0.3$ and structural damping $\eta=0.01$, the underwater sound radiation of the cabin under such vibration excitation is required when the cabin is immersed in sea water.

![Figure 7](image)

**Figure 7** – Underwater vibration and noise radiation model of a ship cabin

To facilitate analysis, according to the requirements of literature [16], infinite elements are laid out on the outside surface of flow field the radius $R=4.5\text{m}$ to improve the calculation accuracy. The mass of rigid body is supposed to be $m_0=100\text{kg}$, $1\text{ton}$, $10\text{ton}$ and $100\text{ton}$, respectively. Responses of underwater vibration and sound radiation of the cabin with different mass $m_0$ are calculated and compared. Results are shown in Fig.9 - Fig.11. Vibration gauges are in Figure7. Sound radiation gauge 5# is located at the center of symmetry of the cabin, while the far-field sound radiation gauge 6# is right below 5# with the radius of $100\text{m}$.

![Figure 8](image)

**Figure 8** – Vibration of cabin with variant mass $m_0$
Fig. 8–Fig. 10 has shown that when the mass of rigid body $m_0$ is less than $10^4$ kg, great error of underwater vibration and sound radiation of the cabin by MBM will occur. And the smaller $m_0$ is, the greater the error is. However, when the mass of rigid body is greater than $10^4$ kg, the underwater vibration and sound radiation of the cabin obtained by MBSM agrees well with that from the original model, and the greater $m_0$ is, the better the results fit with each other. Further analysis also indicates that $m_0 \geq 10^8$ kg is enough to meet the accuracy requirements for most vibration and noise analysis of a ship. Hence, if the vibration acceleration of base a panel below equipment is known, underwater vibration and sound radiation analysis of the ship can be employed indirectly, by transforming the vibration acceleration (obtained from measuring) into excitation loads with MBSM.

From the discussion above we know that MBSM is feasible in engineering field.

4. APPLICATION OF MBSM IN SHIP’S UNDERWATER VIBRATION AND SOUND RADIATION

4.1 Introduction of Ship’s Underwater Vibration and Sound Radiation Model

The pedestal of gearbox locates at the symmetry center, $1/4L$ away from the stern of a ship. The vibration acceleration of the gearbox pedestal is $1m/s^2$, and the frequency band is $20Hz$–$400Hz$. The ship is $62m$ in length and $7.5m$ in breadth. And the displacement of the ship is $675$ ton when floating at the draft of $2.5m$. The loss factor of the ship is $\eta=0.025$. The finite element model of the ship is shown in Figure 12. Underwater vibration and sound radiation of the ship excited by the vibration acceleration from gearbox is required to be calculated.

Considering the excitation is the vibration acceleration of gearbox pedestal, MBSM is thus employed to analyze the underwater vibration and sound radiation of the ship. To ensure the accuracy of analysis, the mass of the rigid body is taken as $m_0=10^8$ kg, the flow field is established according to
literature [16, 17] and divided into inner and outer fluid domain at radius \( R_1 = 10 \text{m} \) and \( R_2 = 25 \text{m} \). Besides, non-reflecting boundary condition is also applied to the outside surface of the outer fluid field.

![Model of ship](image1)

Model of ship

![Vibration and noise prediction model](image2)

Vibration and noise prediction model

**4.2 Underwater Sound Radiation Characteristic of The Ship Structure**

**4.2.1 Underwater Sound Distribution Along The Length of The Ship**

The underwater sound distribution of the ship is shown in Fig.12.

![Underwater sound distribution](image3)

Figure 12 shows that, on one hand, underwater sound radiation of a ship is mainly concentrated in the middle and aft region (at a distance of 3/4L from the bow), where the excitation source, the gearbox pedestal, is located. No matter how the excitation frequency changes, the underwater sound radiation of gearbox pedestal compartment is higher than that of other regions apparently. And the underwater sound radiation is relatively high in the aft and low in the bow. On the other hand, the underwater radiation noise of the ship decreases gradually with the increasing of frequency. This can be illustrated as: since the excitation loads comes from the constant vibration acceleration of the gearbox pedestal, the vibration energy decreases as frequency increases, which lead to a great noise radiation in low frequency band while a low one in high frequency band.

**4.2.2 Underwater Noise Distribution Along Horizontal and Draft Direction**

Fig.13 shows underwater noise distribution along the horizontal and draft direction of the ship.

![Underwater noise distribution](image4)

Figure 13 shows that, when \( f \) is smaller than 100Hz, the underwater noise radiation of ship hull is characterized by the total vibration of ship hull, and it is more uniformly distributed along the ship length, even though the sound radiation near excitation source and the aft is slightly high. In addition, the underwater sound radiation field presents a typical butterfly pattern distribution along the axis of symmetry,
and due to the asymmetry of ship, the distribution of sound field in horizontal direction is slightly asymmetric as well. When the exciting frequency \( f \) is greater than 200Hz, inhomogeneity distribution of ship’s underwater sound radiation comes forth gradually. The directivity of underwater sound radiation field becomes apparently. And moreover, the underwater sound radiation gradually follows sphere attenuation law, which indicates the characteristics of a point source radiation. When \( f \) is greater than 300Hz, the distribution of ship’s underwater sound radiation becomes more inhomogeneous. The propagation of underwater sound radiation turns to be more similar to the propagation ways of rays, where sound radiation are mainly distributed around the compartment where gearbox pedestal is installed.

From Fig.13 (b) we know that, the distribution of underwater sound radiation from ship hull is symmetric to the cross-section in general. However, due to the locally asymmetry of ship hull, underwater sound radiation field shows some asymmetry as well. When exciting frequency is lower than 100Hz, the underwater sound radiation field is symmetric to the cross-section and distributed at both sides of ship hull in a butterfly-pattern. When \( f \) is greater than 200Hz, the symmetry distribution of sound field decreases, and acoustic radiation is mainly concentrated on the area right below the gearbox pedestal. With a further increase of exciting frequency to \( f \) is greater than 300Hz, the underwater acoustic field becomes directional more obviously.

The foregoing analysis shows that the distribution of underwater sound radiation from ship hull is closely related with excitation locations and frequencies. On one hand, underwater radiation noise of the ship is mainly concentrated in the middle and aft region of the ship, where is close to the excitations, while lower in the bow. On the other hand, in low frequency band, underwater sound radiation from ship hull is more evenly distributed along the length of ship hull. However, with the increase of exciting frequency, inhomogeneity distribution of the underwater sound field increases and the directivity becomes more evident.

5. CONCLUSIONS

Based on theory of structural dynamics and taking a pedestal-equipment vibration system as an example, this paper examines the in accordance of kinematical and mechanical boundary conditions in structural dynamic analysis. By constructing “virtual force” to keep the consistency of moving boundaries of structure with dynamic parameters locally unknown, MBSM is proposed to get structural dynamic response of given moving boundary conditions with structural dynamic parameters locally unknown. On this basis, MBSM is employed to analyze the characteristics of the underwater sound radiation of a ship. The foregoing analysis yields:

1. MBSM proposed in this paper is feasible in structural dynamic analysis. Namely, by ensuring the consistency of the moving boundary conditions to fabricate a virtual force, analysis for structural dynamic response can be implemented.

2. MBSM is valid on the premises that \( m_0 \gg m_v \) and \( m_v \omega^2 \gg |k + j\omega| \). And the greater the mass of rigid body \( m_0 \) is, the higher the accuracy will be. If requirements of MBSM are not satisfied with, significant errors will occur.

3. Ship’s underwater sound radiation is closely related with factors such as excitation locations and frequencies. On one hand, ship’s underwater noise radiation is mainly concentrated in the middle and aft region of ship hull which is close to the excitation source, and becomes lower in the bow. On the other hand, in low frequency band, ship’s underwater sound radiation is more evenly distributed along the ship length, while more inhomogeneous and directive with the increasing of exciting frequency.

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