# The study on sound radiation of semi-submerged cylindrical with antisymmetric velocity distribution 

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#### Abstract

The vibro-acoustic characteristics of cylindrical shell are widely studied because of its strong engineering application background, especially when the shell is immersed in infinite fluid field or partially immersed. In the present paper, the method for analyzing sound radiation of semi-submerged finite cylindrical shell with antisymmetric velocity distribution is first proposed. The orthogonality of trigonometric function and sound field distribution characteristic of dipole are utilized to establish the formulas. Sound radiation diagram of representative velocity distribution is depicted. Numerical calculations show that BEM Sysnoise is an effective tool to deal with the sound radiation of semi-submerged cylindrical shell.


Keywords: sound, cylindrical, semi-submerged
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## 1. INTRODUCTION

The vibro-acoustic characteristics of cylindrical shell are widely studied because of its strong engineering application background. The studied cylindrical model contains stiffened cylindrical shell, damped cylindrical shell and so on. Recently twenty years, the sound and vibration of partially immersed cylindrical shell are gotten more and more concerned. Seybert[1] investigated the radiation and scattering of acoustic waves from bodies of arbitrary shape in a three dimensional half space. It is showed that the integrals on the infinite plane may be removed by using a modified Green's function in the Helmholtz integral. Seybert[2] presented a modified version of the Helmholtz integral equation to study radiation and scattering for bodies sitting on an infinite plane. The provided formulation is verified by Numerical results. Chen[3] described a numerical method to compute the response and acoustic radiation for structures partially immersed in fluid with using a coupled BEM/FEM. Pierre[4] presented analytical theories to predict the far-field pressure radiated by a circular cylindrical shell partially immersed in an acoustic fluid at high frequency band. Li[6] presented parametric study on the far-field sound pressure radiated from an infinite semi-submerged cylindrical shell excited by a radial point load. It is showed that the far-field sound pressure is related to the position and frequency of the excited force. Amabili [6-7] presented an analytical method to study the flexural vibration of cylindrical shell partially coupled with external fluid. Ye[8] studied the vibro-acoustic characteristics of semi-submerged cylindrical shell.

In the present paper, the method for analyzing sound radiation of semi-submerged finite cylindrical shell with antisymmetric velocity distribution is first proposed. It is showed that if the velocity of underwater part shell satisfies the boundary condition of four sides, $\theta=0$ or $\theta=\pi, x=0$ or $x=L$, simple supported, or the velocity of shell is antisymmetric velocity distribution about horizontal plane which is through the cylindrical shell axis, the radiation sound field can be computed directly. The typical sound radiation diagram of representative velocity distribution is depicted. Numerical calculations show that BEM is an effective tool to deal with the sound radiation of semi-submerged cylindrical shell.

## 2. THEORY

### 2.1 The Model Introduction

The geometry parameters and the coordinate system of cylindrical shell semi-submerged in field and submerged in infinite fluid domain are showed in Fig.1(a) and Fig.1(b), respectively. Two pairs of antisymmetric force applied to cylindrical shell submerged in infinite fluid domain are depicted in Fig.1(c). The axial, circumferential and radial coordinates are denoted by $x, \theta$ and $r$, respectively. $u, v$ and $w$ are the displacements in the direction of the $x, \theta$ and $r$.


Fig 1 Geometric parameters and coordinate system of cylindrical shell
(a: semi-submerged in field; $b$ : submerged in infinite fluid domain; $c$ : Two pairs of antisymmetric force)
The method for analyzing the sound radiation of semi-submerged cylindrical shell is base on the method of analyzing the sound radiation in infinite fluid domain. So the solving method of cylindrical shell in infinite fluid domain is given firstly.

### 2.2 Governing Equations of Cylindrical Shell

The normal displacement of cylindrical shell with normal force excitation related to displacement mechanical impedance in infinite fluid domain can be expressed as

$$
\begin{equation*}
Z_{m n} W_{m n}^{\alpha}+P_{m n}^{\alpha}=F_{n n}^{\alpha} \tag{1}
\end{equation*}
$$

where $m$ and $n$ are the axial half wave number and circumferential wave number respectively, $W_{m n}^{\alpha}$, $P_{m n}^{\alpha}$ and $F_{m n}^{\alpha}$ are amplitude of normal displacement, sound pressure and normal force, $Z_{m n}$ is displacement mechanical impedance, written by

$$
Z_{m n}=\frac{|L|}{\frac{R^{2}\left(1-v_{1}^{2}\right)}{E_{1} h_{1}}\left|\begin{array}{ll}
L_{11} & L_{12}  \tag{2}\\
L_{21} & L_{22}
\end{array}\right|}
$$

### 2.3 The Pressure in Fluid Field

For the ideal fluid, the time pressure $p_{0}$ satisfies the Helmholtz equation. After take the Fourier transform and utilize orthogonality of trigonometric function, the pressure $P_{m n}^{\alpha}$ is given by

$$
\begin{equation*}
P_{m n}^{\alpha}=\frac{4 \pi \rho_{0} \omega^{2} m^{2} W_{m n}^{\alpha}}{L^{3}} \int_{0}^{\infty} \frac{\left[1+(-1)^{m+1} \cos (k L)\right]}{\left(k_{m}^{2}-k^{2}\right)^{2}} \frac{H_{n}^{2}\left(\sqrt{k_{0}^{2}-k^{2}} R\right)}{\sqrt{k_{0}^{2}-k^{2}} H_{n}^{\prime 2}\left(\sqrt{k_{0}^{2}-k^{2}} R\right)} \mathrm{d} k \tag{3}
\end{equation*}
$$

where $k_{m}=\frac{m \pi}{L}$.

### 2.4 The External Force

For multiple point forces excitation in radial, the modal expansion coefficient is

$$
\begin{equation*}
F_{n n}^{\alpha}=\sum_{i} \frac{\varepsilon_{n} F_{i}}{\pi L R} \sin \left(n \theta_{i}+\frac{\alpha \pi}{2}\right) \sin \left(k_{m} x_{i}\right) \tag{4}
\end{equation*}
$$

where, $\varepsilon_{n}=1$ for $n=0$, and $\varepsilon_{n}=2$ for $n \geq 1 . x_{i}$ and $\theta_{i}$ are axial and circumferential coordinate of external force.

As all known, the middle surface pressure of dipole is zero, the same circumstance as the free surface of water. According to velocity and phase characteristic of dipole, the method for calculating sound radiation of semi-submerged finite cylindrical with special velocity distribution is established.

For a finite cylindrical shell in field showed in Fig.1(b), two pairs of antisymmetric force are exerted like showed in Fig.1(c), and place of two forces in each pair are symmetrical to the horizontal plane which is through the cylindrical shell axis. The two forces amplitude is equal, but phase is opposite.

It is set that coordinates of one pair of antisymmetric force are $\left(\theta_{1}=\theta_{0}, x_{1}=x_{0}, F_{1}=F_{0}\right)$ and ( $\left.\theta_{2}=-\theta_{0}, x_{2}=x_{0}, F_{2}=-F_{0}\right)$, substitute the two forces into Eq.(4), one can get

$$
\left\{\begin{array}{l}
F_{m n}^{0}=\frac{2 \varepsilon_{n} F_{0}}{\pi L R} \sin \left(n \theta_{0}\right) \sin \left(k_{m} x_{0}\right)  \tag{5}\\
F_{m n}^{1}=0
\end{array}\right.
$$

The Eq.(5) means that a pair of antisymmetric point force can be decomposed into antisymmetric line force.

Because the geometric parameters and boundary conditions are also symmetrical to horizontal plane, so only the antisymmetric sound field $P_{m n}^{0}$ is arose by the antisymmetric force, and the time $p_{0}=0$ for $\theta=0$ or $\theta=\pi$. It means the pressure of horizontal plane is zero. So the cylindrical shell part bellow horizontal plane can be deemed to be semi-submerged in field.

### 2.5 Coupled Equations and Displacement Distribution

Substitute Eq.(2), Eq.(3) and Eq.(5) into Eq.(1), the normal displacement $W_{m n}^{\alpha}$ is obtained. It is showed that $W_{m n}^{0}$ is only existed. The normal displacement in time domain can be written by

$$
\begin{equation*}
w=\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{m n}^{0} \sin (n \theta) \sin \left(k_{m} x\right) \tag{6}
\end{equation*}
$$

As to Eq.(9), when $\theta=0$ or $\theta=\pi, \quad x=0$ or $x=L$, then $w=0, \partial w^{2} / \partial \theta^{2}=0, \partial w^{2} / \partial x^{2}=0$. It means that the underwater part of cylindrical is simple supported about $\theta=0, \theta=\pi, x=0$ and $x=L$.

### 2.6 Sound Radiation

Once the displacement $W_{m n}^{\alpha}$ and pressure $P_{m n}^{\alpha}$ are obtained, radiation sound power is given by

$$
\begin{equation*}
W=\frac{\pi R L}{2} \operatorname{Re}\left(\sum_{\alpha=0}^{1} \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \frac{1}{\varepsilon_{n}} P_{m n}^{\alpha} \dot{W}_{m n}^{\alpha^{*}}\right) \tag{7}
\end{equation*}
$$

The sound power level is related to

$$
\begin{equation*}
L_{p}=10 \log \left(\frac{W}{W_{0}}\right) \tag{8}
\end{equation*}
$$

where $W_{0}=10^{-12} \mathrm{~W}$.

## 3. NUMERICAL CALCULATIONS AND DISCUSSIONS

### 3.1 Geometric and Material Parameters

The geometric parameters $R=0.6 \mathrm{~m}, L=1.8 \mathrm{~m}$; sound speed $c_{0}=1500 \mathrm{~m} / \mathrm{s}$, mass $\rho_{0}=1000 \mathrm{~kg} / \mathrm{m}^{3}$; Young's modus $E_{1}=2.1 \times 10^{11} \mathrm{~N} / \mathrm{m}^{2}, \quad \eta_{1}=0.008, v_{1}=0.3, \quad \rho_{1}=7800 \mathrm{~kg} / \mathrm{m}^{3}$.

### 3.2 The validation of Proposed Method and BEM (Boundary Element Method)

The proposed method for sound radiation of semi-submerged cylindrical shell is validated by BEM; meanwhile, the effectiveness of BEM to analyze sound radiation of shell with free water surface is checked. The procedures of comparing the sound power of proposed method and BEM are:
(1) Applied pairs of antisymmetric force to cylindrical shell submerged in infinite fluid domain, analyze the vibration and sound of semi-submerged cylindrical shell theoretically;
(2) Divide the underwater shell part into regular quadrangle region (mesh), extract the normal velocity of mesh and edit velocity data to a file which can be imported into BEM Sysnoise;
(3) Import the mesh file and velocity file into BEM Sysnoise, set the antisymmetric sound filed boundary condition, and then get the sound power;
(4) Compare the theoretical and simulation result.

For a typical example, BEM mesh and antisymmetric boundary is showed Fig.3; the pressure distribution of sphere point is showed in Fig.4. It is seen that pressure of antisymmetric surface is zero.


Fig 3 BEM mesh and antisymmetric boundary


Fig 4 Pressure distribution of sphere point

The sound field antisymmetric boundary conditions leads to the pressure amplitude symmetrical to the free surface such as showed in Fig.5, while the real or imaginary part antisymmetric to the free surface such as showed in Fig.6.


Fig 5 Pressure amplitude of shell cross section (left: logarithmic coordinate; right: linear coordinate)


Fig 6 Pressure distribution of shell cross section (left: real part; right: imaginary part)

The comparisons of sound power with a pair of antisymmetric force excitation and two pairs of antisymmetric force excitation analyzed by shell theory and BEM respectively are showed in Fig. 7 and Fig.8. Two figures both are validated that the proposed method and BEM both are effective method to deal with the sound radiation of semi-submerged cylindrical shell with velocity distribution in Eq.(6).



Fig . 7 Sound power with a pair of antisymmetric force Fig . 8 Sound power with two pairs of antisymmetric force

### 3.3 Radiation Sound Power with Velocity Distribution

From the Eq.(6) one can get, if the shell velocity distribution $q_{0}(\theta, x)$ satisfies the boundary condition of four sides simple supported, the $q_{0}(\theta, x)$ can be decomposed with

$$
\begin{equation*}
q_{0}(\theta, x)=\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} Q_{m n} \sin (n \theta) \sin \left(k_{m} x\right) \tag{9}
\end{equation*}
$$

Multiply Eq.(9) both sides with $\sin (q \theta) \sin \left(k_{p} x\right)$, and double integral in the integral interval $[0, \pi]$ and $[0, L]$, one can get

$$
\begin{equation*}
Q_{p q}=\frac{4}{\pi L} \int_{0}^{\mathrm{L}} \int_{0}^{\pi} q_{0}(\theta, x) \sin (q \theta) \sin \left(k_{p} x\right) \mathrm{d} \theta \mathrm{~d} x \tag{10}
\end{equation*}
$$

Substitute Eq.(10) into (3), the pressure $P_{p q}$ is obtained, and then take the $Q_{p q}$ and $P_{p q}$ into Eq.(7), the radiation sound power can be solved. So we can conclusion:

For the semi-submerged cylindrical, if the velocity of underwater part shell satisfies the boundary condition of four sides simple supported, or the velocity of shell is antisymmetric velocity distribution about horizontal plane which is through the cylindrical shell axis, the radiation sound power of shell can be computed by Eq.(9), Eq.(10),Eq.(3) and Eq.(7) directly.

In fact, at the velocity distribution of Eq.(9), the sound power is equal with the semi-submerged and in fluid field.

Fig. 9 show the sound power with different $n$ and $m$ with $Q_{m n}=0.001$.


Fig. 9 sound power with different $m$ and $n$

## 4. CONCLUSIONS

The sound radiation of semi-submerged cylindrical is investigated theoretically. The orthogonality of trigonometric function and sound field distribution characteristic of dipole are utilized to establish the formulas. Through comprehensive research, some conclusions are obtained:
(1) The method for analyzing sound radiation of semi-submerged finite cylindrical shell with antisymmetric velocity distribution is first proposed.
(2) BEM is an effective tool to deal with the sound radiation of semi-submerged cylindrical shell.
(3) If the velocity of underwater shell part satisfies the boundary condition of four sides, $\theta=0$ or $\theta=\pi, \quad x=0$ or $x=L$, simple supported, the radiation sound field can be computed directly.
(4) The sound radiation diagram given in the paper can be reference as a criterion to check the veracity of other method in antisymmetric velocity distribution.

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