An approach to optimal sensor placement for vibration tests on large structures

Chunhui YUAN[1]; Junjie ZHANG[2]

[1] National Key Laboratory on Ship Vibration & Noise, China

[2] China Ship Development and Design Center, China

ABSTRACT

The critical issue of vibration test on hull structures with large size is the numerous amounts of sensors. Too many sensors lead to excessive consumption whereas notable error would come from reducing the number of sensors. An approach is proposed to reconstruct the complete structural vibration via much fewer sensors. This iterative progress eliminates the sensor location that contributes the most significantly to the condition number of modal matrix in each cycle. Along with the iteration, the condition number goes down quickly to a certain level, but rises suddenly after lots of calculating cycles. The optimal number of sensors is the one before the condition number zooming. The corresponding sensor locations are also optimal. An experiment on cylindrical shell demonstrates that vibration responses reconstructed from the data of 26 optimal sensors are consistent with the responses measured by 200 initial sensors. The vibration error is only 2.4 dB. This approach may be applied to vibration test and measurement on large structures.

Keywords: Vibration, Test, Sensor placement I-INCE Classification of Subjects Number(s): 54.1

1. INTRODUCTION

Vibration measurement on large-sized structures is difficult as far as the uphill work is concerned. Hundreds of sensors have to be placed on the surface of the structure by hands, together with long signal cables generally. This means not only hard work, but also added mass to the structure that may influence its dynamic characteristic. However, insufficient sensors on the surface are risky for measurement with respect to data discrepancies. Thus optimal sensor placement for vibration test on large structure is necessary. One can reconstruct the complete vibration response field of the structure using limited data measuring by these chosen sensors. This method can substitute the way that measuring every inch responses over the surface to get sufficient spatial resolution of information about the structure.

Lots of methods of optimal sensor placement have been proposed. One common measure to judge the suitability of sensor positions is the Fisher information matrix (FIM) using mode shapes of the structure. Kammer proposed this Effective Independence (EFI) method to quantify the contributions of response measurements so that the modal states of targeted modes can be optimally observed (1). Similar studies of FIM are proposed to choose sensor locations by maximizing the determinant of the FIM, by maximizing the smallest eigenvalue of the FIM, by minimizing the trace of inverse of the FIM, by maximizing the norm of the FIM, or by minimizing the condition number of the FIM. Another criterion to judge combinations of sensor positions within single setup configurations is the modal kinetic energy. Heo et al. derived the Kinetic Energy Optimization Technique (EOT) with the formulation similar to EFI (2), and the difference lies in the quantity that is optimized. The EFI method maximizes the Fisher′s information matrix while the EOT optimizes the kinetic energy matrix. As the kinetic energy is only a mass weighted version of the Fisher information matrix, the connection to the effective independence method is obvious. However, EOT eliminates the problem of EFI that the sensor locations with low energy content may be selected. A similar way to solve the problem is the EFI-DPR (Driving Point Residue) method, which multiplying the candidate sensor contribution of the

[1] sunny_wuhan@163.com

[2] zhangjunjie0103@163.com

EFI method with the corresponding DPR coefficient (3). Another criteria is derived from the modal assurance criterion (MAC) originally introduced by (4), whereas the off diagonal terms of the MAC matrix need to be minimized.

Condition number is also been used for condense sensor group, which is well known as a measure of robustness of the system. The criterion can be the condition number of the observability matrix of the linearized tangent model of the discretized model of the process (5), the spectral condition number of the Hankel matrix (6), or the condition number of the frequency response function (7). In this article, the condition number of the modal shape matrix of the structure is employed to optimize sensor locations. Experiment demonstrates the accuracy of this approach and its results are illustrated later.

2. APPROACH OF CONDITION NUMBER

2.1 Principle

Vibration responses of the structure can be expressed in mode shapes:

$$V = \phi A \tag{1}$$

where V is the vector of responses, ϕ is the matrix of mode shapes, A is the vector of modal coordinates.

In fact, not all degrees of freedom (DOFs) can be given in mode shapes or placed sensors. In sensor location optimization, a limited number of sensors are placed on the chosen measurable degrees of freedom. Therefore, the chosen DOFs are given in the formulation, and secondary ones are omitted. This is the purpose of optimization that condensing enormous sensor candidates into much smaller group so that the latter can be applied to engineering practice.

If only the primary DOFs are employed in Eq. (1), it can be rewritten as:

$$V_E = \phi_E A \tag{2}$$

where $V_E = [v_1, v_2, \ldots, v_E]^T$ represents the chosen velocity responses of E locations, and ϕ_E is the corresponding matrix of mode shapes at the chosen locations.

According to Eq. (2), there is:

$$A = \phi_E{}^{+} V_E \tag{3}$$

where '+' denotes the Moore-Penrose generalized inverse matrix.

Substituting Eq. (3) into Eq. (1), one gets

$$V = \phi(\phi_E{}^{+} V_E) \tag{4}$$

Eq. (4) means that one can measure only few responses of the structure and reconstructs responses anywhere over the structure (suppose all mode shapes are available). However, these few sensor locations are not chosen freely. The combination of sensor positions dominates the accuracy of inverse problem as well as the accuracy of response reconstruction. Herein the approach of condition number is employed to choose the combination of sensor locations for optimization.

The condition number of ϕ_E dominates the accuracy of the matrix inverse in Eq. (3). The bigger the condition number of ϕ_E the bigger the error is. So one can chooses the group of sensor locations leading to the smallest condition number of ϕ_E, then the optimization of sensor placement comes to realization. Herein the condition number of ϕ_E is the criteria of optimal sensor placement, it can be expressed in Frobenius norm:

$$cond(\phi_E) = \|\phi_E\|_F \, \|\phi_E{}^{+}\|_F \tag{5}$$

where $\| \; \|_F$ denotes Frobenius norm.

Another question is: how can we determine the value of E or the number of optimal locations? At least E should be bigger than the number of vibration mode participated in the calculation. In engineering practice more sensors should be placed for measurement as various uncertainties are concerned. In the approach of condition number proposed in this article, one can get this final optimal number of location according to the rising speed of condition number during the iteration. This will be explained in detail later in the results of the experiment.

2.2 Iteration Procedure

Assuming that the purpose of optimization is choosing E sensor locations from N location candidates, the better choice is iteration instead of accomplish optimization in one step, considering the remarkable consumption of calculation. The procedure of iteration is that as follows:

(1) Calculate the condition number of mode shape matrix ϕ_N composed by all N locations (initial candidates), which is symbolized by $cond(\phi_N)$;

(2) Attempting to eliminate a row (a sensor location) from ϕ_N to get a submatrix ϕ_{N-1}, then get the condition number of ϕ_{N-1} symbolized by $cond(\phi_{N-1})$.

The number of all different $cond(\phi_{N-1})$ are N because there are N rows in ϕ_N. One can using $cond(\phi_{N-1})_1$ to express that this ϕ_{N-1} is the result from eliminating the first row of ϕ_N, and $cond(\phi_{N-1})_2$ from eliminating the second row of ϕ_N and so on. All these $cond(\phi_{N-1})$ compose a group { $cond(\phi_{N-1})_1, cond(\phi_{N-1})_2, \cdots, cond(\phi_{N-1})_N$ }.

(3) Finding the smallest condition number, e.g. $cond(\phi_{N-1})_i$. This instance means that the row no. i contributes the most significant to the condition number of matrix ϕ_N. As the last step of this cycle in the iteration procedure, eliminate row i to get ϕ_{N-1}.

(4) Substitute ϕ_{N-1} for ϕ_N, then repeat the above steps from (1) to (3) to get matrix ϕ_{N-2} with the smallest condition number. Continue substituting ϕ_{N-2} for ϕ_N, ϕ_{N-3} for ϕ_N and so on, until one gets ϕ_E finally.

In the end, all E chosen locations are available, which means the accomplishment of the optimization of sensor placement. One can continue to reconstruct the field of responses of the structure using Eq. (4).

3. EXPERIMENT AND RESULTS

3.1 Experimental set-up

In order to demonstrate the approach of condition number mentioned above, an experiment is carried out on cylindrical shell model with dimensions of ϕ1200mm×L1800mm (shown in Fig. 1). The thickness of the shell is 8mm. 200 sensors are placed uniformly over the inner surface of the cylindrical shell. Responses of the structure are excited by an electromagnetic vibration generator isolated from the shell by four isolators. The cylindrical shell is sealed and sunk into the water. All experiments are carried out in a deep detention reservoir, where the shell vibration is measured 6 meters under the water surface.

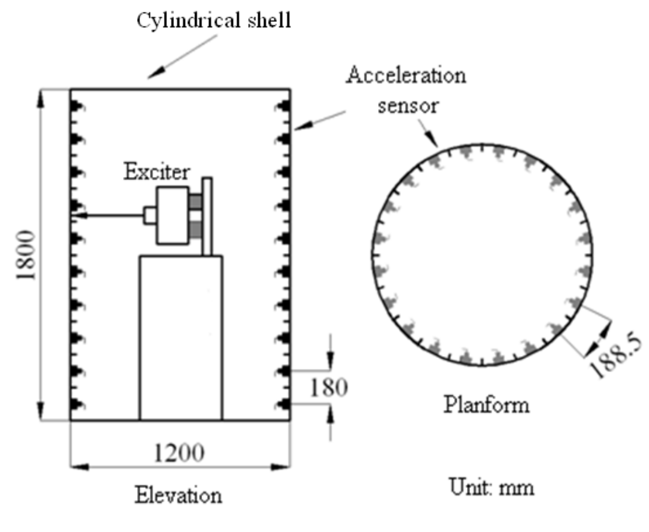


Figure 1 – Sketch of experimental structure and sensor placement

The aim of the experiment is that measuring vibration responses of the cylindrical shell by limited sensors at optimal locations, and then reconstructing the complete responses of the structure.

The reconstructive results are compared with the real data measuring by 200 sensors in uniform placement to verify the consistencies. The highest frequency of concern is 200 Hz.

3.2 Experimental results

Vibration modes in frequency band of concern are simulated numerically by ANSYS. The first 12 modes (shown in Tab. 1) are taken part in the calculation of optimization.

Table 1 – Frequencies of the first 12 modes

Order	Modal frequency, Hz	Order	Modal frequency, Hz
1	29.1	7	155.4
2	68.7	8	167.3
3	71.6	9	172.2
4	89.6	10	178.0
5	92.7	11	194.1
6	127.9	12	208.9

According to the procedure described in section 2.2, a Matlab program is designed to carry out optimization. Along with the iteration, the condition number goes down quickly to about 6, and then keeps steady approximately until rising suddenly after deep iterations. Since the more iteration cycles the fewer the optimal sensor locations left, one should accept results after the more cycles the better. Meanwhile a smaller condition number should be guaranteed. Thus one can choose to stop iteration before the condition number zooming shown in Fig. 2, and the corresponding number and locations of the left DOFs (where to place sensors finally in practice) are optimal. In Fig. 2 we choose 26 final sensor locations after 174 iteration cycles, and the corresponding condition number of submatrix of mode shapes is about 6.2.

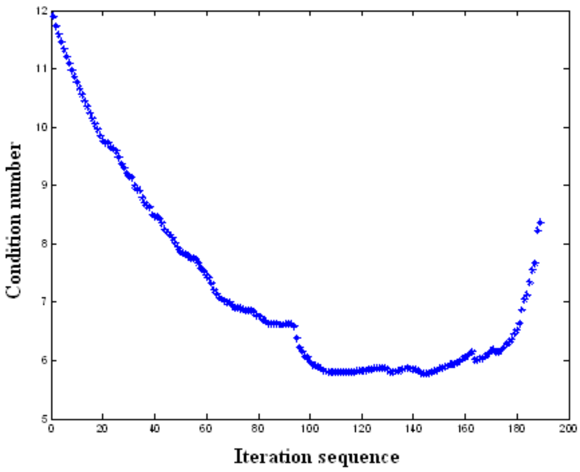


Figure 2 – Condition numbers during iteration

Responses of the structure are reconstructed using Eq. (4), and the results are compared with the measuring data. It is shown that the error is only 2.4dB according to vibration levels of RMS in the frequency band lower than 200 Hz, averaged over the surface of the structure. Typical comparing curves illustrated in Fig. 3 for magnitudes and Fig. 4 for phases. It is noticed in Fig. 4 that phase angles have large errors at several frequencies, but this is not the truth. The errors shown in Fig. 4 are almost because of the separation between +180° and -180°, whereas these two angles are the same and superposed in phase space.

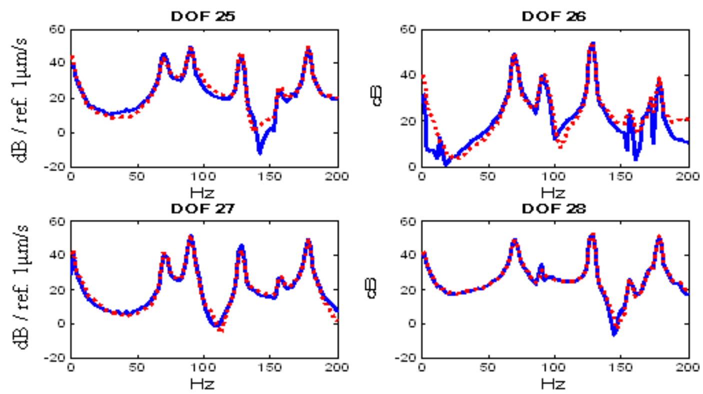


Figure 3 – Magnitude comparison between reconstructed (solid curve) and measured (dashed) vibrations at four positions (DOF 25 - DOF 28)

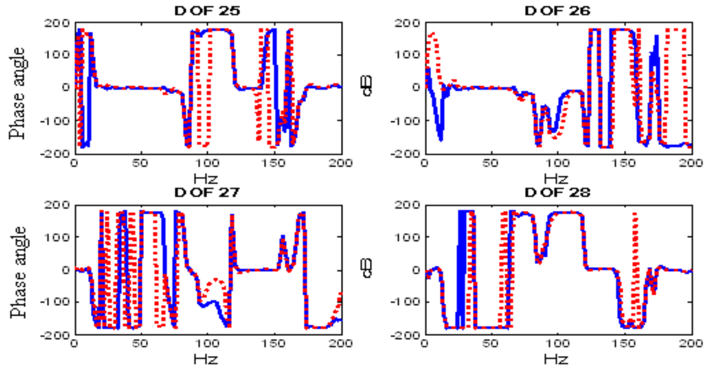
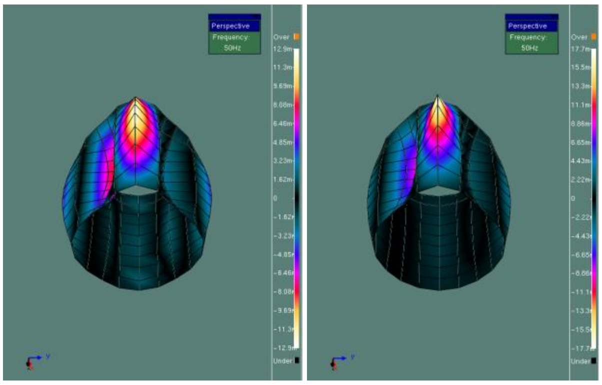
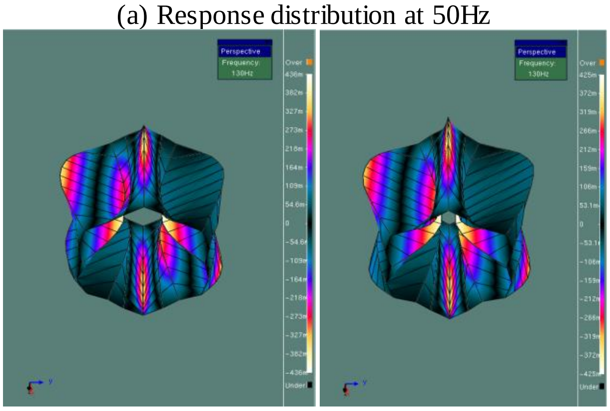


Figure 4 – Phase comparison between reconstructed (solid curve) and measured (dashed) vibrations at four positions (DOF 25 - DOF 28)

The vibration response distributions on the surface of the cylindrical shell are compared between reconstructed and measured results. Typical illustrations are shown in Fig. 5 demonstrating good consistencies of response distribution.



(a) Response distribution at 50Hz



(b) Response distribution at 130Hz

Figure 5 – Typical comparison between reconstructed (left) and measured (right) response distribution

4. CONCLUSIONS

The approach of condition number for optimal sensor placement proposed in this article is effective and validated by the experiment, which reconstructs the complete structural vibration via much fewer sensors. It eliminates sensor locations that contribute significantly to the condition number of modal matrix in iteration. The reconstructive error is only 2.4 dB comparing 26 optimal sensor placement with 200 initial sensor placement. The distributions over the surface of structure are also consistent between reconstructive and measuring data. This approach may be applied to vibration test and measurement on large structures.

REFERENCES

1. Kammer DC. Sensor placement for on-orbit modal identification and correlation of large space structures. Journal of Guidance, Control, and Dynamics. 1991;14 (2):251-259.
2. Heo G, Wang ML, Satpathi D. Optimal transducer placement for health monitoring of long span bridge. Soil Dynamics and Earthquake Engineering. 1997;16 (7-8):495-502.
3. Meo M, Zumpano G. On the optimal sensor placement techniques for a bridge structure. Engineering Structures. 2005;27 (10):1488-1497.
4. Allemang RJ, Brown DL. A correlation coefficient for modal vector analysis. Proceedings of the 1st International Modal Analysis Conference; Orlando, Florida 1982. p. 110-116.
5. Dochain D, Tali-Maamar N, Babary JP. On modelling, monitoring and control of fixed bed bioreactors. Computers Chem. Engng. 1997;21(11):1222-1266.
6. Li YY, Yam LH. Sensitivity analyses of sensor locations for vibration control and damage detection of thin-plate systems. Journal of Sound and Vibration. 2001; 240(4):623-636.
7. Choi HG, Thite AN, Thompson DJ. Methods for selecting sensor locations for improving indirect force determination. Proceedings of the Institute of Acoustics. 2004; 26(2): 265-276.